Online Appendix
Financial Crises and Exchange Rate Policy

Luca Fornaro

CREI, Universitat Pompeu Fabra

and Barcelona GSE

November 2014

A Implementation of flexible wage equilibrium

In this appendix I show that the central bank can implement the flexible wage equilibrium by setting the exchange rate according to:

$$S_t = \bar{S} z_t^{\xi_z}, \tag{A.1}$$

with $\xi_z \equiv (1-\omega)/(\omega-\alpha_L)$.

Let us start by deriving the labor allocation under flexible wages. With flexible wages, equation (9) holds in any date and state. Using the functional forms assumed, this gives the labor supply equation:

$$L_t^{\omega} = \frac{\sigma-1}{\sigma}\frac{W_t}{S_t}L_t. \tag{A.2}$$

Firms' labor demand is

$$\alpha_L L_t^{\alpha_L-1} K^{\alpha_K} z_t = \frac{W_t}{S_t}. \tag{A.3}$$

Combining these two expressions gives the equilibrium labor under flexible wages:

$$L_t = \left(\frac{\sigma-1}{\sigma}\alpha_L K^{\alpha_K} z_t\right)^{\frac{1}{\omega-\alpha_L}}. \tag{A.4}$$

This equation shows that under flexible wages TFP shocks are the only drivers of movements in equilibrium labor.

Now consider the economy with nominal wage rigidities. Let us guess that in every period t anticipating the policy rule $(A.1)$ households set the wage:

$$W_t = \bar{S}\left[(\alpha_L K^{\alpha_K})^{\frac{1-\omega}{\omega-\alpha_L}}\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1-\alpha_L}{\omega-\alpha_L}}\right]^{-1}. \tag{A.5}$$

Once wages are set, equilibrium labor is determined by firms' labor demand:

$$L_t = \left(\alpha_L K^{\alpha_K} z_t \frac{S_t}{W_t}\right)^{\frac{1}{1-\alpha_L}}. \tag{A.6}$$

The exchange rate that replicates the labor allocation under flexible wages can be found by combining firms' labor demand with equation $(A.4)$:

$$S_t = W_t\left(\alpha_L K^{\alpha_K} z_t\right)^{\frac{1-\omega}{\omega-\alpha_L}}\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1-\alpha_L}{\omega-\alpha_L}} = \bar{S} z_t^{\xi_z}, \tag{A.7}$$

where to derive the second equality I used equation $(A.5)$ to substitute out W_t.

Now we have to show that it is optimal for households to set the wage according to equation $(A.5)$. Given the proposed choices of W_t and S_t equation (9), the wage setting equation, holds in any date and state, implying that it is optimal for households to set wage according to equation $(A.5)$. This verifies our initial guess.

B Numerical solution method

Computing the equilibrium involves finding the functions $W(B^*, z_{-1})$, $L(B^*, z_{-1}, z)$, $C(B^*, z_{-1}, z)$, $B^*_{+1}(B^*, z_{-1}, z)$, $Q(B^*, z_{-1}, z)$, $R(B^*, z_{-1}, z)$ that solve the system:

$$W = \frac{\frac{\sigma-1}{\sigma} E_{-1}\left[\frac{\left(C-\frac{L^\omega}{\omega}\right)^{-\gamma}}{S} L\right]}{E_{-1}\left[\left(C-\frac{L^\omega}{\omega}\right)^{-\gamma} L^\omega\right]} \tag{B.1}$$

$$L = \min\left[\left(z\frac{S}{W}K^{\alpha_k}\right)^{\frac{1}{1-\alpha_L}}, \left(\frac{W}{S}\right)^{\frac{1}{1-\omega}}\right] \tag{B.2}$$

$$C + B^*_{+1} = zL^{\alpha_L}K^{\alpha_K} + R^* B^* \tag{B.3}$$

$$-B^*_{+1} \leq \kappa \frac{Q}{S} K \tag{B.4}$$

$$\left(C - \frac{L^\omega}{\omega}\right)^{-\gamma} = \beta R^* E \left(C_{+1} - \frac{L^\omega_{+1}}{\omega}\right)^{-\gamma} + \mu \tag{B.5}$$

$$R = \frac{R^*}{1 - \mu / \left(C - \frac{L^\omega}{\omega}\right)^{-\gamma}}$$

$$\frac{Q}{S}\left[\left(C - \frac{L^\omega}{\omega}\right)^{-\gamma} - \kappa\mu\right] = \beta E\left[\left(C_{+1} - \frac{L^\omega_{+1}}{\omega}\right)^{-\gamma}\left(\alpha_k L^{\alpha_L}_{+1} K^{\alpha_k - 1} + \frac{Q_{+1}}{S_{+1}}\right)\right], \tag{B.6}$$

for given exchange rate policy $S(B^*, z_{-1}, z)$. The subscripts -1 and $+1$ denote variables referring respectively to dates $t-1$ and $t+1$.

The solution is approximated numerically by applying the time iteration method proposed by Coleman (1990), adapted to address the occasionally binding collateral constraint as in Bianchi and Mendoza (2010). The algorithm follows these steps:

1. Generate a discrete grid for the net foreign asset position $G_{B^*} = \{B^*_1, B^*_2, ...B^*_M\}$ and the productivity shock $G_z\{z_1, z_2, ...z_N\}$. I use 300 points for net foreign assets and interpolate the functions using a piecewise linear approximation.

2. Conjecture $W_J(B^*, z_{-1})$, $L_J(B^*, z_{-1}, z)$, $C_J(B^*, z_{-1}, z)$, $B^*_{J+1}(B^*, z_{-1}, z)$, $Q_J(B^*, z_{-1}, z)$, $R_J(B^*, z_{-1}, z)$, $S_J(B^*, z_{-1}, z)$ at time J $\forall$ $B^* \in G_{B^*}$ and $\forall$ $z \in G_z$.

3. Set $y = 1$.

4. Solve for $W_{J-y}(B^*, z_{-1})$, $L_{J-y}(B^*, z_{-1}, z)$, $C_{J-y}(B^*, z_{-1}, z)$, $B^*_{J-y+1}(B^*, z_{-1}, z)$, $Q_{J-y}(B^*, z_{-1}, z)$, $R_{J-y}(B^*, z_{-1}, z)$, $S_{J-y}(B^*, z_{-1}, z)$ at time $J-x$ using $(B.1)$–$(B.6)$ and $W_{J-y+1}(B^*, z_{-1})$, $L_{J-y+1}(B^*, z_{-1}, z)$, $C_{J-y+1}(B^*, z_{-1}, z)$, $B^*_{J-y+2}(B^*, z_{-1}, z)$, $Q_{J-y+1}(B^*, z_{-1}, z)$, $R_{J-y+1}(B^*, z_{-1}, z)$, $S_{J-y+1}(B^*, z_{-1}, z)$ $\forall$ $B^* \in G_{B^*}$ and $\forall$ $z \in G_z$. To deal with the occasionally binding collateral constraint $\forall$ $B^* \in G_{B^*}$ and $\forall$ $z \in G_z$:

   - Assume collateral constraint $(B.4)$ is not binding. Set $\mu_{J-y}(B^*, z_{-1}, z) = 0$ and solve for $B^*_{J-y+1}(B^*, z_{-1}, z)$ using $(B.5)$.
   - Check whether $-B^*_{+1} \leq \kappa \frac{Q}{S} K$ holds.
   - If constraint is satisfied move to next grid point.
   - Otherwise solve for $\mu(B^*, z_{-1}, z)$ and $B^*_{+1}(B^*, z_{-1}, z)$ using $(B.4)$ with equality and $(B.5)$.

5. Evaluate convergence. If $\sup_{B^*,z_{-1},z} ||x_{J-y}(B^*, z_{-1}, z) - x_{J-y+1}(B^*, z_{-1}, z)|| < \epsilon$ for $x = W,\ L,\ C,\ B^*_{+1},\ Q,\ R,\ S$ we have found the solution. Otherwise, set $x_{J-y}(B^*, z_{-1}, z) = x_{J-y+1}(B^*, z_{-1}, z)$ and $y \rightsquigarrow y+1$ and go to step 4.

C Model with non-traded goods

This section describes in detail the economy with non-traded goods considered in section 4. The basic structure is the same as the benchmark model, but here the economy produces and consumes a tradable good and a non-tradable one. Variables referring respectively to the tradable and the non-tradable sector are denoted with the superscripts T and N.

Firms. A continuum of mass 1 of firms operate in each sector. The production functions are:

$$Y_t^T = z_t \left(L_t^T\right)^{\alpha_T} K^{\alpha_k} \tag{C.1}$$

$$Y_t^N = \left(L_t^N\right)^{\alpha_N}, \tag{C.2}$$

where Y^T and Y^N denote respectively the output of traded and non-traded good and $0 < \alpha_T < 1$, $0 < \alpha_K < 1$, $\alpha_T + \alpha_K < 1$ and $0 < \alpha_N < 1$. For simplicity, I assume that labor L^N is the only factor of production employed in the non-tradable sector, while, similar to firms in the benchmark model, firms in the tradable sector employ labor L^T and land K.

As in the benchmark model, each household supplies a differentiated labor input. L_t^j for $j = T, N$ is a CES aggregate of the differentiated labor services:

$$L_t^j = \left[\int_0^1 L_t^{ij\,\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}},$$

where L_t^{ij} denotes the labor input purchased from household i by firms in sector j and $\sigma > 1$. I assume that labor is freely mobile across sectors, implying that household i charges the same wage W^i to firms in both sectors. Hence,the minimum cost of a unit of aggregate labor L_t^j is the same in the two sectors and given by:

$$W_t = \left[\int_0^1 W_t^{i\,1-\sigma} di\right]^{\frac{1}{1-\sigma}},$$

which can be taken as the aggregate nominal wage. Using this definition, profit maximization implies equality between factor prices and marginal productivities:

$$W_t = S_t z_t \alpha_T \frac{Y_t^T}{L_t^T} \tag{C.3}$$

$$R_t^K = S_t z_t \alpha_K \frac{Y_t^T}{K_t} \tag{C.4}$$

$$W_t = P_t^N \alpha_N \frac{Y_t^N}{L_t^N}, \tag{C.5}$$

where S and P^N denote respectively the domestic currency price of a unit of traded and non-traded good, while R^K denote the rental rate of land in units of domestic currency. Finally, cost minimization gives the demand for household's i labor from sector j:

$$L_t^{ij} = \left(\frac{W_t}{W_t^i}\right)^{\sigma} L_t^j.$$

Households. There is a continuum of mass 1 of households. I focus on symmetric equilibria in which per capita and aggregate variables are the same. Hence, to simplify notation, I omit the superscripts i. Each household derives utility from consumption C_t and experiences disutility from labor effort L_t, where $L_t = L_t^T + L_t^N$. The lifetime utility of a generic household is given by:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right)\right]. \tag{C.6}$$

In this expression, $E_t[\cdot]$ is the expectation operator conditional on information available at time t and β is the subjective discount factor. The period utility function $U(\cdot)$ is assumed to be:

$$U\left(C_t, L_t\right) = \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\gamma} - 1}{1-\gamma},$$

with $\omega \geq 1$ and $\gamma \geq 1$.

Consumption is a CES aggregate of tradable C^T and non-tradable C^N consumption goods:

$$C_t = \left(\psi\left(C_t^T\right)^{1-\frac{1}{\xi}} + (1-\psi)\left(C_t^N\right)^{1-\frac{1}{\xi}}\right)^{\frac{\xi}{\xi-1}}, \tag{C.7}$$

with $\xi \geq 0$ and $0 \leq \psi \leq 1$. Household optimization implies that the domestic currency price of a unit of consumption basket is:

$$P_t = \left(\psi^{\xi} S_t + (1-\psi)^{\xi}\left(P_t^N\right)^{1-\xi}\right)^{\frac{1}{1-\xi}}, \tag{C.8}$$

where:

$$P_t^N = S_t \frac{1-\psi}{\psi}\left(\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\xi}} \tag{C.9}$$

As in the benchmark model, households have access to domestic and foreign bonds denominated in units of foreign currency. The budget constraint of household i in terms of the domestic currency can be written as:

$$P_t C_t + S_t\left(B_{t+1}^* + B_{t+1}\right) + Q_t(K_{t+1} - K_t) = W_t L_t + R_t^K K_t + S_t\left(R^* B_t^* + R_{t-1} B_t\right) + \Pi_t. \tag{C.10}$$

Moreover, households are subject to the collateral constraint:

$$-B_{t+1}^* \leq \kappa \frac{Q_t}{S_t} K_{t+1}. \tag{C.11}$$

Nominal wages are set at the start of the period. Optimal wage setting implies:

$$-E_{t-1}\left[U_L(C_t, L_t) L_t\right] = \frac{\sigma-1}{\sigma} W_t E_{t-1}\left[\frac{U_C(C_t, L_t)}{P_t} L_t\right]. \tag{C.12}$$

Once wages are set, households are willing to satisfy firms' labor demand as long as:

$$\frac{W_t}{P_t} \geq -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)}. \tag{C.13}$$

Given the pre-set wage and the realization of the productivity shock, each period the household chooses C_t, B_{t+1}^*, B_{t+1} and K_{t+1} to maximize the expected present discounted value of utility ($C.6$), subject to the budget constraint ($C.10$) and the collateral constraint ($C.11$). The optimality conditions are:

$$\frac{S_t U_C(C_t, L_t)}{P_t} = \beta R_t E_t\left[\frac{S_{t+1} U_C(C_{t+1}, L_{t+1})}{P_{t+1}}\right] \tag{C.14}$$

$$\frac{S_t U_C(C_t, L_t)}{P_t} = \beta R^* E_t\left[\frac{S_{t+1} U_C(C_{t+1}, L_{t+1})}{P_{t+1}}\right] + \mu_t \tag{C.15}$$

$$\mu_t\left(\kappa\frac{Q_t}{S_t}K_{t+1}+B^*_{t+1}\right)=0 \tag{C.16}$$

$$\frac{Q_t}{P_t}U_C(C_t,L_t)=\beta E_t\left[U_C(C_{t+1},L_{t+1})\frac{R^K_{t+1}+Q_{t+1}}{P_{t+1}}\right]+\frac{Q_t}{S_t}\kappa\mu_t. \tag{C.17}$$

Market clearing and equilibrium. Market clearing implies:

$$C_t+B^*_{t+1}=Y^T_t+R^*B^*_t. \tag{C.18}$$

$$C^N_t=Y^N_t \tag{C.19}$$

$$B^*_t=0 \tag{C.20}$$

$$L_t=L^T_t+L^N_t \tag{C.21}$$

$$K_t=K, \tag{C.22}$$

where the last condition derives from the assumption of a fixed endowment of land.

We are now ready to define a rational expectations equilibrium as a set of stochastic processes $\{Y^T_t, L^T_t, K_t, Y^N_t, L^N_t, W_t, P^N_t, R^K_t, C_t, C^T_t, C^N_t, P_t, Q_t, B^*_{t+1}, B_{t+1}, R_t, \mu_t\}^{\infty}_{t=0}$ satisfying $(C.1)$-$(C.5)$, $(C.7)$-$(C.9)$ and $(C.12)$-$(C.22)$, given the exogenous process $\{z_t\}^{\infty}_{t=0}$, the central bank's policy $\{S_t\}^{\infty}_{t=0}$ and initial conditions B^*_0 and z_{-1}.

Derivation of equations (23) **and** (24)**.** To derive equation (23) combine equations $(C.15)$ and $(C.17)$. Equation (24) is obtained by combining $(C.8)$ and $(C.9)$.

References

Bianchi, J. and E.G. Mendoza (2010) "Overborrowing, Financial Crises and Macro-prudential Taxes," NBER Working Paper No. 16091.

Coleman, W. J. (1990) "Solving the stochastic growth model by policy-function iteration," *Journal of Business Economic Statistics*, Vol. 8, No. 1, pp. 27–29.