Optimal Redistributive Policy in a Labor Market with Search and Private Information*

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October 2012

Abstract

We study the design of optimal policies in a frictional model of the labor market with private information about skills. Heterogeneous, risk-averse agents look for a job in a labor market characterized by an aggregate matching technology. Firms post vacancies but cannot observe each agent's productivity. This paper emphasizes the importance of general equilibrium effects in policy design and focus on the non-observability of workers' underlying skills as the main information friction. Our mechanism design approach shows that the constrained optimal allocation can be implemented by policy instruments such as a non-linear tax on wages, a non-constant unemployment insurance and firm subsidies. We calibrate our model to the US economy and characterize the welfare gains from the optimal policy and its effects on output, employment and the wage distribution. Our findings suggest that the optimal policy under a utilitarian government features a negative income tax, a more generous unemployment insurance for low-skilled workers and higher marginal tax rates, which results in a higher participation in the labor market and a lower unemployment rate. This paper also shows that a government with a higher taste for redistribution would favor policies with more *European* characteristics: heavier taxation and more generous unemployment insurance, which result in a lower output and slightly higher unemployment rate.

JEL Classification: E24, H21, J65

*Email addresses: edouard.schaal@nyu.edu and mathieu@wharton.upenn.edu. We thank Nobu Kiyotaki, Esteban Rossi-Hansberg, Thomas Sargent and seminar participants at Princeton, the Society for Economic Dynamics 2010 meeting, the Philadelphia Workshop in Macroeconomics 2012 and the Penn Search and Matching Workshop. Mathieu Taschereau-Dumouchel is grateful for the financial support of the Fonds Québécois de la Recherche sur la Société et la Culture.

1 Introduction

Labor market policies are extremely diverse across countries. A quick look at a survey of these policies Nickell (2006) reveals that, in 2000, countries such as France had a minimum wage as high as 60% of the median income, while Spain's was at a mere 32%. Scandinavian countries, on the other hand, did not have a legal minimum wage, but relied on employer groups and unions to establish a minimum level of earnings for workers. The differences across countries are also important for tax rates. For instance, the 2000 marginal tax rate of a single person with no children earning the average wage was of 21% in New Zealand but of 58% in Germany. The contrast in policies is also significant for union laws, employment protection and unemployment insurance. This leads to two questions. First, what is an optimal labor market policy? What instruments should be used by governments? Second, is it possible to explain the observed discrepancies in terms of government and national preferences?

This paper provides some answers to the above questions. We present a search and matching model of the labor market under asymmetric information where heterogeneous, risk-averse agents and firms meet randomly and negotiate the terms of employment. Several important elements distinguish our paper from existing literature. First, the key information friction analyzed in previous studies of optimal unemployment insurance was the inability for governments to monitor job search effort. We choose to abstract from this dimension to focus on the unobservability of workers' productivity. Such a friction is especially relevant in a setup where insurance creates disincentives to work, and where skilled workers prefer to shirk at low-productivity jobs to enjoy more generous transfers from the government. Second, our design carefully accounts for general equilibrium effects of labor market policies. Changes in income taxation and unemployment insurance can indeed result in adverse movements in wages, market participation and employment that limit the scope of government intervention. Third, we solve a mechanism design in which workers tell their types to firms and firms report it to the insurance agency. We then provide a way to implement the optimal allocation using simple policy instruments. This method allows us to describe what instruments should be part of an optimal policy without imposing much constraints on the initial problem.

After defining the problem of the social planner, we show that the constrained optimal allocation can be implemented in an economy where the wage is observable with simple policy instruments: a non-linear income tax, unemployment insurance based on the previously earned wage and possibly firm subsidies. A minimum wage can also be used to simplify the tax schedule. Simulations of the model show that a negative income tax for low-productivity workers is often optimal.

We calibrate our model on the US economy and solve for the optimal policy under different welfare criteria, including the utilitarian case. Our baseline optimization in the benchmark case suggests that applying the optimal policy could increase welfare by 17.5% in consumption equivalent, reduce unemployment by 1.3%, and increase labor market participation. In particular, the optimal policy features a close to linear income tax that becomes negative at the lower-end of the income distribution and an unemployment insurance with high replacement ratios. The optimal policy thus achieves a delicate balance between encouraging agents to work, while still supplying high unemployment benefits.

Finally, we investigate how changes in the planner's preferences influence the optimal policy. We show that an increase in the government's taste for redistribution leads to policies that display features similar to those that have been used in Europe during the last decades: higher tax rates and a more generous unemployment insurance. The drawbacks are a slightly higher unemployment level as well as a reduced output level. These findings go against the point of view that generous redistributive policies are suboptimal.

1.1 Related literature

This paper is related to previous literature on the optimal design of labor market institution and policy. Our approach is most closely related to Blanchard and Tirole (2008), who examine the joint design of unemployment insurance and employment protection by solving a mechanism design problem in a simple model of the labor market. The authors provide a way to implement the optimal allocation using unemployment benefits, layoff and payroll taxes. Our paper follows a similar mechanism design approach, but focuses on different trade-offs including in particular the management of incentives for job creation and labor market participation under private information.

Mortensen and Pissarides (2002) investigate the effects of taxes and subsidies on labor market variables and characterizes the optimal policy in the labor market. Their paper restricts the set of policy instruments to a linear payroll tax, a job destruction tax and unemployment compensation. Based on a similar model, our paper improves on their approach by solving the optimal mechanism problem and providing an implementation result. Furthermore, their model describes an economy with risk neutral agents where the Hosios condition holds (Cf. Hosios (1990)). The equilibrium is therefore efficient. We extend their model by studying an economy with risk-averse heterogeneous workers where the Hosios condition is no longer satisfied and characterize more general constrained Pareto optimal allocations.

Our paper also draws on the optimal unemployment insurance literature as in Shavell and Weiss (1979), Wang and Williamson (1996) and Hopenhayn and Nicolini (1997). These articles mostly focus on the moral hazard problem that arises from the inability for the insurer to monitor the job search effort and job performance of the worker. These papers deliver important results on the optimal timing of benefits and their negative relationship with unemployment duration. As these issues have already been studied to a certain extent, we put the search effort dimension aside, but keep the job performance dimension as our model emphasizes the importance of both extensive and intensive margins of labor.

Our study uses to a large extent methods and techniques developed in public finance literature for the analysis of optimal direct taxation under imperfect information. The mechanism design approach that we use draws its inspiration from works such as Mirrlees (1971), Atkinson and Stiglitz (1980), or Stiglitz (1988) but rejects the assumption of a frictionless labor market. In that sense, this paper is much closely related to recent studies on optimal taxation of imperfect labor markets. Hungerbühler et al. (2006) uses a similar search model with risk-neutral heterogeneous agents, but focuses on the redistributive aspects of taxation. The Hosios condition is immediately assumed, so that taxation only appears to balance a trade-off between redistribution and efficiency. Their paper develops interesting insights on the optimal tax schedule by modeling explicitly the entry/exit decision of workers and firms in a labor market with frictions and their general equilibrium effects on employment. Our paper considers a similar model away from the Hosios condition and adds an intensive margin for labor. Hence, direct taxation is not only a tool for redistribution, but also appears as a tool for the government to fine-tune the agents' incentives to work.

2 The Economy

We consider the problem of a government designing labor market policies in an economy characterized by search frictions and private information. This government seeks to achieve the right balance between efficiency and welfare. Efficiency related issues arise because of the need for policy to correct labor market frictions. It also concerns the provision of right incentives to elicit participation and work effort from agents. From the point of view of welfare, the government aims at providing some insurance against unemployment risk and redistributing wealth across agents.

We build a search and matching model along the lines of Pissarides (2000), where an aggregate matching function governs match creation. We extend the standard model to allow for *ex-ante* heterogeneous workers and describe their decisions on the extensive (whether to work or not) and intensive (how much to work) margins. The latter has been somewhat neglected in the search-and-matching literature even though evidence suggests its importance in differences across countries vis-a-vis total number of hours worked (Rogerson, 2006). It should be noted that agents with different skills respond to changes in policy in radically different ways. As evidenced by Saez (2000), low-productivity workers tend to react with their extensive margins, while high-productivity ones rather respond with their intensive margin by changing the time they work. These effects are present in our model and greatly matter for policy design.

In this model, agents need to actively look for a job to receive offers at a hazard rate determined by aggregate labor market conditions. Agents can decide whether to search or not, while firms have to post vacancies to attract job candidates. When a firm and an agent meet, the firm makes a *take-it-or-leave-it* offer to the worker, who can accept or reject it. The offer specifies a wage and an amount of output to be produced.

The policy maker's ability to achieve his objectives is limited by an information asymmetry. Each worker's specific productivity and work effort cannot be observed directly neither by the government nor by the firms. The policy design needs to take into account that agents and firms will try to take advantage of that situation. For example, if unemployment benefits are high for low-skill people or if their entry on the market is subsidized, high-skill workers may accept low paid jobs to obtain high benefits and substitute leisure for work. Such situations limit the government's ability to insure people.

We first define the competitive equilibrium of our economy under taxation. A government provides unemployment benefits $b(w)$ based on the wage the agent received when he was previously employed and a transfer b_0 to agents that have never been employed. Firms receive a constant transfer T when matched with a worker. The government balances its budget by levying a non-linear income tax on workers $\tau(w)$, and can also impose a minimum wage $\underline{w}$. The following sections will show that this set of instruments is an optimal one. Other instruments are unnecessary or redundant[1].

2.1 Population and Technology

There is a unique consumption good. The economy is populated by a continuum of mass 1 of ex-ante heterogeneous agents that differ only in their productivity level s. The cumulative distribution of productivity is $G(\cdot)$, where G is a continuous, increasing function on $\mathbb{R}_+^*$. We denote the corresponding probability density

[1]For instance, firing costs and hiring subsidies can be given as lump sum transfer included in other instruments. In a model with endogenous job destruction, firing costs become useful.

function by $g(\cdot)$. Productivity is constant over time and agents know their own productivity parameter. Agents live forever and time is continuous. If hired by a firm, each agent can provide an amount of labor $h \in [0, \infty)$ to his employer. Agents are assumed risk-averse with life-long preferences

$$\mathcal{U}(c,h) = \int_{t=0}^{\infty} e^{-rt} \left[u\left(c(t)\right) - v\left(h(t)\right) \right] \mathrm{d}t$$

where $u'(\cdot) > 0$, $v'(\cdot) > 0$, $u''(\cdot) < 0$ and $v''(\cdot) > 0$ and where c is the consumption flow.

There is a continuum of identical firms endowed with a production technology f that uses labor as sole input. A firm that employs an agent s working an amount of time h produces $f(sh)$ of the consumption good. We assume that f is concave, strictly increasing, and $f(0) = 0$. The firms are owned by risk-neutral entrepreneurs who only care about profit maximization. Entrepreneurs cannot work and only participate in the labor market as firm owners. The number of firms is not fixed and there is free-entry.

2.2 Labor market

There is a unique market where firms post vacancies at a cost $\kappa > 0$ per unit of time. Agents decide whether to search for a job or not. Firms do not observe the types of the agents on the market before they meet. In other words, they cannot direct their search to a specific type of worker. Matches occur at a certain Poisson rate given by aggregate labor market conditions. An exogenously given function $m(\cdot, \cdot)$ determines the rate of matching between agents and firms. In a market with U unemployed workers searching for a job and V vacancies, $m(U, V)$ pairs of worker-firm per unit of time meet and decide whether or not to stay together and produce. The function $m(\cdot, \cdot)$ is homogeneous of degree 1.

For convenience, define $\theta \equiv V/U$, the labor market tightness and $q(\theta) \equiv m(U, V)/V$. Therefore, the firm meets a worker at rate $m(U, V)/V = q(\theta)$ and, similarly, the probability rate at which an unemployed worker finds a firm is $m(U, V)/U = \theta q(\theta)$. Jobs are destroyed at an exogenous rate $\delta > 0$, identical across jobs. These assumptions imply that the probability of finding a job is independent of an agent's productivity and the unemployment rate is the same for everyone.

In this economy, frictional unemployment arises because information about job opportunities disseminates slowly, and match creation takes time. As usual in the matching literature, this model is subject to a *congestion* externality. The decision of agents to participate and search for a job has an adverse effect on the job finding probability for other agents. The more people of a certain type are in the job market, the less likely they can find a partner. In our model, this externality is further amplified by a *composition* effect due to the heterogeneity of workers. Entry decisions by different groups of agents have a differential impact on the economy, which requires the use of specific policies. This effect is present in our model because search is *random* — not *directed* — and because there is a unique pool of applicants. Although this assumption is a little strong, we choose to maintain it in order to study this composition effect. As suggested in Ljungqvist and Sargent (2007), the assignment of workers into different pools may have a huge impact on the equilibrium outcome and needs to be studied with care. With this caveat in mind, we focus on the optimal policy in a single labor market.

2.3 Contracts

Upon meeting in the labor market, an agent s and a firm play a three-stage game in which the firm has imperfect information about the worker's real skill level. Figure 1 shows the game. At node A, the worker tells a type $\tilde{s}$ to the firm and the firm then makes a *take-it-or-leave-it* offer $\{w(\tilde{s}), y(\tilde{s})\}$ specifying a wage and an amount of goods to be produced. The worker accepts or rejects the offer. We consider stationary Perfect Bayesian Nash equilibria in pure strategies of this game. Workers have complete information, while firms ignore the types of agents and form beliefs $p(\cdot)$ about them. Both players know the structure of the game and firms' information set has to be derived using Bayes rule, wherever it applies.

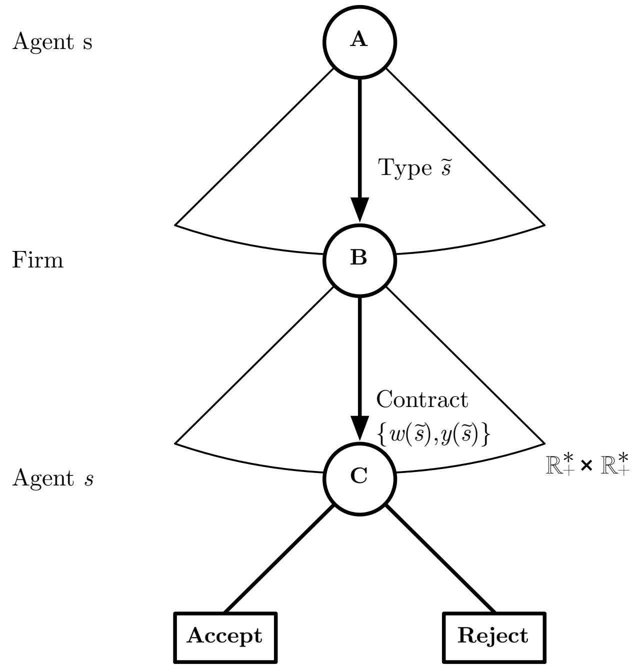

Figure 1: Contract setting game

2.4 Worker's problem

A worker of type s decides whether to enter the labor market or not. If he enters and meets a firm, his individual skill being private information, he can pretend to be of a different type than he really is. Given the firm's strategy, he knows that if he declares a type $\tilde{s}$ the offered contract will be the wage $w(\tilde{s})$ and the production of $y(\tilde{s})$. In our setup, an agent s can act as any other agent $s' \in \mathbb{R}_+^*$. However, pretending to be an agent with productivity $s' \gg s$ may require a large work effort.

We define $W_u(s, \tilde{s})$ as the life-long expected utility of an unemployed agent of type s looking for a job and who told his previous employer that he was of type $\tilde{s}$. At the current time, this agent receives $b(w(\tilde{s}))$ as an unemployment compensation from the government. His job finding rate is $\theta q(\theta)$. His value function is

$$rW_u(s, \tilde{s}) = u\big(b(w(\tilde{s}))\big) + \theta q(\theta) \left[W_e\left(s\right) - W_u(s, \tilde{s})\right] \tag{1}$$

where $W_e\left(s\right)$ is the equilibrium value function of a working agent of type s choosing his optimal strategy. Now, define

$$rW_e(s, \tilde{s}) = u\big(w(\tilde{s}) - \tau(w(\tilde{s}))\big) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \delta\left[W_u(s, \tilde{s}) - W_e(s, \tilde{s})\right]. \tag{2}$$

The last equation says that a working agent of type s declaring a type $\tilde{s}$ receives a wage $w(\tilde{s})$ and pays taxes $\tau(w(\tilde{s}))$ in the current period. He loses his job with rate probability δ.

Agent s declares the type $\tilde{s} \in \mathbb{R}_+^*$ that maximizes its expected utility. Define the equilibrium value functions and optimal strategy of agents as follows:

$$\begin{cases} s^*(s) = \arg\max_{\tilde{s}} W_e(s, \tilde{s}) \\ W_e(s) \equiv \max_{\tilde{s}} W_e(s, \tilde{s}) = W_e(s, s^*(s)) \\ W_u(s) \equiv W_u(s, s^*(s)) \end{cases}$$

where $s^*(s)$ is the type declared by an agent of type s. In a truth-telling equilibrium, $s^*(s) = s$ for all s. More explicitly, this agent s solves:

$$\max_{\tilde{s}} \frac{1}{r+\delta} \left[u\big(w(\tilde{s}) - \tau(w(\tilde{s}))\big) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \frac{\delta}{r+\theta q(\theta)} \big(u(b(w(\tilde{s}))) + \theta q(\theta) W_e(s)\big) \right]. \tag{3}$$

Some workers, let us call them *inactive*, prefer not to participate in the labor market. They remain unemployed forever and their sole income, b_0, is provided by the government. The corresponding life-long utility is

$$rW_u(s) = u(b_0). \tag{4}$$

2.5 Firm's problem

The value function of a firm employing an agent[2] who pretends to be of type s is

$$rJ_e(s) = y(s) - w(s) + T + \delta \left[J_u - J_e(s)\right] \tag{5}$$

where $y(s) = f(sh(s))$ is the production generated by a worker of productivity s and J_u is the value function of a firm with a vacant position. More explicitly,

$$rJ_u = -\kappa + q(\theta) \left[\mathrm{E}\left(J_e(s)\right) - J_u\right] \tag{6}$$

where κ is the rate cost of posting a vacancy and where $\mathrm{E}(\cdot)$ is the expected value operator over the distribution of workers types *in* the labor market.

We impose a free-entry condition that drives the profit of posting a vacancy to zero, $J_u = 0$. Therefore,

$$\frac{\kappa}{q(\theta)} = \mathrm{E}\left(J_e(s, h(s), w(s))\right). \tag{7}$$

Note that the last equation implies that labor market tightness, and therefore the rate probability of being matched, is a function of the expected profits of firms employing workers. Therefore, in an economy with production, firms generate strictly positive profits.

In a Perfect Bayesian Nash equilibrium, taking the agent's strategy as given, the firm updates its belief on the worker's type using Bayes rule, $p(s|\tilde{s}) = p(\tilde{s}|s)p(s)/p(\tilde{s})$, and offers a contract $\{\tilde{w}, \tilde{y}\} \in \mathbb{R}_+^* \times \mathbb{R}_+^*$ that maximizes its profits:

$$\max_{\{\tilde{w},\tilde{y}\}} \mathrm{E}_s \left(\frac{\tilde{y} - \tilde{w} + T}{r+\delta} \times \mathbb{1} \left(\frac{1}{r+\delta} \left[u\left(\tilde{w} - \tau(\tilde{w})\right) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)} \left(u(b(\tilde{w})) + \theta q(\theta) W_e(s)\right) \right] \geq W_u(s) \right) \middle| \tilde{s} \right)$$

[2]Here it does not matter whether the agent is telling the truth or not. The contract depends only on the declared type.

where the expression in the indicator function equals 1 when the worker accepts the offer $W_e(s, \tilde{w}, \tilde{y}) \geq W_u(s)$, a decision that is made by the worker at node C. It is important to note that workers may not always reveal their types in such a game and that we would need in principle to consider equilibria with partial pooling. Fortunately, as we will see later, under the assumption of differentiability of the contracts, equilibria have to be truth-telling. We focus on this case from now on.

If the worker reveals his type, the firm's information set reduces to $p(s|\tilde{s}) = \mathbb{1}_{s=\tilde{s}}$. The firm chooses a contract that maximizes its profits subject to the worker's acceptance of the offer:

$$J_e(s) \equiv \max_{\tilde{w},\tilde{y}} \frac{\tilde{y} - \tilde{w}}{r + \delta} \tag{8}$$

$$\text{s.t } \frac{1}{r+\delta}\left[u\left(\tilde{w} - \tau(\tilde{w})\right) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r + \theta q(\theta)}\big(u(b(\tilde{w})) + \theta q(\theta) W_e(s)\big)\right] \geq W_u(s). \tag{9}$$

This last constraint simply states the worker's willingness to accept the offer. The left part is the present day utility of working under the terms of the offer plus the discounted utility of losing the job. Note that since the firm always wants a higher production $\tilde{y}$, it is optimal for the firm to offer the worker exactly his alternative valuation and therefore $W_e(s) = W_u(s)$ for all s who reveal their types and therefore $u\big(w(s) - \tau(w(s))\big) - v(h(s)) = u\big(b(w(s))\big)$.

This specific way of setting wages and hours worked is similar to a bargaining setup where the firm has all the bargaining power and receives all the surplus from the match. In this setting, wages are usually set too low to be efficient and tend to induce too much vacancy posting.

The asymmetry of information brings interesting general equilibrium phenomena. Consider for instance an increase in the market tightness $\theta = V/U$ and fix an agent s. This agent could decide to deviate by declaring a type $s' < s$ in which case his utility would be

$$W_e(s, s') = \frac{(r + \theta q(\theta))(u\left(w(s') - \tau(w(s'))\right) - v\left(\frac{f^{-1}(y(s'))}{s}\right)) + \delta u(b(w(s')))}{r(r + \theta q(\theta) + \delta)}.$$

Agent s works a bit less by declaring s' but he also receives lower unemployment payments when he gets fired. An increase in θ changes this trade-off and makes deviating a better alternative. To prevent a deviation from happening, the firm gives the agent better working conditions, which usually implies a higher wage.

2.6 Government

The government maximizes some welfare criterion. It has access to policy tools based on observable variables: employment status of workers, the current wage for employed workers and the previous wage for unemployed ones. This assumption seems indeed reasonable since in practice most governments base their labor market policies on these variables.

The global effect of policy instruments is complicated by the general equilibrium setup. We can however have a sense of the forces at play by looking at the contract setting mechanism. The first relevant inequality is the constraint that provides an agent with a utility at least as high as his reservation utility. The second one is the worker's first-order condition, stating the equality of his marginal disutility of labor and his marginal utility of consumption:

$$\frac{1}{r+\delta}\left[u\left(\tilde{w}-\tau(\tilde{w})\right)-v\left(\frac{f^{-1}(\tilde{y})}{s}\right)+\frac{\delta}{r+\theta q(\theta)}\left(u(b(\tilde{w}))+\theta q(\theta)W_e(s)\right)\right]\geq W_u(s) \tag{10}$$

$$v'\left(\frac{f^{-1}(y)}{s}\right)\frac{1}{sf'(f^{-1}(y)/s)}=u'(w-\tau(w))(1-\tau'(w))+\frac{\delta}{r+\theta q(\theta)}u'(b(w))b'(w). \tag{11}$$

Under the typical policy in this paper[3] the left-hand side of (11) is increasing in y and the right-hand side is decreasing in w.

Consider an increase in the *level* of the tax schedule τ. The first effect is to reduce the utility of the worker. To compensate him, the firm has to lower the production and/or has to increase the wage. The second effect comes from equation (11). A higher tax increases the right-hand side of the equation while leaving the left-hand side unchanged. To reach an equality, the wage and/or the production have to go up. When combining the two effects, we see that an increase in $\tau(w)$ is likely to lead to a higher wage while the effect on production is uncertain. A similar reasoning tells us that an increase in the *marginal* tax rate lowers the wage and the production. On the other hand, an increase in the level of unemployment benefits b tends to lower wages with an ambiguous effect on production, while an increase in the marginal benefits raises both wages and production.

2.7 Stationary competitive equilibrium

Throughout this paper, we focus on stationary equilibrium. Let N be the total number of workers on the labor market (employed and unemployed actively searching). The rate of change over time of the number of unemployed active workers is

$$\dot{U}=(N-U)\delta-U\theta q(\theta) \tag{12}$$

where the first term is the number of employed worker loosing their job and the second term the number of searching unemployed agents leaving unemployment. We only consider steady-states of this economy. Therefore $\dot{U}=0$ and the unemployment rate is

$$\frac{U}{N}=\frac{\delta}{\delta+\theta q(\theta)}. \tag{13}$$

We can now define a competitive equilibrium of this economy with government taxation.

Definition 1. Given taxes $\tau(w)$, unemployment benefits $b(w)$ and b_0, and transfers to firm T, a *stationary competitive equilibrium* in this economy is a strategy for workers $s^*(\cdot)$, a set of contracts offered by firms $\{w(\cdot),y(\cdot)\}$, an equilibrium number of unemployed agents U and a market tightness θ such that,

1. Equilibrium value functions $W_e(\cdot),W_u(\cdot),J_e(\cdot)$ and J_u satisfy the system of equations defined by equations (1)-(7),
2. Workers' strategies, firms' strategies and information sets $p(\cdot)$ form a Perfect Bayesian Nash Equilibrium of the wage setting game,
3. Unemployment is stationary: (13) is satisfied,
4. The government's budget constraint is balanced.

[3]In the optimal policies, $\tau(w)$ is close to linear and $b(w)$ is increasing and concave.

3 Designing the optimal mechanism

This section introduces the problem of the social planner. We characterize some of features of the optimal allocation and show what policy instruments can be used to implement it in a competitive economy.

3.1 Goals of the planner

Three broad objectives guide the planner's decisions: i) insurance against unemployment risk, ii) redistribution and iii) the correction of labor market inefficiencies.

There are however limits to insurance and redistribution. The first one is related to incentives. For instance, a generous unemployment insurance program can deter people from entering the labor market. This effect is strongest at the lower end of the skill distribution. Provided that they can secure a comfortable income while unemployed, workers may decide to stop searching for a job and stay unemployed, lowering the efficiency of the policy. Second, insurance and redistribution payments can drive the bargaining power of the workers up by increasing the value of their alternative option, shifting the wage distribution up. This reduces the rent for the firms, which deters them from posting vacancies and finally raises unemployment. Third, these policies need to be funded by increased taxation on workers and firms. Heavier taxes induce high-skill workers to reduce their job effort and to try to extract transfers from the government by pretending to be of a lower type. Heavier taxes on firms also reduce the rate at which they post vacancies.

Furthermore, the planner tries to correct two inefficiencies. The first one is a *congestion externality* that appears because agents do not internalize the negative impact of their search decision on other agents probability of finding a job. This externality is further magnified by a *composition effect* since agents have ex-ante heterogeneous skills and are all searching in the same pool. The differences in productivity make the marginal social cost of a low-skill searching worker higher than those of a high-skilled worker. As a consequence, the planner might want to keep low-productivity workers away from the labor market. This results in an *optimal segmentation* of the labor market.

The second inefficiency comes from the wage setting mechanism. Since the firm has all the bargaining power, wages tend to be below their optimal level and do not internalize the search externality (an effect similar to the one found in Hosios (1990)). This leads firms to post too many vacancies.

3.2 Limits on the planner

Since governments cannot directly observe workers' productivity nor control their individual decisions, we endow the social planner with the same information set as the one available to policy makers. The planner's main constraints are therefore of physical and informational nature. Subject to search frictions, the planner must rely on the signals received in order to allocate resources and provide incentives for agents to comply with decisions. The social planner is subject to the following limitations:

Matching technology The planner cannot override the search frictions coming from the matching technology. Posting vacancies is costly to the planner, match creation takes time, so workers and firms still receive job offers and candidates at the hazard rate determined by aggregate conditions.

Participation constraint The planner cannot force people into accepting jobs, nor firms into posting vacancies. The entry/exit decisions for both types of agents belongs to them. The optimal allocation will therefore have to satisfy participation constraints.

Incentive constraint The planner cannot observe the underlying productivity parameter of the worker, nor the amount of time spent at work. This limits his ability to provide full insurance to the workers and restricts the set of policy instruments that can be used to implement the optimal policy. In particular, the planner is subject to the way firms and workers negotiate their contracts and choose their optimal reporting strategies. The optimal reporting strategy has to be a Perfect Bayesian Nash equilibrium of the game introduced earlier.

Informational constraint In principle, the planner would want to condition transfers on all the history of past reports by agents about their types. Such a mechanism design problem quickly becomes intractable and may not be of practical relevance. We thus consider a memoryless planner that conditions transfers on current reports only for employment agents.

3.3 Optimal mechanism

We now introduce the set of transfers over which the social planner optimizes. We focus our attention on steady-states of the economy. In other words, the planner maximizes the steady-state level of welfare. For this reason, we impose time-invariance on the set of transfers we study.

The social planner solves directly in terms of allocation. Define the optimal transfers to be: a consumption schedule for employed agents $c_e(s)$, consumption of unemployed but active agents $c_u(s)$ and that of the inactive agents c_0, and transfers $t_f(s)$ to firms employing agents of type s. Equilibrium valuations can be redefined in terms of allocations:

$$\begin{cases} rW_e\left(s,\tilde{s}\right) & = u\left(c_e\left(\tilde{s}\right)\right) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \delta\left[W_u\left(s,\tilde{s}\right) - W_e\left(s,\tilde{s}\right)\right] \\ rW_u\left(s,\tilde{s}\right) & = u\left(c_u\left(\tilde{s}\right)\right) + \theta q\left(\theta\right)\left[W_e\left(s\right) - W_u\left(s,\tilde{s}\right)\right] \\ rJ_e\left(s\right) & = f\left(sh\left(s\right)\right) + t_f\left(s\right) + \delta\left[J_u - J_e\left(s\right)\right] \\ rJ_u & = -\kappa + q\left(\theta\right)\left[E\left(J_e\left(s\right) - J_u\right)\right] \end{cases}$$

Once again, we use the $\tilde{}$ notation to emphasize variables over which deviations by the agents are possible.

We now describe the type of reporting mechanism we focus on. We study economies where the wage is the signal observed by the government. In the wage-setting process we consider, contracts (w, y) are Nash equilibria of the game introduced earlier. In the planner's setting, contracts do not specify a wage and a production $\{w(\tilde{s}), y(\tilde{s})\}$ but a pair $\{s^R(\tilde{s}), y(\tilde{s})\}$ specifying a type to report and a production objective. The planner optimizes in the space of allocation $\{c_e(s^R), c_u(s^R), t_f(s^R)\}$, where R stands for a reporting strategy. We thus focus on mechanisms where the signal is jointly chosen by the firm-worker pair as a solution of a game similar to the one of figure 1. The game is as follows: at the first stage, the worker declares a type $\tilde{s}$, the firm then proposes a production $y(\tilde{s})$ and a signal $s^R(\tilde{s})$ to report to the planner. The worker accepts or rejects this offer. We consider Perfect Bayesian Nash equilibria in pure strategies of this game.

3.4 The planner's problem

The social planner optimally designs transfers $\{c_e(s), c_u(s), c_0, t_f(s)\}$ to adjust the amount of time an agent provides $h(s)$, labor market tightness θ and unemployment rate U/N. To keep track of the agents that participate in the labor market, we define the following indicator function χ:

$$\chi(s) = \begin{cases} 1 \text{ if agents } s \text{ is employed or unemployed but looking for a job} \\ 0 \text{ if the agent is inactive} \end{cases}$$

The social planner maximizes a sum of a concave transformation of the valuations of workers. This summation is weighted by the respective sizes of employment and unemployment pools. Firms are owned by risk-neutral agents whose utility is valued with coefficient λ by the planner. The social welfare criterion is

$$\begin{aligned} \max_{\substack{c_e(.), c_u(.), c_0, t_f(.), \\ \chi(.), y(\cdot), \theta, U}} \quad & \left(1 - \frac{U}{N}\right) \int \chi(s) \Phi\Big(W_e\big(c_e(s), c_u(s), y(s)\big)\Big) g(s) ds && \text{(employed)} \quad \text{(SP)} \\ & + \frac{U}{N} \int \chi(s) \Phi\Big(W_u\big(c_e(s), c_u(s), y(s)\big)\Big) g(s) ds && \text{(unemployed)} \\ & + \int (1 - \chi(s)) \Phi\left(\frac{u(c_0)}{r}\right) g(s) ds && \text{(inactive)} \\ & + \left(1 - \frac{U}{N}\right) \lambda \int \chi(s) J_e(t_f(s)) g(s) ds && \text{(firms)} \end{aligned}$$

where $N = \int \chi(s) g(s) ds$ is the total size of the labor market, and $\Phi(\cdot)$ is an increasing concave function that describes the planner's taste for redistribution. Notice that this social welfare function encompasses the utilitarian case if $\Phi(\cdot)$ is linear. The first two integrals in SP are the valuations of employed and unemployed workers. The third integral is the utility of agents that stay out of the labor market. The fourth term is the utility of the firms' owners.

The planner's design is subject to the flow condition (13), the free entry condition ($J_u = 0$) and to the resource constraint

$$\begin{aligned} & \left(1 - \frac{U}{N}\right) \int \chi(s) c_e(s) g(s) ds + \frac{U}{N} \int \chi(s) c_u(s) g(s) ds + \int (1 - \chi(s)) c_0 g(s) ds \qquad \text{(RC)} \\ & + \left(1 - \frac{U}{N}\right) \int \chi(s) t_f(s) g(s) ds \leq 0. \end{aligned}$$

The following participation constraints also apply. All agents that are participating in the labor market must be willing to search for a job, and stay on the market. Namely,

$$\chi(s) = 1 \Leftrightarrow \begin{cases} W_e(s) \geq W_u(s) \\ W_e(s) \geq \frac{u(c_0)}{r} \\ y(s) + t_f(s) \geq 0 \end{cases} \qquad \text{(PC active)}$$

The top inequality in set of questions (**??**) implies that, in equilibrium, unemployed agents want to look for a job. The bottom one implies that a firm is willing to hire them. Similarly, inactive agents must be prevented from entering the labor market. Therefore, for all s such that $\chi(s) = 0$ and for all $\tilde{s}$ such that $\chi(\tilde{s}) = 1$,

$$\frac{u(c_0)}{r} \geq \frac{(r + \theta q(\theta))(u(c_e(\tilde{s})) - v(f^{-1}(y(\tilde{s}))/s)) + \delta u(c_u(\tilde{s}))}{r(r + \delta + \theta q(\theta))}. \qquad \text{(PC inactive)}$$

This inequality simply states that no existing job offer can make inactive workers better off.

As stated before, the strategy-belief profile for firms and workers must form a Perfect Bayesian Nash equilibrium of the wage-setting game. Hence, neither workers nor firms can have an incentive to deviate.

More precisely, the transfer scheme proposed by the planner $\{c_e(\cdot), c_u(\cdot), c_0, t_f(\cdot)\}$, the optimal strategy for the worker $s^*(s)$, and contracts $\{s^R(\tilde{s}), y(\tilde{s})\}$ have to solve the worker's problem

$$s^*(s) = \underset{\tilde{s} \text{ s.t } \chi(\tilde{s})=1}{\operatorname{argmax}} \frac{1}{r+\delta}\left[u\left(c_e\big(s^R(\tilde{s})\big)\right) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \frac{\delta}{r+\theta q(\theta)}\left(u\Big(c_u\big(s^R(\tilde{s})\big)\Big) + \theta q(\theta) W_e(s)\right)\right]. \tag{14}$$

Remember that after reporting a certain type $\tilde{s}$, the worker gets a transfer $c_e(s^R(\tilde{s}))$ during employment and a transfer $c_u(s^R(\tilde{s}))$ while unemployed until he meets with a new firm and starts negotiating again. Therefore, all deviations need to take into account that transfers while unemployed also depend on the currently reported type.

Similarly, the firm's strategy $\{s^R(\tilde{s}), y(\tilde{s})\}$ has to be optimal for the firm, given the worker's strategy and beliefs. Beliefs $p(s|\tilde{s})$ have to be derived using Bayes' rule wherever it applies. In particular, if the worker reveals his type ($s^*(s) = s$ and $p(s|s) = 1$), the firm's problem becomes

$$\begin{aligned} J_e(s) &\equiv \max_{s^R, \tilde{y}} \frac{\tilde{y} + t_f(s^R)}{r+\delta} \qquad (15) \\ \text{s.t } &\frac{1}{r+\delta}\left[u\left(c_e(s^R)\right) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)}\left(u(c_u(s^R)) + \theta q(\theta) W_e(s)\right)\right] \geq W_u(s). \end{aligned}$$

3.5 Solution approach

Solving the social planner's problem as stated above is in general quite complicated, and we must impose additional restrictions in order to deal with the incentive constraints. We substitute workers' and firms' first-order conditions for their incentive constraints. This approach enables us to treat these constraints in a very tractable way. This method is however known since Rogerson (1985) to produce sometimes invalid results. To ensure the accuracy of our results, we check our numerical simulations so that these first-order conditions are indeed sufficient. Our solution strategy nevertheless requires that the optimal transfers $\{c_e(s), c_u(s), c_0, t_f(s)\}$ and production offers $\{y(s)\}$ are differentiable functions of s. This in turn allows us to focus on separating equilibria, as the following lemma shows.

Lemma 1. *If the social planner's optimal transfers $\{c_e(\tilde{s}), c_u(\tilde{s}), c_0, t_f(\tilde{s})\}$ and firm's offered production $y(\tilde{s})$ are differentiable functions of $\tilde{s}$, then it is optimal for the worker to reveal his type.*

Proof. See appendix. □

The rest of the paper assumes from now on that equilibria are fully revealing. Truth-telling allows us to characterize some properties of the constrained optimal allocation.

Lemma 2. *In the constrained optimal allocation:*

1. *If there exists a s such that $\chi(s) = 1$, then $\chi(s') = 1$ for all $s' \geq s$. We define $\underline{s}$ as the smallest s such that $\chi(s) = 1$.*
2. $W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))} v'\left(\frac{f^{-1}(y(s))}{s}\right)\frac{f^{-1}(y(s))}{s^2}$ *and $W_e(s)$ is increasing*
3. $W_e(s) = W_u(s)$ *for all s*
4. $J_e'(s) = 0$ *and therefore, in equilibrium,* $J_e(s) = \kappa/q(\theta)$
5. $W_e(\underline{s}) = u(c_0)/r$

6. $c_u(s)$ *is increasing*

Proof. See appendix. □

The first result tells us that it is optimal for the planner to let all agents above a certain threshold $\underline{s}$ participate in the labor market, while keeping less productive agents out of it. This arises as a consequence of the congestion externality; an increased participation of low-skill agents does not compensate for the negative impact on the job matching rate of other agents. This also reflects worker's individual rationality: as wages reach 0, workers are better off enjoying leisure and social benefits b_0.

To understand (ii), note that without information asymmetry in this economy, firms would have all the bargaining power, and could drive wages down to the reservation utility $\frac{u(c_0)}{r}$. This does not happen in our setup as agents have their own skill as private information. Firms therefore need to compensate them for possible deviations and need to raise their offers in order to have workers reveal their type. (ii) illustrates this fact as W_e has to be increasing at a rate equal to the marginal utility of deviating. (iii) is a direct consequence that in a truth-telling equilibrium firms can extract all the surplus from workers, under the condition that their incentive constraints are satisfied. Combining these two results, (vi) tells us that unemployment insurance $c_u(s)$ is increasing. (iv) is derived from the Envelope theorem on the firm's problem in a truth-telling equilibrium. Firms choose production and wages optimally for each s. As a result, firms' profit do not vary directly with s and J_e is constant. (v) simply states that the utility of the first type to enter the labor market has to be equal to the reservation utility of receiving social benefits $\frac{u(c_0)}{r}$.

3.6 Implementation of the optimal allocation

The previous sections have presented the social planner's problem as a static truth-telling mechanism. It matters in practice to find an easily implementable tax/transfer system that will effectively achieve the optimal allocation. In particular, we will make explicit the way in which policy instruments make agents and firms choose a wage that will reveal their types.

The following proposition shows that the optimal mechanism is implementable in an economy where the wage is observable.

Proposition 1. *If wages are observable, an optimal allocation in this economy is implementable using a non-linear income tax on workers* $\tau(w)$*, unemployment benefits* $b(w)$*, a transfer* b_0 *to inactive agents, and a uniform subsidy to firms* T*.*

Proof. See appendix. □

This proposition shows that the optimal mechanism can be implemented by simple instruments already used by many governments, such as non-linear income taxes on workers and an unemployment insurance system based on the previously earned wage. In principle, the transfers designed by the social planner should depend on all available information, which explains why all transfers are functions of the underlying skills and employment status of workers. This requires in practice the use of some non-linear instruments such as an income tax $\tau(w)$ or a tax on firms $\tau_f(w)$. In our setup, however, imposing a non-linear tax on firms is unnecessary, as the two tax schedules (on workers and firms) then become redundant. Other tax systems that include taxes on firms may also implement the optimal mechanism.

A minimum wage policy can also be part of an optimal tax system but is not essential, as the optimal transfers can separately shift the general level of wages up or down. Therefore, a minimum wage policy cannot improve welfare. It can be used, however, as a device to simplify the optimal tax. Indeed, when expressed as a function of w, the optimal tax may not be defined on the entire real axis. To prevent agents from considering out-of-equilibrium wages (e.g. below the lowest equilibrium wage) the planner can either impose a large tax, or simply impose a minimum wage equal to this lowest wage.

One may notice that firing costs and hiring subsidies are absent from the optimal policy. This is not a robust feature of the model and is actually a consequence of exogenous job destruction. In the current setup, employment protection policies like firing costs are redundant with the existing transfers for firms, $t_f(s)$[4].

3.7 Optimal control

The optimal allocation being implementable with instruments $\{\tau(\cdot), b(\cdot), b_0, T\}$, it is possible to state the social planner's problem as a function on these instruments. We show in this subsection that the social planner's problem can be cast into an *optimal control* problem. In this new formulation, we treat $W_e(\cdot)$ as the state variable and production $y(\cdot)$ as the control variable. This formulation allows us to use robust resolution techniques to solve the problem.

The optimal control problem is

$$\max_{\theta,\underline{s},b_0,h(\cdot)} \int_{\underline{s}}^{s_{max}} \Phi\left(W_e(s)\right) g(s)ds + \int_0^{\underline{s}} \Phi\left(\frac{u(b_0)}{r}\right) g(s)ds + \lambda \frac{\theta q(\theta)}{\delta + \theta q(\theta)} \frac{\kappa}{q(\theta)} (1 - G(\underline{s})) \tag{16}$$

$$\text{s.t} \begin{cases} W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))} v'\left(\frac{f^{-1}(y(s))}{s}\right) \frac{y(s)}{s^2} \\ W_e(\underline{s}) = \frac{u(b_0)}{r} \\ \frac{\theta q(\theta)}{\delta+\theta q(\theta)} \int \chi(s)\tau(s)g(s)ds - \frac{\delta}{\delta+\theta q(\theta)} \int \chi(s)b(s)g(s)ds - G(\underline{s})b_0 - (1-G(\underline{s}))\frac{\theta q(\theta)}{\delta+\theta q(\theta)} T \geq 0 \end{cases} \tag{17}$$

where

$$\begin{cases} b(s) & \equiv u^{-1}(rW_e(s)) \\ \tau(s) & \equiv y(s) - \frac{\kappa(r+\delta)}{q(\theta)} - u^{-1}(rW_e(s) + v\left(\frac{f^{-1}(y(s))}{s}\right). \end{cases}$$

The first term in (16) is the welfare contribution of workers participating in the labor market (workers with productivity above $\underline{s}$). The second term is the contribution of inactive workers, while the last one is the firm's expected profits weighted by parameter λ (remember $E[J_e(s)|s \geq \underline{s}] = \frac{\kappa}{q(\theta)}$). Concerning the constraints (17), the first equation is the incentive constraint for active workers, the second one combines the incentive constraint for inactive agents and agent $\underline{s}$, while the last one is the planner's resource constraint.

The traditional way of solving this type of problem is to use Pontryagin's maximum principle to derive necessary conditions. To do so, we write the Hamiltonian[5]

$$\mathcal{H}(s, W_e, y) = g(s)\Phi(W_e) + \mu(s)\frac{r + \theta q(\theta)}{r(r + \delta + \theta q(\theta))} v'\left(\frac{f^{-1}(y(s))}{s}\right) \frac{f^{-1}(y(s))}{s^2} \tag{18}$$
$$+ \nu g(s)\left[\theta q(\theta)\left(y(s) - u^{-1}\left(rW_e + v\left(\frac{f^{-1}(y(s))}{s}\right)\right)\right) - \delta u^{-1}(rW_e)\right]$$

[4]The decision to destroy the job or not being exogenous, firing costs cannot influence the firing decisions of firms. It only affects the number of posted vacancies by reducing firms' expected profits. One can show that a tax system with constant transfers to firm is in this case equivalent to any tax system based on firing costs.

[5]For simplicity, we include only the terms that are relevant for the partial differential equations system.

where $\mu(\cdot)$ is the costate variable of $W_e(\cdot)$ and where ν is the Lagrange multiplier on the resource constraint.

The necessary conditions are

$$\frac{\partial \mathcal{H}}{\partial W_e} = -\mu'(s) \quad \frac{\partial \mathcal{H}}{\partial \mu(s)} = W_e'(s) \quad \frac{\partial \mathcal{H}}{\partial h} = 0 \tag{19}$$

and the boundary conditions are

$$\begin{cases} W_e(\underline{s}) = \frac{u(b_0)}{r} \\ \mu(s_{\max}) = 0 \end{cases} \tag{20}$$

The first boundary condition comes from lemma 2. The second one is the appropriate transversality condition since $W_e(\cdot)$ is assumed free at s_{max}. Once the optimal control problem is solved for $W_e(\cdot)$ and $y(\cdot)$, finding the optimal $\underline{s}$, b_0 and θ is a simple maximization problem.

4 Optimal Policy in the United States

It has now been established that the government can implement the optimal mechanism using an income tax on workers $\tau(w)$, an unemployment benefit schedule $b(w)$, a subsidy to firms T and a transfer to *inactive* agents b_0. In this section, we calibrate the model on the US economy and solve for the optimal policy. We highlight characteristics of the optimal policy and then explain how the various policy instruments influence the allocation.

4.1 Functional forms

We use the following functional forms. Worker preferences are given by

$$\mathcal{U}(c_i, h_i) = \int_{t=0}^{\infty} e^{-rt}[u(c_i(t)) - v(h_i(t))]\mathrm{d}t.$$

We use the following CRRA utility function and disutility of labor:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma}, \text{ if } \gamma > 0, \gamma \neq 1 \\ \ln c, \text{ if } \gamma = 1 \end{cases} \quad , \quad v(h) = \rho \frac{h^{1+1/\varphi}}{1+1/\varphi}$$

where $1/\gamma$ is the intertemporal elasticity of substitution for consumption, φ is the Frisch elasticity of labor supply, and $\rho > 0$ is the weight on the disutility of labor.

The production function is

$$f(s \cdot h) = A(s \cdot h)^{\alpha}$$

with $0 < \alpha < 1$, and $A > 0$.

For the labor market variables, we assume that the aggregate matching function is a constant return-to-scale Cobb-Douglas function

$$m(U, V) = MU^{\mu}V^{1-\mu}$$

with $0 < \mu < 1$, and $M > 0$.

Most of the empirical literature has used Pareto and log-normal skill distribution. For our purpose, we choose a log-normal distribution with parameters (μ_s, σ_s^2) as it provides a better fit for the calibration.

In its current form, the model is not identified; a change in the value of A can be achieved by an equivalent and independent change in (μ_s, σ_s^2). Hence, we normalize the average skill in the population to 1. Therefore, $e^{\mu_s+\sigma_s^2/2} = 1$.

4.2 Calibration

The idea behind the calibration is as follows: given a set of inputs (parameters, policies and a skill distribution) the model produces a wage distribution and a time-at-work distribution. We therefore try to set these parameters such as to minimize the difference between the calibrated distributions (given by the model) and the empirical ones (observed in the data).

We use a loss function that takes the square of the area between the empirical and the calibrated wage distributions, and we add to it the square of the difference between the empirical and calibrated means for time-at-work.[6] We weight these two quantities such that their contributions to the loss are of the same magnitude. Therefore,

$$\text{LOSS} = \int \left[g_{(\text{w calibrated})}(w) - g_{(\text{w empirical})}(w)\right]^2 dw + \lambda_{weight}\left[\text{E}(g_{(\text{h calibrated})}) - \text{E}(g_{(\text{h empirical})})\right]^2$$

where g is the designated distribution, λ_{weight} is the weight given to the second term and E is the operator that gives the mean of the distribution.

The empirical data for the wage and time-at-work distributions comes from the *Current Population Survey* (CPS). To avoid calibrating the model in a turbulent period, we pick the data from January 2005. To estimate the hourly wage distribution, we use the reported weekly earnings and weekly hours actually worked at all jobs. We drop all the reported hourly-wage rates below the Federal minimum wage ($5.15 in January 2005). Also, as their density is insignificant, we drop all observations above $100 per hour. We also take the weekly hours worked data from the CPS for the distribution of h. The time unit is a year. Total income per year is stated in thousands of US dollars.

We use 2005 data from the US *Bureau of Labor Statistics* (BLS) for aggregate labor market variables, such as the unemployment rate, total labor force, and the average duration of unemployment.

According to Petrongolo and Pissarides (2001), estimates for the aggregate matching function elasticity μ range from 0.5 to 0.7. We arbitrarily set $\mu = 0.5$. We also set the risk aversion $\gamma = 2$ and $\varphi = 0.5$.

Data on vacancies in the US have been available since 2000 from the *Job Openings and Turnover Survey* (JOLTS). We use the January 2005 estimate of job openings rate as a proxy for the number of vacancies V.

The official federal minimum wage was $5.15/hour in 2005. We introduce it in the contract setting process as a lower bound on the set of hourly wage rates. The tax schedule is the 2005 federal income tax from the Internal Revenue Service for a single person.

A critical step is the choice of the unemployment compensation schedule b. In our model, $b(w)$ is constant over the unemployment period, which is not the case in the United States. In most states, unemployment compensation is a fraction of previous income received during a predetermined number of weeks. For example,

[6]The natural thing to do here would have been to also take the difference in the empirical and calibrated density of time-at-work. The distribution of time-at-work is however very messy and properly matching it with the smooth distribution given by the model is impossible.

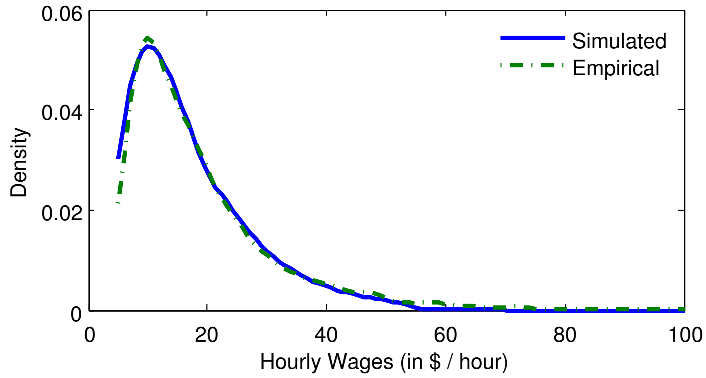

Figure 2: Empirical and calibrated hourly wage distribution

in the state of New York, unemployed workers receive a fraction of 1/26 of their previous quarter income during 26 weeks. This amounts to a flow of 50% of the previous wage for 26 weeks. Since the average duration of unemployment was 19.4 weeks in January 2005, and therefore below 26 weeks, we calibrate b to be 50% of the previous wage.

The life-long utility of the lowest skill entering the labor market is $u(b_0)/r$. We calibrate b_0 to match the expected utility of a worker earning the minimum wage and working the average hours worked in the economy, given aggregate conditions and US policy. The cutoff $\underline{s}$ is then computed to be the first type s entering the market given b_0.

Finally, there is no available data on the rate cost of posting a vacancy κ. Mortensen (1994) calibrates it to be a fraction 0.3 of the production of the highest skilled agent in a given period. We follow Ljungqvist and Sargent (2007) in assuming that κ corresponds to 6 months of the average wage in the economy.

Table (1) summarizes the choice of parameters.

Parameter	Origin/Target moment	Empirical moment	Parameter value
γ	Empirical studies		2
φ	Empirical studies		0.5
r	Interest rate		0.02
U	Bureau of Labor Statistics		7,759,000
V	JOLTS	2.9% total empl.	4,067,130
θ	V/U		0.5242
μ	Petrongolo and Pissarides (2001)	[0.5,0.7]	0.5
M	Avg. duration of unemp. $1/M\theta^{1-\mu}$	19.4 weeks	3.7129
δ	Unemployment rate $\delta/(\delta+\theta q(\theta))$	0.052	0.1475
$\underline{w}$	Federal minimum wage		$5.15/h
κ	6 months of avg. wage	1/2 × $36,840/yr	18.42

Table 1: Pre-calibration parameters

We calibrate the model by minimizing the loss function. Figure 2 shows the calibrated and the empirical hourly earnings distributions.

Table 2 shows the optimal parameters. From $var(g)$, the variance of the skill density, we can recover the

α	A	var(g)	ρ	$\underline{s}$	b_0
0.4431	72.83	3.290	3.016	0.0474	7.246

Table 2: Results of the calibration

	$\underline{s}$	b_0	θ	Unemployment rate
Calibrated	0.047	7.246	0.5242	5.2%
Optimal	0.001	12.69	0.9575	3.9 %

Table 3: Optimal policy in the US

parameters of the log-normal distribution $\mu_s = -0.728$ and $\sigma_s^2 = 1.456$.

4.3 Optimal Policy

Using the parameters found in the calibration, we now solve the optimal control problem and find the optimal labor market policy for the US.[7]

Table 3 summarizes some of the results. Global welfare goes from -3.11 to -2.7062. This is equivalent to an increase of each agent's consumption by 17.5%. This number is large but it is important to note that, since the government is the only source of insurance in this model, the gain would be smaller if agents had access to savings. Still, the bulk of welfare gains goes to low-skill agents, those that might have difficulties hedging against unemployment by savings.

The planner reduces $\underline{s}$ from 0.047 to 0.001. This shows the inefficiency of minimum wage policies since they prevent low-skill agents from entering the labor market. Instead, the planner prefers a progressive tax schedule to redistribute wealth and to subsidize their entry on the labor market. Also, the unemployment compensation to inactive agents, b_0, is increased by 75%, going from \$7250 to \$12690 per year. The labor market tightness goes from 0.5242 to 0.9575. This results in an unemployment rate of 3.9%, lower than the 5.2% observed in the data for that period.

Figure 3 summarizes the economy. Panel (a) shows the before-tax wage, the after-tax wage and the unemployment benefits $b(s)$. Panel (b) presents the welfare $W_e(s)$ of the workers. It is increasing and concave and its precise shape is given by the incentive constraint.

Panel (c) shows the tax schedule. It features a *negative income tax* for the lowest wages. This is a robust feature of the model. A negative tax makes searching for a job more attractive for low-skill agent and therefore tends to lower the number of inactive agents. Furthermore, the optimal policy reduces the unemployment insurance payments made by the government while increasing global output.

The tax is almost linear for the biggest part of the wage distribution but flattens for higher value. This flattening is a consequence of the higher bound on the skill distribution and the transversality condition. At this point, the incentive constraint is less binding (an increase in $W_e(s_{max})$ does not require a change in other agent's W_e to respect the IC constraint). The planner therefore seeks to minimize the distortionary effects of the tax schedule by flattening it. The optimal tax schedule is much more progressive than the 2005 US tax schedule.

[7] The Matlab code used to find the optimal allocation is available at the authors' webpages.

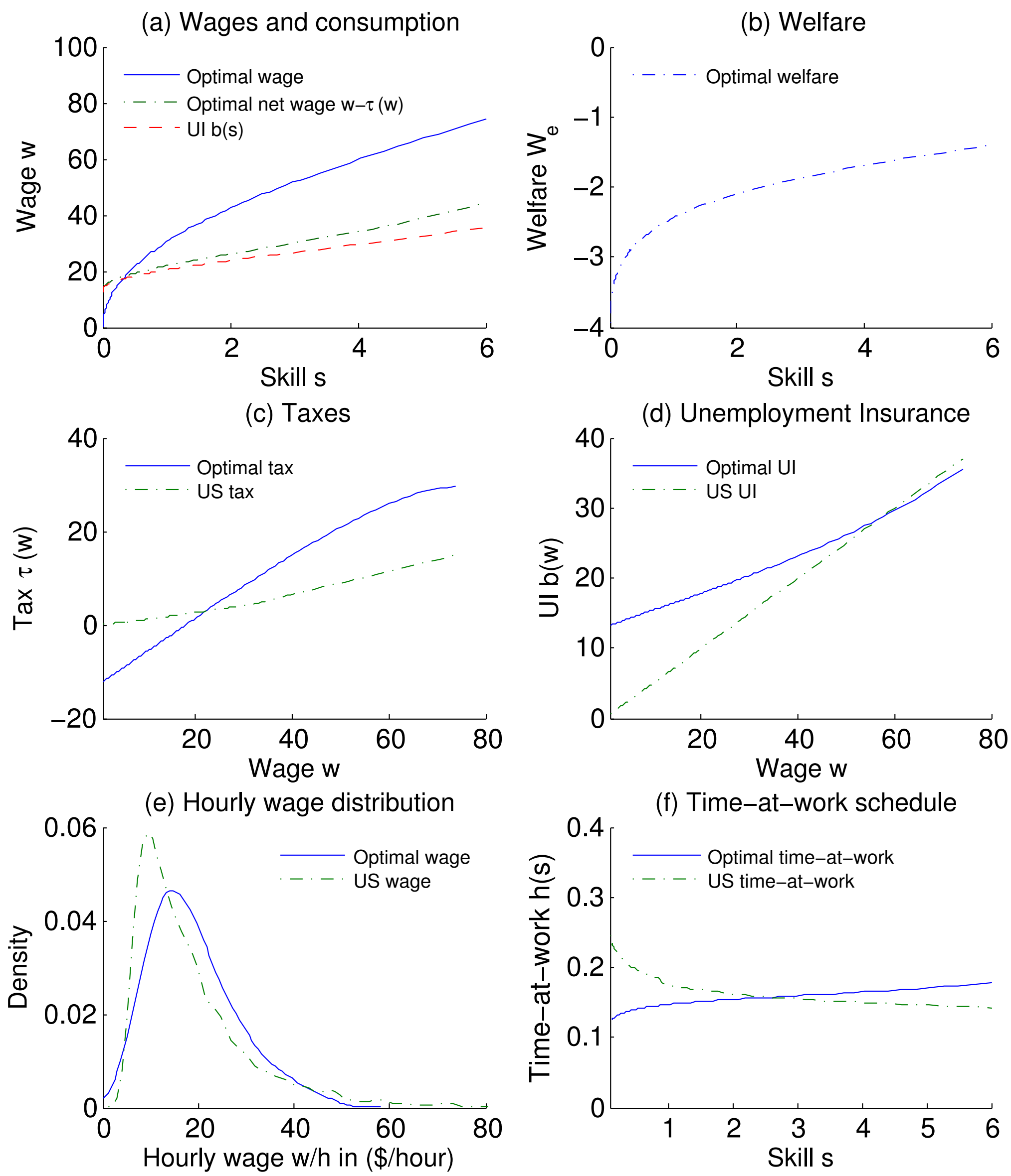

Figure 3: Comparing the optimal policy with the US policy

Panel (d) shows the optimal unemployment insurance schedule $b(w)$ and compares it to the one used by the state of New York. Both schedules compensate workers with high wages in a similar way while the optimal one is more generous to workers with low wages. Panel (e) shows the hourly wage distributions. The optimal one is shifted to the right and less skewed than the actual one. Notice also that in the optimal setup many workers earn very low wages. They are however compensated by the negative income tax such that their total consumption is higher.

Panel (f) presents the difference between the optimal time-at-work schedules and the one coming from the calibration. In the optimal case, high-skill workers are incentivized to be more productive. Two phenomena are at play here. The first one is a wealth effect: richer agents substitute consumption for leisure which gives a concave shape to the optimal schedule. The second effect comes from the fact that these agents are highly productive and that the planner wants them to work as much as possible, and so aligns the appropriate incentives for that to happen. In fact, the incentive structure manages to change the monotonicity of the curve.

4.4 Economic forces at work

The optimal policy presents several robust features. First, $W_e(s)$, $w(s)$ and $b(s)$ are increasing concave functions of s (Figure 3). This is a direct consequence of the incentive constraints. Because high productivity workers can deviate and pretend to be low-skilled, the social planner needs to compensate them to reveal their true type. Remember from lemma 2 that the incentive constraint is

$$W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))} v' \left(\frac{f^{-1}(y(s))}{s} \right) \frac{f^{-1}(y(s))}{s^2}.$$

Since the gain of a deviation from the equilibrium is high for low s, W_e is steep at the lower-end of the skill distribution and its slope progressively decreases as the skill increases. This explains the concave shape of the functions.

Also, production y and hours worked h increase with s. Production goes up with s simply because agents are increasingly productive and it becomes less costly for them to produce larger amounts. This is another interesting feature of the optimal policy: it is designed to induce productive workers to reveal their types and to produce more. A more striking element is the fact that working hours also increase with s. This suggests that the optimal tax is set such that the income effect is weak and the marginal rate of substitution between consumption and leisure is distorted so that productive agents actually work more. This can be explained by the following mechanism. Because the disutility of labor is convex, an increase in the hours worked of a low-skill agent s_l induces a large cost for him. However, if an high-skill agent were to claim the same contract, s_l, he would face a relatively lower change in his disutility of labor; the deviation becomes more profitable. Hence, the planner needs to compensate him more to prevent the deviation, which makes increasing the number of hours worked by low-skill agents very costly.

As a consequence of the incentive constraint, the equilibrium consumption and wage schedule are increasing and concave. Wages are very low, however, for unproductive agents. This comes from the fact that firms only offer contracts that yield positive profits. As production of low-productivity workers goes to 0, wages also have to go to 0. This is harmful in terms of welfare. Therefore, the optimal policy raises the utility of low-skill agents by increasing their unemployment benefits and by subsidizing their entry on the labor market by a *negative income tax*. As a result, the equilibrium tax $\tau(w)$ (Figure 3) is quite negative for low wages, and the unemployment benefits $b(w)$ is relatively higher for low-productivity workers.

Notice finally that the optimal tax appears to be close to linear for workers ranging in the middle of the skill distribution. This result is well known in public finance, but is not a robust feature of the optimal policy as it mainly depends on the functions used for the preferences. The curvature of the optimal tax actually changes with the preference parameters, or with other utility functions, as our sensitivity analysis will show.

4.5 Impact of policy instruments

Because of the general equilibrium setup, it is a difficult task to isolate precisely the effect of a single policy instrument on the whole economy. In order to get some intuition, we constrain the planner's optimization along some dimensions by fixing certain parameters to see how the optimal policy would adjust. For instance, by fixing $\underline{s}$ and θ to arbitrary values, it is possible to vary b_0 to have an idea of the effects of the benefits to inactive agents on the allocation. We do this exercise for these three parameters $\underline{s}$, θ and b_0.

Extensive margin of the labor supply $\underline{s}$: The extensive margin of labor supply matters for several reasons. As argued in section 3, the social planner wants to induce more people to work to increase production, raise more tax revenue and reduce his spending on welfare transfers. In order to do so, a negative income tax can be used to make people participate more in the labor market. On the other hand, the planner may prefer to limit workers entry on the labor market to ease the *congestion externality*. This can be done by lowering the subsidies at the lower-end of the skill distribution and by raising b_0 to encourage low-skill agents to stay out of the labor market. Figure 4 presents the simulated optimal policy for different values of $\underline{s}$, keeping the other parameters at their optimal values. Interestingly, the tax and unemployment insurance change very little with $\underline{s}$. However, the planner increases the minimum wage observed in this economy. This amounts to put a very high tax on the lowest wages.

Unemployment and market tightness θ: The planner can also affect the unemployment rate U/N by acting on the firm's profits. This comes from the free-entry condition (equation (7)) and the steady-state condition (equation (13)). An increase in expected profits $\mathrm{E}\left(J_e(s)\right)$ leads to more vacancy posting, a higher labor market tightness θ and thus to a lower unemployment rate U/N. However, this raise in profits must either come from more hours worked or from a smaller wage, both of which lower utility for the agents. To change the level of profits, the social planner can vary the lump-sum transfer T and affect the tax level. Figure 5 shows the optimal simulated policy for different values of θ (different unemployment rates). We can see that the social planner prefers to reduce the tax level. As a result, wages go down, while production stays about the same. Profits however go up, increasing the equilibrium level of θ.

Transfers to inactive agents b_0: The transfer b_0 matters not only for the extensive margin, but most importantly because it sets the lowest level of utility in the economy. By raising b_0, workers' utility while inactive goes up. Therefore, to avoid deviations, equilibrium consumption schedules have to increase to compensate. Figure 6 shows that increasing b_0 requires an increase in taxes, so that the planner's resource constraint is balanced. As a consequence, production is discouraged and wages go down.

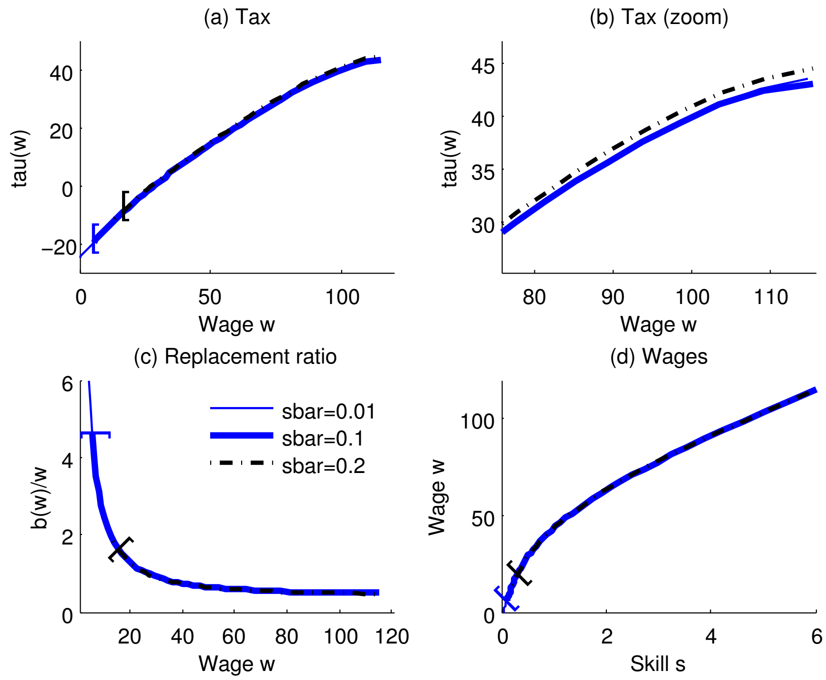

Figure 4: Optimal tax and unemployment insurance for $\underline{s} = 0.01, 0.1, 0.2$ keeping $\theta = 0.1$ and $b_0 = 25$

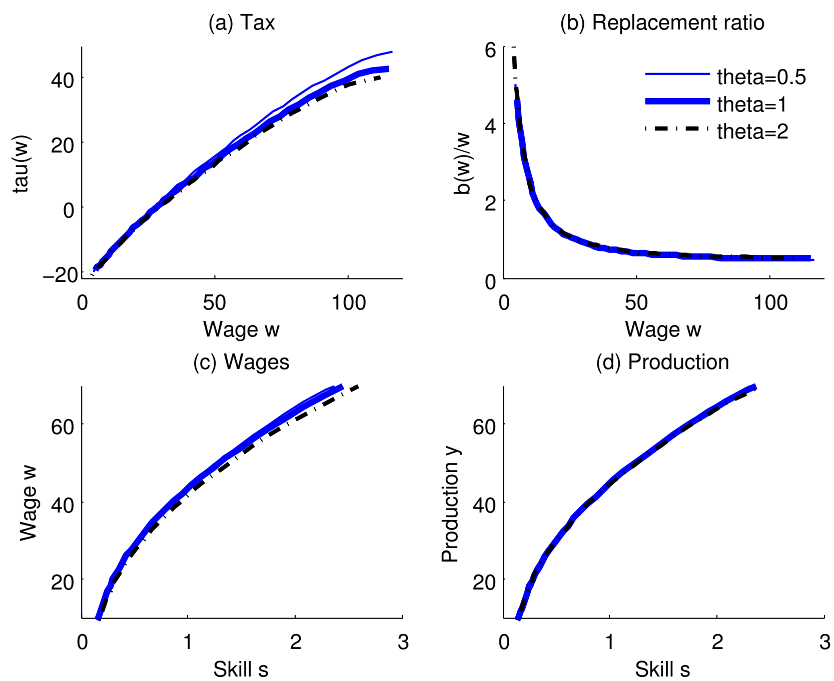

Figure 5: Optimal tax and unemployment insurance for $\theta = 0.5, 1, 2$

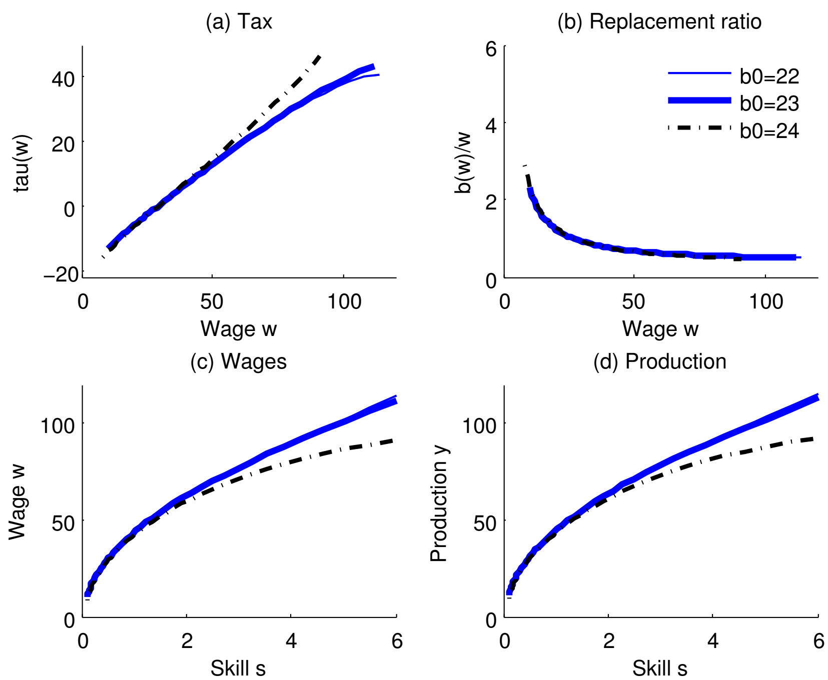

Figure 6: Optimal policy for $b_0 = 22, 23, 24$

	$\underline{s}$	b_0	θ	Unemployment rate
Calibrated	0.047	7.246	0.5242	5.2%
Optimal ($\xi = 1$)	0.001	12.69	0.9575	3.9 %
Optimal ($\xi = 4$)	0.0014	14.64	0.9170	3.98%

Table 4: Optimal policy under different welfare preferences

4.6 Increasing the planner's preference for redistribution

We investigate what happens to the optimal policy if the planner has a higher aversion to inequality. To do so, we increase the concavity of $\Phi(\cdot)$. For computational purposes, we set $\Phi(x) = -(-x)^{\xi}$ and vary ξ to modify the preference for redistribution. The utilitarian case of the last section corresponds to $\xi = 1$. The result of the optimization with $\xi = 4$ are presented in table 4 and figure 7.

The planner pushes the skill threshold $\underline{s}$ from 0.001 to 0.0014, a small difference. The planner does not want to limit access to the labor market. The unemployment payments to inactive agents, b_0, goes from \$12690 to \$14640, as expected by increasing the taste for redistribution.

Panel (a) of figure 7 shows that high-skill agents are more taxed with $\xi = 4$ while low-skilled ones receive a higher subsidy. The same is true for unemployment benefits, as shown in panel (b). Panel (c) shows the wage schedules. High-skill workers receive a higher wage but are taxed more heavily and work longer hours (panel (d)). Low-skill agents, on the opposite, enjoy much more leisure. Panel (e) show the welfare schedules. Although it is not clearly visible on the graph, the two schedules intersect. As expected, low-skill agents are better off with $\xi = 4$ while high-skilled ones are worse-off. The final panel, (f), shows the wage distributions being shifted to the left with the more redistributive policies.

Notice that the policies observed with $\xi = 4$ share some features with European economies: more progressive taxation and generous benefits, which results in fewer hours worked and lower total production. However, the increase in the unemployment rate, from 3.9% to 3.98%, is not representative of the observed discrepancies between Europe and the US. Obviously, these numbers represent a comparison of economies under *optimal policies* — there are indeed reasons to believe that observed policies depart from optimality. Therefore, this last exercise should not be understood as an attempt to account for cross-country differences but instead to emphasize general features that characterize more egalitarian policies. Generous unemployment benefits and heavier taxation should not always be considered sub-optimal. Good policy designs may in fact keep the unemployment level low even with highly redistributive policies.

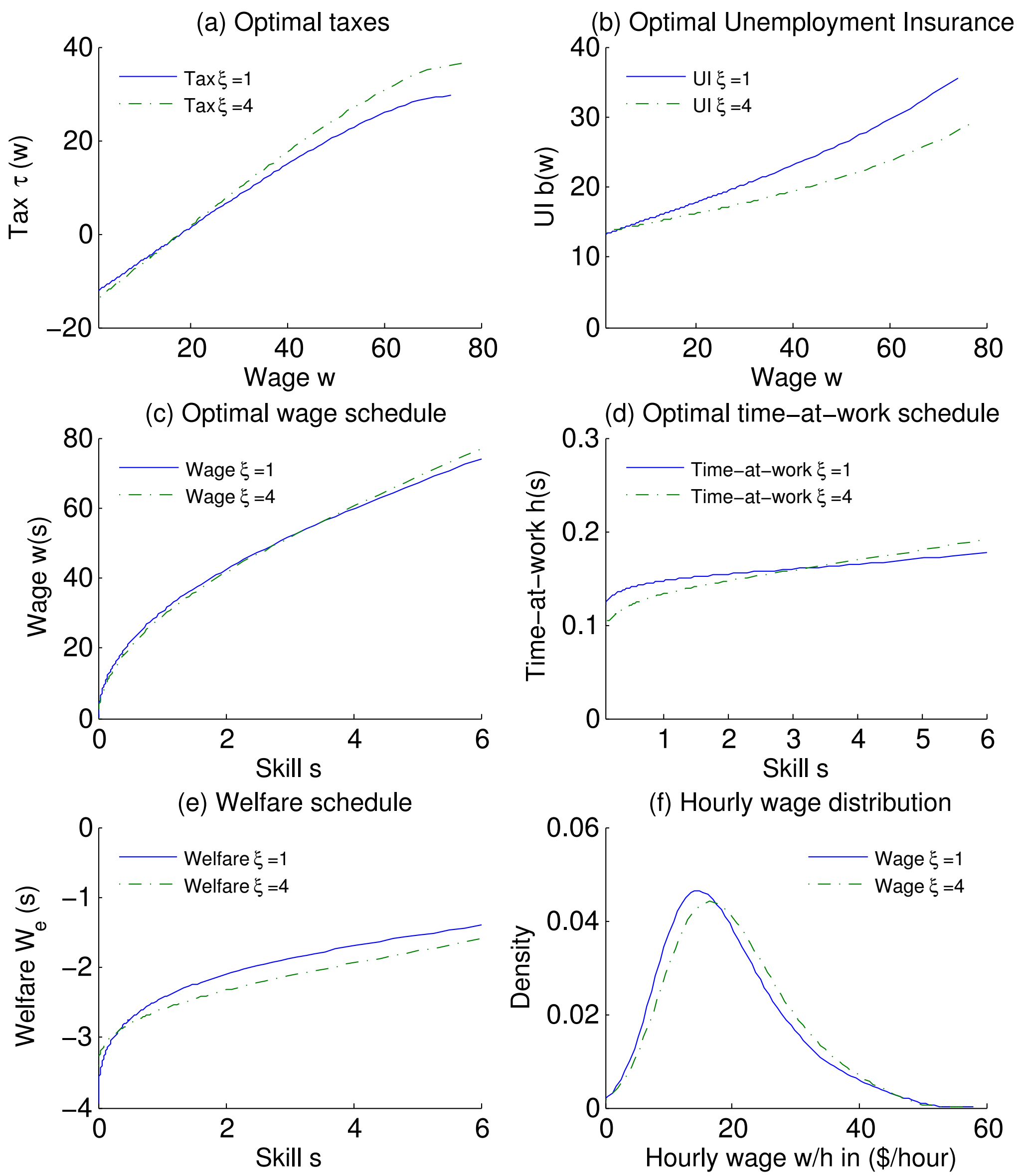

Figure 7: Increasing the preference for redistribution: optimal policies with $\xi = 1$ and $\xi = 4$

5 Conclusion

This paper studies the optimal design of policies in a frictional labor market with private information. Heterogeneous workers with unobservable skills search for jobs in a unique labor market governed by an aggregate matching function. When they meet, firms and workers play a signaling game where workers first reveal a type and firms then offer a contract. We study the optimal mechanism for a general class of welfare functions (including utilitarian) and show that the constrained optimal allocation is implementable in an economy where wages are observable by a non-linear tax on wages, a non-linear unemployment insurance and firm subsidies. Our baseline calibration suggests that switching to the optimal policy would reduce unemployment, increase labor market participation, and increase welfare by 17.5% in consumption equivalent. Our findings also suggest that optimal policies often feature a negative income tax and generous replacement ratios for low-skilled people. Also, increasing the planner's taste for redistribution yields policies that display some European features: higher marginal tax rates, more generous benefits and lower hours and production.

The findings in this paper could be extended in several dimensions. First of all, there is no credit market. The government is therefore the only provider of insurance. This feature tends to exaggerate the welfare gains of implementing the optimal policy, and explains why we obtain such a generous unemployment insurance. Also, we have abstracted from possible frictions like imperfect monitoring of search effort, which would also limit the capacity of the government to insure people.

A last extension worth exploring would be to study the full dynamic model and allow for time-dependent policies. However, such an extension would require a tractable dynamic model of the labor market with heterogeneous agents. Another interesting dimension could be to introduce human capital and investigate how policies could influence agents' decisions to look for jobs in industries with higher risks but with better learning opportunities.

References

Atkinson, A. B. and J. E. Stiglitz (1980): *Lectures on public economics / Anthony B. Atkinson, Joseph E. Stiglitz*, McGraw-Hill Book Co., London, New York.

Blanchard, O. and J. Tirole (2008): "The Joint Design of Unemploment Insurance and Employment Protection: A First Pass," *Journal of the European Economic Association*, 6, 45–77.

Hopenhayn, H. A. and J. P. Nicolini (1997): "Optimal Unemployment Insurance," *The Journal of Political Economy*, 105, 412–438.

Hosios, A. J. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *The Review of Economic Studies*, 57, 279–298.

Hungerbühler, M., E. Lehmann, A. Parmentier, and B. V. D. Linden (2006): "Optimal Redistributive Taxation in a Search Equilibrium Model," *Review of Economic Studies*, 73, 743–767.

Ljungqvist, L. and T. J. Sargent (2007): "Understanding European unemployment with matching and search-island models," *Journal of Monetary Economics*, 54, 2139 – 2179.

Mirrlees, J. A. (1971): "An Exploration in the Theory of Optimum Income Taxation," *The Review of Economic Studies*, 38, 175–208.

Mortensen, D. T. (1994): "Reducing supply-side disincentives to job creation," *Proceedings*, 189–237.

Mortensen, D. T. and C. A. Pissarides (2002): "Taxes, Subsidies and Equilibrium Labor Market Outcomes," CEP Discussion Papers 0519, Centre for Economic Performance, LSE.

Nickell, W. (2006): "The CEP-OECD Institutions Data Set (1960-2004)," CEP Discussion Papers CEPDP0759, CEP.

Petrongolo, B. and C. A. Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39, 390–431.

Pissarides, C. A. (2000): *Equilibrium Unemployment Theory - 2nd Edition*, The MIT Press.

Rogerson, R. (2006): "Understanding Differences in Hours Worked," *Review of Economic Dynamics*, 9, 365–409.

Rogerson, W. P. (1985): "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 53, 1357–1367.

Saez, E. (2000): "Optimal Income Transfer Programs: Intensive Versus Extensive Labor Supply Responses," NBER Working Papers 7708, National Bureau of Economic Research, Inc.

Shavell, S. and L. Weiss (1979): "The Optimal Payment of Unemployment Insurance Benefits over Time," *The Journal of Political Economy*, 87, 1347–1362.

Stiglitz, J. E. (1988): "Pareto Efficient and Optimal Taxation and the New New Welfare Economics," *SSRN eLibrary*.

Wang, C. and S. Williamson (1996): "Unemployment insurance with moral hazard in a dynamic economy," *Carnegie-Rochester Conference Series on Public Policy*, 44, 1–41.

A Sensitivity analysis

We proceed to some sensitivity analysis to see how the shape of the optimal policy functions depend on the model's parameters. We perform a similar kind of comparative statics as in the previous section: fixing $\underline{s}$, θ and b_0, we solve for the optimal control problem and present the resulting policy functions. Graphs of the optimal policies are presented in the appendix.

Intertemporal elasticity of substitution $1/\gamma$: The optimal policy changes dramatically even for small variations of γ, keeping all other parameters constant (b_0, θ and $\underline{s}$). Figure 8 presents the optimal policy for $\gamma = 1.8$, 1.9 and 2. We can see that increasing γ makes our policy functions more concave. Note that γ is also the relative risk aversion of agents in our economy. Also, the tax tends to increase with γ, while production and wages tend to decrease. All these effects come from the marginal utility of low-skill workers. The planner allocates more resources to them and finances these transfers with higher taxation on high-skill agents.

Frisch elasticity of labor supply φ: Figure 9 presents the optimal policies for $\varphi = 0.4$, 0.45 and 0.5. Results are very similar to the case where γ varies. Part (ii) of lemma 2 states that the slope of W_e is increasing with φ (since $h(\cdot)$ is strictly lower than 1 in all the simulations). This shows that an increase in φ makes deviations more profitable. To prevent low-skill workers from deviating upward, the planner reduces the wage and the production of high-skill agents. This is done through the tax scheme.

Matching elasticity μ: Remember that the matching function is $m(U,V) = MU^{\mu}V^{1-\mu}$. Figure 10 shows the optimal policy when the matching elasticity is $\mu = 0.5$, 0.6, and 0.7. The results are a similar to those obtained when θ varies. The explanation is simple: μ only changes the rate at which agents are matched in a steady-state. Raising μ when $\theta = 0.5$ increases the rate at which agents find a job in the economy, while decreasing the matching rate for firms. A higher matching rate for agents increases the slope of $W_e(s)$ and requires the planner to compensate more for possible deviations. Since an increased unemployment insurance can put pressure on the resource constraint, the planner actually reduces the equilibrium production, therefore lowering agents' claims for unemployment benefits. As a consequence, the wage schedule also decreases.

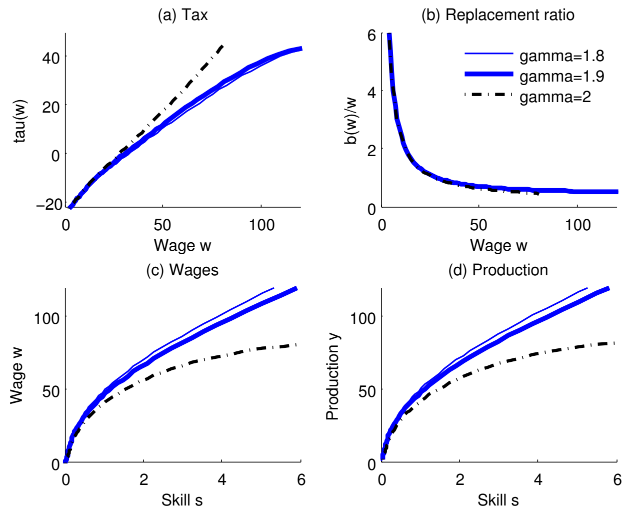

Figure 8: Optimal policy for $\gamma = 1.8, 1.9, 2$ keeping $\theta = 1$, $b_0 = 25$ and $\underline{s} = 0.01$

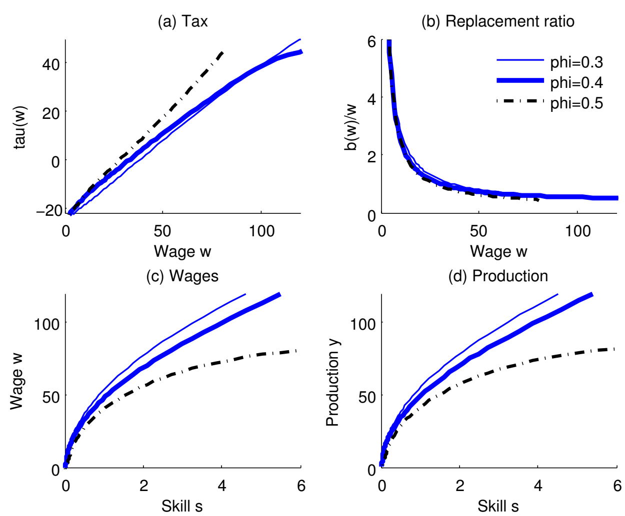

Figure 9: Optimal policy for $\phi = 0.4, 0.45, 0.5$ keeping $\theta = 1$, $b_0 = 25$ and $\underline{s} = 0.01$

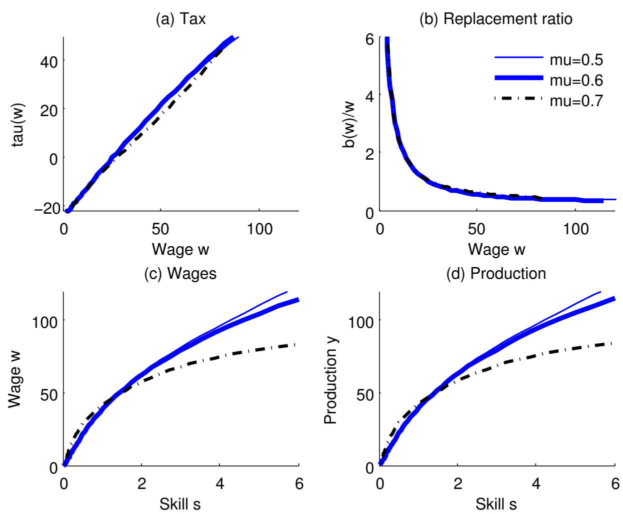

Figure 10: Optimal policy for $\mu = 0.5, 0.6.0.7$ keeping $\theta = 0.5$, $b_0 = 25$ and $\underline{s} = 0.01$

B Proofs

Here are the proofs from the previous sections.

Lemma 1. *If the social planner's optimal transfers* $\{c_e(\tilde{s}), c_u(\tilde{s}), c_0, t_e(\tilde{s}), t_u\}$ *and firm's offered production* $y(\tilde{s})$ *are differentiable functions of* $\tilde{s}$, *then it is optimal for the worker to reveal his type.*

Proof. Taking the first-order condition for a worker of type s willing to work in equation (14) yields:

$$u'\big(c_e(s^*)\big)c_e'(s^*) - v'\left(\frac{f^{-1}(y(s^*))}{s}\right)\frac{y'(s^*)}{sf'(f^{-1}(y(s^*)))} + \frac{\delta}{r+\theta q(\theta)}u'(c_u(s^*))c_u'(s^*) = 0$$

The left-hand side of the equation is a strictly increasing function of s. Therefore, this condition can be satisfied by at most one type s. Workers never choose the same contract. □

Lemma 2. *In the second-best optimal allocation:*

[(i)]

1. *If there exists a* s *such that* $\chi(s) = 1$, *then* $\chi(s') = 1$ *for all* $s' \geq s$. *We define* $\underline{s}$ *as the smallest* s *such that* $\chi(s) = 1$.
2. $W_e'(s) = \frac{r+\theta q(\theta)}{r(r+\delta+\theta q(\theta))}v'\left(\frac{f^{-1}(y(s))}{s}\right)\frac{f^{-1}(y(s))}{s^2}$ *and is therefore increasing*
3. $W_e(s) = W_u(s)$ *for all* s
4. $J_e'(s) = 0$ *and therefore, in equilibrium,* $J_e(s) = \kappa/q(\theta)$
5. $W_e(\underline{s}) = u(c_0)/r$
6. $c_u(s)$ *is increasing*

Proof. Proceed as follows:

1. Suppose not. Therefore, there is $s' > s$ such that $\chi(s') = 0$ and $\chi(s) = 1$. From equation (PC inactive), we have

$$\begin{aligned} u(c_0) &\geq \frac{(r+\theta q(\theta))(u(c_e(s)) - v(f^{-1}(y(s))/s')) + \delta u(c_u(s))}{r+\delta+\theta q(\theta)} \\ &> \frac{(r+\theta q(\theta))(u(c_e(s)) - v(f^{-1}(y(\tilde{s}))/s)) + \delta u(c_u(s))}{r+\delta+\theta q(\theta)} \\ &= rW_e(s) \end{aligned}$$

   But, since $\chi(s) = 1$, equation (**??**) states that $rW_e(s) \geq u(c_0)$, which is a contradiction.

2. Recall that

$$W_e(s) \equiv \max_{\tilde{s}} \frac{1}{r+\delta}\left[u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \frac{\delta}{r+\theta q(\theta)}\left(u(c_u(\tilde{s})) + \theta q(\theta)W_e(s)\right)\right]$$

   which can be rewritten

$$W_e(s) = \max_{\tilde{s}} \frac{\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right)\right)(r+\theta q(\theta)) + \delta u(c_u(\tilde{s}))}{r(r+\theta q(\theta)+\delta)}$$

   Applying the envelope theorem on this last expression gives the result.

3. The result comes directly from the firm's problem. An increase in $\tilde{y}$ yields to an increase in profit and reduces the left-hand side of the agent's participation constraint. Therefore, the constraint binds.

4. Remember that in a truth-telling equilibrium, the firm's problem is

$$J_e(s) = \max_{\tilde{y},\tilde{s}} \frac{\tilde{y} + t_f(\tilde{s})}{r+\delta}$$
$$s.t. \frac{1}{r+\delta}\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)}(u(c_u(\tilde{s})) + \theta q(\theta) W_e(s))\right) \geq W_u(s)$$

Since $W_e(s) = W_u(s)$, the Lagrangian is

$$\mathcal{L}(s) = \tilde{y} + t_f(\tilde{s}) + \lambda\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) - r\frac{r+\theta q(\theta)+\delta}{r+\theta q(\theta)} W_e(s)\right)$$

The envelope theorem with part *(ii)* yield $J_e' = 0$. In equilibrium, recall from equation (7) that

$$\frac{\kappa}{q(\theta)} = \frac{\int_{\underline{s}}^{\infty} J_e(s) g(s) ds}{N} \tag{21}$$

Since $J_e(s)$ is constant, the result follows.

5. Since $\underline{s}$ is the first working agent, we know that $rW_e(\underline{s}) \geq u(c_0)$. Suppose that the inequality is strict and consider the agents of type $(\underline{s} - \epsilon)$ for $\epsilon > 0$ small. Since $\underline{s} - \epsilon < \underline{s}$, $\chi(\underline{s} - \epsilon) = 0$ and

$$u(c_0) \geq \frac{(r+\theta q(\theta))\left(u(c_e(\underline{s})) - v\left(\frac{f^{-1}(y(\underline{s}))}{\underline{s}-\epsilon}\right)\right) + \delta u(c_u(\underline{s}))}{r+\delta+\theta q(\theta)} \equiv \Gamma_{\underline{s}}(\epsilon)$$

Since v is continuous, we can take the limit $\lim_{\epsilon \to 0} \Gamma_{\underline{s}}(\epsilon) = W_e(\underline{s})$. This contradicts the supposition $rW_e(\underline{s}) > u(c_0)$ and completes the proof.

6. We have $c_u(s) = u^{-1}(rW_e(s))$ and the result follows since $W_e(\cdot)$ is increasing.

□

Proposition 1. *If wages are observable, an optimal allocation in this economy is implementable using a non-linear income tax on workers $\tau(w)$, unemployment benefits $b(w)$, a transfer b_0 to inactive agents, and in some cases supplemented by a uniform subsidy to firms T.*

Proof. Implementing the equilibrium means we can find an equilibrium wage schedule $w(s)$ and functions $b(w)$, $\tau(w)$ and a subsidy T such that the following holds:

$$\begin{cases} c_e(s) = w(s) - \tau(w(s)) \\ c_u(s) = b(w(s)) \\ t_f(s) = -w(s) + T \end{cases}$$

The first thing to notice is that the level of the firm subsidy T is indeterminate. The existence of such a subsidy is only to prevent wages to be negative in equilibrium. Indeed, $t_f(s)$ can in principle be positive in equilibrium. Therefore, one needs to choose a subsidy high enough so that wages $w(s) = T - t_f(s)$ are always positive. Otherwise, its absolute level does not matter as wages and taxes can simply be adjusted to yield the same allocation. We are going to construct a tax system that implement the mechanism. Pick any

$T > \max_s(t_f(s))$. The only critical part of the proof is to show that the wage can be a sufficient statistic for the skill, so that we can base transfers on the wage with functions $\tau(w)$ and $b(w)$. We are going to show that $t_f(s)$ is different for all s. Assume by contradiction that there exists $s < s'$ such that $t_f(s) = t_f(s')$.

Remember that in a truth-telling equilibrium, the firm solves the following problem:

$$\max_{\tilde{y},\tilde{s}} \frac{\tilde{y} + t_f(\tilde{s})}{r+\delta}$$
$$s.t. \frac{1}{r+\delta}\left(u(c_e(\tilde{s})) - v\left(\frac{f^{-1}(\tilde{y})}{s}\right) + \frac{\delta}{r+\theta q(\theta)}(u(c_u(\tilde{s})) + \theta q(\theta)W_e(s))\right) \geq W_u(s)$$

Since $t_f(s) = t_f(s')$, the firm picks the type $\tilde{s}$ that gives the highest utility to the worker:

$$\max_{\tilde{s}\in\{s,s'\}} u(c_e(\tilde{s})) + \frac{\delta}{r+\theta q(\theta)}u(c_u(\tilde{s}))$$

as it enables it to increase production. In a truth-telling equilibrium, types s and s' have to be chosen, so the firm must be indifferent between both:

$$u(c_e(s)) + \frac{\delta}{r+\theta q(\theta)}u(c_u(s)) = u(c_e(s')) + \frac{\delta}{r+\theta q(\theta)}u(c_u(s')) \tag{22}$$

Now, turn to the worker's problem. Define:

$$V(s,\tilde{s}) = u\big(c_e(\tilde{s})\big) - v\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right) + \frac{\delta}{r+\theta q(\theta)}u(c_u(\tilde{s}))$$

A worker of type s solves the following problem:

$$\max_{\tilde{s}\in\mathbb{R}_+^*} V(s,\tilde{s})$$

Transfers for agents s and s' yield the same expected utility. Thus, workers always choose to declare the type with the smallest production $y(s)$ or $y(s')$. In a separating equilibrium, both contracts must be chosen. This can only happen if workers are indifferent between the two contracts: $y(s) = y(s')$. This will however lead to a contradiction. Given the differentiability assumption, we can write the following derivative for agent s:

$$\frac{\partial}{\partial\tilde{s}}V(s,\tilde{s}) = u'\big(c_e(\tilde{s})\big)c_e'(\tilde{s}) - v'\left(\frac{f^{-1}(y(\tilde{s}))}{s}\right)\frac{y'(\tilde{s})}{sf'(f^{-1}(y(\tilde{s})))} + \frac{\delta}{r+\theta q(\theta)}u'(c_u(\tilde{s}))c_u'(\tilde{s}) \tag{23}$$

In a separating equilibrium, agents reveal their true types. Thus: $\frac{\partial}{\partial\tilde{s}}V(s,s) = 0$ and $\frac{\partial}{\partial\tilde{s}}V(s',s') = 0$. Note that (23) is a strictly increasing function of s, the true type of the worker. Therefore, given that the first-order condition is satisfied for agent s' at $\tilde{s} = s'$, i.e $\frac{\partial}{\partial\tilde{s}}V(s',s') = 0$, we must have:

$$\frac{\partial}{\partial\tilde{s}}V(s,s') < 0$$

The same function evaluated at s instead of s' is strictly negative. That means: there are contracts just below s' that yield a higher expected utility to agent s. Given his indifference between claiming types s and s', agent s would rather deviate and claim a type just below s' than choose his own contract. This cannot be true in equilibrium. The initial assumption that there exists $s \neq s'$ with $t_f(s) = t_f(s')$ was wrong. We conclude that all wages in equilibrium are different and reveal the worker's type. Thus, set:

$$w(s) = T - t_f(s)$$

It is therefore possible to find functions $\tau(w)$ and $b(w)$ that implement the optimal allocation, i.e:

$$\begin{cases} \tau(w(s)) = w(s) - c_e(s) \\ b(w(s)) = c_u(s) \end{cases}$$

To prevent agents from choosing out-of-equilibrium wages $\tilde{w}$, the government can set a high tax $\tau(\tilde{w}) = 1$. $\square$