

# WEALTH DISTRIBUTION AND HUMAN CAPITAL: HOW BORROWING CONSTRAINTS SHAPE EDUCATIONAL SYSTEMS

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## Abstract

This paper provides a theory of how the wealth distribution of an economy affects the optimal design of its educational system. The model features two key ingredients. First, agents are heterogeneous both in their ability and wealth levels, neither of which is observable. Second, returns to schooling depend on the ability-composition of agents attending each school tier, for example, because of choices of common curricula. An educational system is characterized by an assignment rule of agents to schools and by endogenous sizes of tiers. I find that a benevolent planner seeking to maximize economic efficiency implements “elitist” educational systems in economies with poor, borrowing-constrained, agents. Compared to the first best, the optimal solution features (i) relatively low-ability, rich agents selecting into higher education and (ii) higher education schools with less capacity. The same qualitative results obtain when only two commonly used instruments are available to the planner: school fees and exams. In addition, I show that economies with relatively tighter borrowing constraints rely more extensively on exams, and that agents performing better on exams are rewarded with lower school fees.

*Keywords:* Human Capital, Educational Systems, Inequality.

*JEL Classification:* I21, I23, J24, O15.

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# 1 Introduction

Educational systems in developing and rich countries differ in many respects. In particular, higher education in developing countries has lower attendance rates and lower educational achievement as measured by international tests. Moreover, access to higher education relies much more extensively on gate-keeping exams. A conventional view explaining these differences is that "...in many developing countries governments lack either the financial resources or the political will to meet their citizens' educational needs..."<sup>1</sup> These differences in the provision of human capital have lead many observers to emphasize the role of educational systems in developing countries as a means of generating and perpetuating the ruling elites (e.g., Engerman and Sokoloff (2000, 2002)).

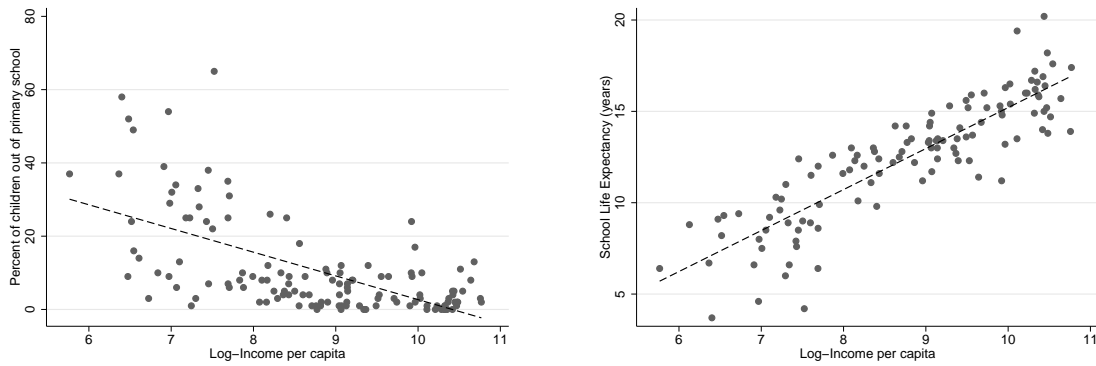
This paper presents an alternative theory for which these same differences in educational systems occur even when governments seek to maximize aggregate welfare. The purpose of this theory is to emphasize that, when there are borrowing-constrained agents with private information on their valuation of education and wealth, there are economic forces pushing benevolent governments to design seemingly elitist educational systems. I illustrate this by showing that, even when education can be provided at *no* cost, a benevolent social planner implements a system in which higher education features (i) low attendance rates, (ii) a dampened ability-composition of agents attending higher education, and (iii) an allocation process for higher education that relies extensively on gate-keeping exams and rationing for poor agents in the form of lotteries.

The intuition for this result stems from the fact that access to education is a source of rents for agents. The combination of borrowing constraints and private information makes it difficult to separate true valuations from willingness to pay. Thus, in order to separate low-ability, unconstrained agents from high-ability but constrained agents, the educational system adopts additional screening mechanisms. This results in the usage of lotteries for poor people and extensive reliance on gate-keeping exams to access higher education (because it is less costly for high ability types to prepare exams). However, these are imperfect screening mechanisms. Therefore, in equilibrium, the ability-composition of agents that select into higher education is worse and the capacity of the higher education tier is reduced relative to an economy without borrowing-constrained agents.

The three stylized facts presented above, which emerge as the solution of the planner's problem in the presence of borrowing constraints, are well-documented features of educational systems in developing countries. First, there is ample evidence on more extensive use of gate-keeping exams in developing countries relative to richer countries, especially in Africa and Latin America, (Al-Samarrai and Peasgood, 1998; Kellaghan and Greaney, 1992;

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<sup>1</sup>This excerpt is from an article by Hillman and Jenkner prepared for the IMF publication "Economic Issues", <http://www.imf.org/external/pubs/ft/issues/issues33/index.htm>



(a) Percent of children not enrolled in primary school in 2005. (Source: UNESCO)

(b) School life expectancy (in years) conditional on attending primary school. (Source: UNESCO)

Figure 1: Differences in School Enrollment

Kellaghan, 2004; Lockheed and Mete, 2007; Mete, 2004). For example, Kellaghan (2004) and Kellaghan and Greaney (1992) document that in most African countries, three (if not more) major examinations are required to complete secondary education.<sup>2</sup> Kellaghan emphasizes the role of examinations as gatekeepers and argues that this is reflected in the large numbers of students who fail exams and repeat their grade. This is consistent with UNESCO's data for 2005, which shows that the repetition rate at fifth grade (before accessing secondary education) in developing countries is 8.7% on average, versus 1.9% in rich countries.

Second, another difference between developing and rich countries is enrollment rates. Figure 1a shows that the fraction of children not enrolled in primary school is higher in low income countries. These differences in attendance rates are exacerbated as one moves forward in the education system. Figure 1b shows that the expected number of years children stay in school conditional on having some primary schooling is increasing in income per capita.<sup>3</sup>

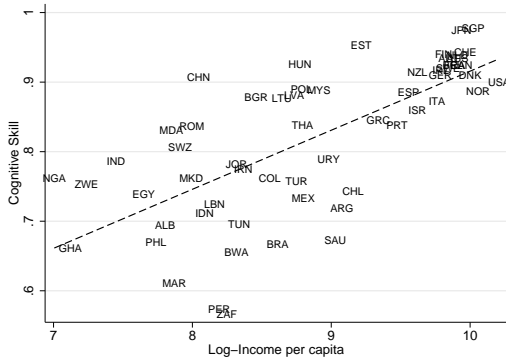
Third, differences in school quality are documented by Hanushek and Woessmann (2008, 2009) for a cross-section of countries. The authors construct an index based on results on a set of international tests to proxy for school quality and find significant cross-country differences.<sup>4,5</sup> Figure 2a shows how their measure of education quality is positively correlated

<sup>2</sup>These examinations are typically at the end of primary schooling, after two or three years in secondary school and around the end of secondary school. Kellaghan and Greaney (1992) document that in Francophone African countries students tend to be subject to even more exams. In particular, additional examinations are administered during primary school and, also, a competitive examination, termed the *concours*, is used to select pupils for the next education level.

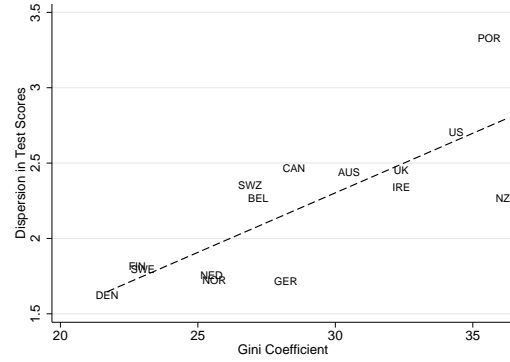
<sup>3</sup>The implicit assumption in this argument is that poorer countries have more borrowing constrained agents.

<sup>4</sup>Measures of school quality have been developed prior to these studies. For example, Hanushek and Kimko (2000) use a similar approach. Note that this concept of school quality differs from the approach of Barro and Lee (1993).

<sup>5</sup>Hanushek and Woessmann construct a cross-country comparable measure of acquired cognitive skills to proxy for education quality. The international student achievement tests they use include the following. Trends



(a) Cognitive Skill Score constructed from international tests and income per capita. (Source: Hanushek and Woessmann (2009))



(b) Education quality dispersion and inequality. Dispersion is the ratio 95th to 5th percentile of the average score of quantitative and prose in IALS. (Source: Nickell (2004))

Figure 2: Differences in School Quality

with income per capita. Nickell (2004) documents an additional correlation between wealth inequality and dispersion in school quality. Countries with a more unequal wealth distribution tend to have more dispersion in quality measures. This is shown in Figure 2b.<sup>6</sup>

Next, I discuss in more detail the main elements and results of the paper. The theory presented rests on two central elements. The first element is heterogeneity in agents' characteristics, ability and wealth, both of which are private information. The second element is the existence of complementarities in human capital formation across agents with the same level of education. A natural explanation for this complementarity is that the curriculum requirement of each education tier adjusts to students' ability.

In this context, an educational system is characterized by an assignment rule of agents to schools, and capacities of tiers. I characterize the educational system that a social planner would design and its decentralization under perfect capital markets and borrowing constraints. First, I show that in economies in which there are no borrowing constraints, private information alone does not prevent the educational system from being first best. In these economies, the educational system is meritocratic in the sense that agents are matched to different school tiers according to their ability. Moreover, the first best educational system can be decentralized through a market for schooling, even in the presence of private information.

in International Mathematics and Science Study, Programme for International Student Assessment and Progress in International Reading Literacy Study. In order to establish a baseline to compare the performance in different tests, the authors use the United States National Assessment of Education Progress. The reason is that this is the only test that has been administered consistently over a large period of time.

<sup>6</sup>Note that in this case the sample is limited to a particular international test in order to have a clear interpretation of the variance in the data. The method in Hanushek and Woessmann (2009) is not designed to generate comparable second moments.

Then, I turn to the study of the main object of interest of the paper: economies with borrowing constraints. Borrowing constraints generate a wedge between private valuation of education and ability to pay, as agents are constrained in the maximal transfer they can make. This distorts the matching of agents to schools because of the inability of agents to effectively signal their true valuations. I find that the optimal mechanism involves randomization in access to schooling for high-ability, poor agents, while high-ability, rich agents do not face any randomness in allocation. The capacity of the higher education tier is reduced relative to the economy without borrowing constraints. This is consistent with the evidence presented of low attendance rates in developing countries.

The comparative statics on wealth distribution show that in poorer countries the average ability of agents selecting into higher education is reduced. Thus, due to the complementarities in human capital formation, this endogenously reduces the human capital obtained in higher education in developing countries – which is consistent with worse performance in international tests. Changes in wealth dispersion have opposite effects depending on whether or not the median wealth type can afford higher education with certainty. If the original equilibrium features an allocation in which only agents above the median wealth can afford higher education without resorting to lotteries, an increase in wealth dispersion makes it optimal to restrict even further access to higher education, making the educational system more exclusive. Analogous comparative statics results obtain in the case that the social planner can only use school fees as instruments.<sup>7</sup>

Finally, I study an environment in which both school fees and a signaling technology (exams) can be used. There is a trade-off in using exams. They involve wasteful spending in order to be prepared, but allow for an additional screening mechanism because it is less costly for high ability agents to pass an exam. The optimal mechanism is such that agents that perform better in an exam are rewarded with a lower school fee. Thus, this mechanism resembles a scholarship scheme. The comparative statics on wealth distribution show that poorer countries use relatively more exams, and that exams are particularly used in the range in which there is more wealth inequality. These results fit with the evidence presented on extensive use of gate-keeping exams in developing countries.

**Related Literature** This paper emphasizes the role of asymmetric information and borrowing constraints to explain differences in the design of an educational system, and, ultimately, human capital provision. In this sense, while I focus on a different set of factors, this paper shares the approach of [Banerjee \(1997\)](#) and [Esteban and Ray \(2006\)](#) of focusing on asymmetric information and borrowing constraints to rationalize differences in provision of goods.

The paper relates to a rich and diverse literature on the determinants of human capital

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<sup>7</sup>In fact, Section 6 shows that if the social planner cannot commit to exclude some high-ability poor agents from education once they reveal their type, the only credible instrument the planner can use are school fees.

acquisition. To the best of my knowledge, however, this is the first attempt to provide a theory of the design of an optimal educational system that focuses on the role of private information and borrowing constraints in matching of agents to schools.

Fernández and Galí (1999) and Fernández (1998) are the closest papers in terms of the framing of the problem. They study a matching problem with borrowing constraints and compare two alternative mechanisms (prices and exams). This paper differs from theirs in that it takes a mechanism design approach, thus endogenizing the usage of different instruments and the size of tiers. Moreover, this paper provides comparative statics results on the wealth distribution. Another important difference is that, in this paper, educational standards are set endogenously. In this respect, Costrell (1994) and Betts (1998) provide alternative theories on the determinants of educational standards, but they emphasize political economy reasons rather than private information and borrowing constraints.

The problem of allocating heterogeneous agents to schools studied in this paper can be interpreted as an extension of the assignment Roy's model (Satterthwaite, 1993), in which private information and borrowing constraints are introduced. With the exception of the aforementioned work of Fernández and Galí, the literature has typically analyzed other imperfections in the assignment process. For example, Legros and Newman (2007) and Durlauf and Seshadri (2003) study conditions under which monotone matching obtains in environments with non-transferabilities and endogenous coalition sizes. In this paper, the complementarity between ability and school tier and the fact that agents appropriate all the surplus from the match makes positive assortative matching efficient.<sup>8</sup>

The mechanism design problem considered in the presence of borrowing constraints constitutes a bi-dimensional screening problem. This type of problem has been studied in auction design by Che and Gale (1998, 2000) and Lewis and Sappington (2000, 2001).<sup>9</sup> The problem analyzed in this paper differs in that the objective function of the principal is not to maximize profit but welfare, education is an indivisible good and there complementarities in payoffs across agents. These two features make the solution differ from these papers. For example there are bunching regions that would not be otherwise present. The conditions on the wealth distribution that I find for uniqueness of the solution using the first order approach in section 6 (MLRP and increasing hazard rate of the wealth distribution) are similar to the results derived in an auction setting without non-convexities in Che and Gale (2000) and Blackorby and Szalay (2007), respectively.

The optimal educational system implies a particular wage distribution and wage levels. The role of the educational system as a determinant of inequality, income per capita

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<sup>8</sup>Indeed, this is only a convenient simplification of reality (and a common benchmark used in the literature). There are examples of reasonable technologies, as Kremer and Maskin (1996), that exhibit complementarities and fail to satisfy positive assortative matching in general.

<sup>9</sup>Condoirelli (2009) and Che and Gale (2009) compare the performance of market and non-market mechanisms.

and growth has been studied by many authors. For example, Bénabou studies in a series of papers (Bénabou, 1993, 1996a,b) patterns of community formation and its implications for inequality and growth. In these papers he emphasizes the role of human capital formation and complementarities within types in the same community. This paper differs from Bénabou's in that the available mechanisms are endogenized and only a static problem is considered.<sup>10</sup> Finally, this paper abstracts from the role of taxation and financing of education. This has been analyzed, for example, in De Fraja (2002) , Fernández and Rogerson (2003) and Bénabou (2002).

The rest of the paper is organized as follows. Section 2 presents a detailed outline of the paper and summarizes the main results. Section 3 lays out the baseline model and Section 4 characterizes the first best educational system. Section 5 shows that under perfect capital markets, asymmetric information on agents' ability does not preclude the optimal mechanism to attain the first best educational system and how it can be decentralized. Section 6 studies how the educational system changes under borrowing constraints and presents the core results of the paper. Section 7 presents extensions of the baseline model to show the robustness of the results and Section 8 concludes.

## 2 Outline of the Paper

Section 3 lays out the baseline economic environment. As discussed before, agents are heterogeneous in both ability and initial wealth levels. These two characteristics are agents' private information. Agents obtain (linear) utility from a consumption good. The initial wealth endowment is in terms of this consumption good, so it can be consumed if desired. Additionally, final good is privately produced one-for-one with human capital and production cannot be observed by the planner.

Agents obtain human capital attending school. The human capital an agent obtains is jointly determined by its own ability, the school tier she attends and a spillover within the types that attend the same school tier. In the baseline model, these spillovers take the form of a "least common denominator", i.e., they are determined by the lowest ability type attending a school tier. A rationale for this is that curriculum requirements adjust to accommodate the lowest skill agent in a given school tier. Thus, this constitutes an endogenous margin of adjustment by which the curriculum taught at a given school tier, and ultimately, school quality, can differ from one economy to another. Moreover, there is a complementarity between ability and school tier: higher ability agents obtain relatively more human capital in higher tier schools. Finally, to focus solely on the matching problem of agents to schools, the

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<sup>10</sup>Epple and Romano (1998), Fernández and Rogerson (1996, 1998) and Glomm and Ravikumar (1992) provide analysis of alternative (exogenously given) educational systems and their effect on growth, inequality, community formation and the public-private provision dichotomy.



baseline environment abstracts from any costs of school provision.

In the baseline model, there are only two school tiers: basic and higher education.<sup>11</sup> An educational system is characterized by (i) an allocation rule that maps types of agents to school tiers and (ii) school tier capacities. The linearity in the utility function implies that the social planner seeks efficiency and abstracts from any redistributive concerns when designing the educational system. In other words, the social planner implements the educational system that maximizes aggregate consumption and, thus, final good production and aggregate human capital.

I study the social planner problem and its decentralization in a variety of environments, which are summarized in Table 1. Section 4 characterizes the first best educational system. In this case, the allocation rule of agents to schools depends exclusively on ability. The mechanism is such that agents announce their ability type and have to make a (negative) transfer conditional on the announced type. I refer to negative transfers as school fees in Table 1. As discussed before, given the linearity in consumption of the objective function, the goal of the social planner is to achieve efficiency in human capital production (because this maximizes final good production). Given that the marginal cost of school provision is zero, the educational system can be decentralized by setting a negative transfer (tax) conditional on school attended, so that the spillover is internalized. These results are summarized in line (1) of Table 1.

Section 5 shows that in the absence of borrowing constraints, private information alone does not preclude the social planner from achieving the first best educational system. This result obtains because there is single-crossing in the human capital production function. As a result, high ability agents have higher valuation of higher education and, thus, a simple school fee can implement the first best educational system with private information. Moreover, the educational system can be decentralized. Similar to the first best case, in the decentralized equilibrium, the spillover in human capital is priced using school-contingent taxes. Thus, the allocation of agents to schools in an environment with private information coincides with the first best. This is, lines (1) and (2) in Table 1 implement the same allocation of agents to schools.

Section 6 contains the main results of the paper. It studies an economy with an extreme form of borrowing constraints: financial autarky. This introduces a potential wedge between private valuation of education and ability to pay, as agents can be constrained in the maximal transfer they can make. I characterize the optimal schooling system in the presence of borrowing constraints in Subsection 6.1. Agents announce their type (ability and wealth) and are assigned to schools according to a probability rule and a transfer conditional on their (bi-dimensional) type. I show that conditional on an ability level, richer agents are offered a

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<sup>11</sup>A more general form of the complementarity, a model with education costs and more than two tiers are introduced in Section 7.



Table 1: Summary of the Cases Considered in the Paper

Problem	Planner Implementation	Underlying Space	Decentralization (prices and taxes)
(1) Perfect Information	School Fees	$\mathcal{A} \times S$	✓
(2) Private Information	School Fees	$\mathcal{A} \times S$	✓
(3) Private Information with Borrowing Constraints			
(3.1) Constrained Optimum	Lotteries	$\mathcal{A} \times \Phi \times S$	+ Wealth Market
(3.2) No Commitment	School Fees	$\mathcal{A} \times \Phi \times S$	✓
(3.3) Private Outside Option	School Fees	$\mathcal{A} \times \Phi \times S$	✓
(3.4) No Comm. w/ Exams	Fees, Exams	$\mathcal{A} \times \Phi \times \mathcal{T} \times S$	+ Scholarships

higher probability of accessing higher education. The intuition for the use of lotteries is that they effectively allow to relax borrowing constraints. By offering a lower school fee with a corresponding lower probability of access to school, the social planner ensures that this lottery generates the same ex-ante payoff as a certainty-equivalent transfer. The constrained optimal mechanism has two important features compared to the first best: it admits agents with lower ability into higher education and it reduces the total mass of agents accessing higher education. Given the spillover in human capital formation across agents, this implies an endogenous degradation of the human capital obtained for all agents attending higher education.

In a comparative statics exercise, I show that the degradation in higher education quality due the selection of low ability types into higher education increases in poorer countries.<sup>12</sup> The flip side of this result is that, conditional on an ability and wealth level, the probability of accessing higher education is higher in poorer countries. Thus, this resonates with two features of educational systems in developing countries presented earlier: lower school quality and lower capacity of higher education schools. Changes in wealth dispersion have opposite effects depending on whether or not the median wealth type can afford higher education with certainty. If the original equilibrium features an allocation in which only agents above the median wealth can afford higher education without resorting to lotteries, an increase in wealth dispersion makes it optimal to restrict even further access to higher education, making the educational system more exclusive. The converse is true if the original equilibrium features agents with income below the median being able to afford higher education with certainty. Finally, I show that the constrained optimal mechanism can be decentralized through a market for wealth, in which agents play lotteries with each other over their wealth.

The optimal mechanism with borrowing constraints requires the commitment of the so-

<sup>12</sup>An economy is defined as being poorer than another if the wealth distribution of the poorer economy is MLRP-dominated by the richer.

cial planner to exclude from higher education some poor, high-ability agents that engage in lotteries. Note that once agents that select into lotteries to access higher education have revealed their types, the social planner would like to modify the allocation rule ex-post and allocate *all* these agents into higher education. The reason is that they have an ability (weakly) higher than the lowest ability of unconstrained (rich) agents that attend higher education. If the social planner cannot commit to exclude some of these agents from attending higher education once they have revealed their type, then the only credible mechanism that can be used are school fees (without lotteries).<sup>13</sup> Subsection 6.2 shows that this environment delivers results that are qualitatively analogous to the environment with commitment. For example, the optimal educational system admits agents with lower ability into higher education institutions. The comparative statics results are also analogous to the case with commitment: there is endogenous degradation of human capital formation and less capacity of higher education institutions in poorer countries. Decentralization in this case does not require a market for wealth. A school-contingent tax in addition to prices of school suffices.

Finally, in this environment without commitment, Subsection 6.3 studies the design of the schooling system when the social planner has access to a signaling technology, which I interpret as exams. The usage of exams introduces a trade off: exams involve wasteful spending but allow for additional screening power because the cost of obtaining a particular score is decreasing in agents' ability. The optimal mechanism consists of a schedule of school fees and test scores contingent on agents' reported type. Agents that perform relatively better in an exam are rewarded with a lower school fee. Thus, the decentralization of the mechanism involves a set of taxes conditional on test scores that resemble a scholarship scheme.

When the exam technology is merely of a fail/pass-type, I show that the solution of the planner problem only makes use exams in sufficiently poor countries. The intuition for this result is simple. Once exams are put in place, all agents above a particular ability (regardless of whether they are rich or poor) take the exam. Thus, all agents that take the exam incur a wasteful spending. The benefit of using exams only comes through the additional mass of agents that access higher education thanks to exams. As a result, if borrowing constraints are not very severe, the additional screening power gained by using exams may be too costly to use. When the planner has access to a richer signaling technology, in which different test scores can be obtained, the comparative statics of the optimal schooling system on the wealth distribution show that, at any ability level, poorer countries use relatively more exams (i.e., require a higher score) and that the overall access cost to higher education (school fee plus exam cost) is lower in poorer countries. Similarly, for changes in the dispersion in the wealth distribution, I show that exams are relatively more used at the levels where inequality in-

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<sup>13</sup>If instead of no-commitment, agents have limited communication and cannot announce their types, then only a singleton can be used to allocate agents to higher education. If the planner has also limited communication, then only fees can be used.

creases.

Section 7 relaxes some of the assumptions of the baseline model and shows that the main insights derived from the baseline model hold in more general set-ups. In Subsection 7.1, I allow for more general spillovers (for example, the case in which the spillover is determined by the average type attending a school) and a CES production function for the final good. Subsection 7.2 considers the case in which, within each school tier, there are a continuum of sub-tiers. This extension allows for more robust predictions on the mass of agents attending schools. Moreover, it provides a simple framework to study how the presence of an unregulated private provider of schooling may constrain the planner's problem. I show that the existence of a private unregulated sector undermines the capacity of the social planner to provide education to borrowing constrained agents.

### 3 The Environment

This section describes the fundamentals of the economies studied in the paper.

*Endowments.* The economy is populated by a unit mass of agents. Each agent is endowed with ability,  $a \sim G(a)$ , and initial wealth,  $\phi \sim F(\phi)$ . Initial wealth is distributed over the support  $\Phi = [0, \bar{\phi}]$  with cumulative distribution function  $F(\phi)$ , and associated density  $f(\phi)$ . The upper bound on wealth,  $\bar{\phi}$ , can be either finite or infinite. Ability is distributed uniformly over a support  $\mathcal{A} = [\underline{a}, \bar{a}] \subseteq [0, 1]$ . Ability and wealth are uncorrelated across agents and are private information.

*Technologies.* In this economy, two technologies are used: a final good and a human capital production function. Final good is produced one-for-one with human capital  $H$ . Human capital is produced by the schooling system. There are  $S$  school tiers in the economy  $s = \{0, 1, \dots, S-1\}$ , with associated capacities  $c(s)$ . School 0 provides the minimal, mandatory, level of education required for all the population, while schools  $s > 0$  provide further education. The two main cases of analysis in the paper will be case of two and infinite schools. The marginal cost of school provision per student is  $\kappa(s)$ .

An agent with ability  $a$  attending school tier  $s$  obtains human capital  $H(a, s)$ , which is determined by the combination two different factors,  $A(s)$  and  $h(a, s)$ , according to

$$H(a, s) = A(s)h(a, s). \quad (1)$$

The first factor is an intrinsic human capital production function  $h(a, s)$  associated with each school tier. This production function is (weakly) increasing and concave in ability  $a$  and (weakly) increasing in school index  $s$ . There is complementarity between schools and ability. Let  $s > \tilde{s}$ , then

$$\frac{\partial(h(a, s) - h(a, \tilde{s}))}{\partial a} \geq 0 \quad \text{for all } s, \tilde{s} \in S.$$

This means that high ability agents benefit relatively more from high index schools.

In addition to the intrinsic human capital production function, there is an spillover at each school tier level,  $A(s)$ . In the baseline model, this is modeled as an extreme complementarity between types

$$A(s) = \min_a \{a \in s\}. \quad (2)$$

This can arise because the social planner cannot commit to exclude from education students attending each education tier, and has to accommodate the curriculum level of each tier to the ability of its students. With this interpretation in mind, it is natural to have the lowest ability student attending a particular tier determining the spillover effect, because the curriculum level of each tier has to adjust to its “least common denominator.” This implies that if the spillover component,  $A(s)$ , differs across countries, the human capital obtained by an agent of ability  $a$  attending school tier  $s$  can differ across countries. It is in this sense that the model rationalizes differences in education quality across countries. In Section 7.1, I discuss how the results extend to more general type of spillovers, which include, among others, the mean type attending school  $s$ , rather than the minimum.

*Preferences and Aggregate Welfare.* An agent chooses actions so as to maximize her utility from consumption. Utility is linear in consumption and equals wage income, plus initial wealth and a possible lump-sum transfer from the government, minus any expenditures incurred to educate. Aggregate welfare is defined as the sum of the utilities of the agents in the economy. Throughout the paper, the social planner is assumed to be utilitarian and having as objective to maximize aggregate welfare. The linearity of the objective function implies that the social planner is concerned only by production efficiency and abstracts from any distributional consideration. Moreover, as the only input for production is human capital, this implies that the social planner objective is to maximize aggregate human capital and, thus, implement a schooling system based on efficiency considerations alone.

From section 4 to 6, I analyze the case in which there are two school tiers,  $S = 2$ . The main insights from the paper are obtained by this simple two-tier school model. As discussed before, the lower tier,  $s = 0$ , represents the basic or mandatory schooling, while  $s = 1$  represents additional, non-compulsory education. Moreover, to highlight the frictions arising from private information, the cost of provision is assumed to be zero,  $\kappa(s) = 0$ . For the sake of brevity, I refer to  $s = 1$  as higher education. Section 7 extends the model to allow for a richer production function, more general spillovers and a continuum of tiers within basic and higher education.

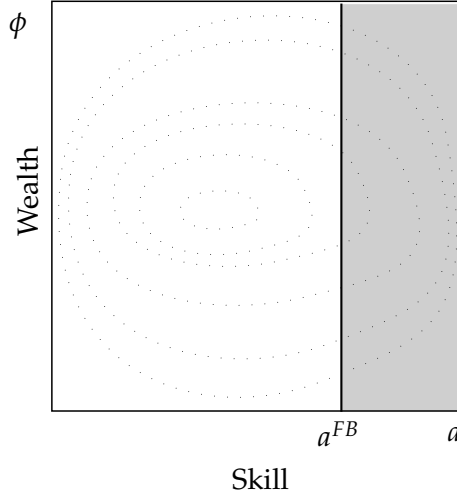


Figure 3: First Best Allocation. Agents are segregated by skill only.

## 4 First Best

In this section, I characterize the optimal schooling system when there is no private information and assignments to basic ( $s = 0$ ) and higher education ( $s = 1$ ) can be made contingent on types. Given the complementarity between skills and schools, segregation by skill is optimal. To see this, consider two agents with abilities  $a_1$  and  $a_0$  with  $a_1 > a_0$ . It cannot be optimal that agent 1 is in school 0 and agent 0 in school 1, as  $H(a_1, 1) - H(a_0, 0) > H(a_1, 0) - H(a_0, 1)$ . Given that it is mandatory to provide basic education to all agents in the economy, the problem of the social planner is to choose the lowest ability  $\tilde{a}$  in  $s = 1$ ,

$$\max_{\tilde{a}} \int_{\tilde{a}}^{\tilde{a}} \tilde{a} h(a, 1) dG(a) + \int_{\underline{a}}^{\tilde{a}} \underline{a} h(a, 0) dG(a),$$

which has as implicit solution<sup>14</sup>

$$\underbrace{\int_{a^{FB}}^{\tilde{a}} h(a, s) da}_{\text{spillover benefit}} = \underbrace{a^{FB} h(a^{FB}, 1) - \underline{a} h(a^{FB}, 0)}_{\text{marginal output cost}}. \quad (3)$$

The social planner balances the spillover (quality) effect that is improved by increasing the ability of the marginal type attending school 1, with the reduction in the mass of agents that attend school 1. Note that the allocation is independent of agents' wealth, see Figure 3.

Conceivably, it could be the case that it would be optimal to have all agents attending higher education. For example, this could happen if the differences in ability in the popula-

<sup>14</sup>Sufficiency of the First Order Condition is shown in Appendix 10.

tion where low (imagine the extreme case in which everybody has the same ability). This is a pathological case of no interest for the discussion as, in this case, no theory of educational systems would be needed. So, in what follows, I shall focus the discussion on the cases in which both, basic and higher education coexist. Simple conditions that would ensure that both school tiers are used are that either  $\underline{a} = 0$  or  $h(a, 0) = 0$ .

## 5 Private Information without Borrowing Constraints

In this section, I show that private information alone does not preclude the social planner from achieving the first best educational system.<sup>15</sup> As in [Fernández and Galí \(1999\)](#), I assume perfect capital markets and that the interest rate paid by agents is constant and normalized to one. This market operates when the educational system is put in place, so that agents, if they desire to, can borrow and repay after production. Trades in this market cannot be monitored by the planner.

The planning problem now is constrained by the fact that the assignment to schools cannot be conditioned directly on agents' ability. The planner problem is divided in two stages. First, the social planner announces an assignment rule and school tier capacities. Second, the economy unfolds: conditional on the educational system chosen in the first stage, agents decide which school to attend (borrowing if necessary) and obtain human capital. Then, they supply their human capital in a competitive labor market, obtain a wage, repay debt (if any) and consume. Given the linearity of utility, the only concern of the social planner is to achieve efficiency by matching agents to schools. Thus, the attention is restricted to the design of the educational system.

Before proceeding, and to avoid the discussion of many cases, I shall make the expositional assumption that there are always "rich enough" agents in the economy.

**Assumption 1** (*Expositional simplifying assumption*) *There is a positive mass of rich agents that can afford paying for extreme segregation,*

$$\bar{\phi} > \bar{a}h(\bar{a}, 1) - \underline{a}h(\bar{a}, 0).$$

This assumption implies that the spillover effect  $\bar{a}$  in school  $s = 1$  is pinned down by the choice of unconstrained agents for any  $a \in \mathcal{A}$ .

In this environment, using school fees as an allocation device is enough to achieve the first best allocation. The reason is that agents' choices satisfy a single-crossing condition. Let  $\psi_s$  denote the school fee of attending school  $s$ , the indirect utility  $u(a) \equiv \max_s H(a, s) - \psi_s$  is

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<sup>15</sup>This result is analogous to [Fernández and Galí \(1999\)](#) for the case in which the spillover effect  $A(s)$  is shut down, i.e.,  $A(s) = 1$  for all  $s \in S$ .

increasing in  $a$ . Moreover, the social planner cannot exclude agents from school 0 ( $\psi_0 = 0$ ). As a result, the choice variable of the planner is to choose fee  $\psi_1 \equiv \psi$  for school 1. The social planner problem can be written as

$$\max_{\psi} \int_{\tilde{a}(\psi)}^{\bar{a}} \tilde{a}(\psi) h(a, 1) dG(a) + \int_{\underline{a}}^{\tilde{a}(\psi)} \underline{a} h(a, 0) dG(a), \quad (4)$$

where the marginal type obtaining higher education  $\tilde{a}$  is implicitly defined by

$$\tilde{a} h(\tilde{a}, 1) - \underline{a} h(\tilde{a}, 0) = \psi. \quad (5)$$

Note that the fees incurred by agents to attend higher education are just transfers and, as such, they are not wasted. The revenue that the social planner obtains from the transfers is redistributed ex-post back to agents. The specifics of the redistribution do not matter given the linearity of utility. For concreteness, I assume that revenue is redistributed back to agents in a lump-sum manner.

**Proposition 1** *The marginal type obtaining education  $\tilde{a}(\psi)$  is strictly increasing in  $\psi$  for all  $\psi \in \Psi = (\underline{a}(h(\underline{a}, 1) - h(\underline{a}, 0)), \bar{a} h(\bar{a}, 1) - \underline{a} h(\underline{a}, 0))$ .*

*The First Best schooling system can be implemented under private information by setting a school fee of  $\psi^{PF} = H(a^{FB}, 1) - H(a^{FB}, 0)$  to agents attending  $s = 1$ , where  $a^{FB}$  is implicitly defined in (3).<sup>16</sup>*

This proposition implies that there is a one-to-one mapping from the school fee  $\psi$  (in the relevant margin) to the marginal type selecting into higher education,  $\tilde{a}(\psi)$ . Using this property, it follows that the First Best schooling system can be implemented by setting a school fee of  $\psi^{PF} = H(a^{FB}, 1) - H(a^{FB}, 0)$ . Section 7 shows that this result extends to the case of  $S > 2$  schools. Indeed, if there was no complementarity within agents in higher education, it would be optimal to make everyone attend higher education.

*Decentralization.* Next, I discuss how the optimal educational system can be decentralized. One can imagine the social planner running a procurement auction for both basic and higher education tiers, and there being a competitive pool of firms willing to enter the market. Firms would undercut each other and this would result in firms willing to supply education at its marginal cost, which is zero in this case. A price of education equal to zero coincides with the optimal school fee for basic education. For higher education, the social planner would set a tax contingent on attending higher education equal to  $H(a^{FB}, 1) - H(a^{FB}, 0)$ . This ensures that the demand of schooling from the agents coincides with the first best allocation of agents to schools. Thus, this discussion shows how the first best educational system can be decentralized in the presence of private information.

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<sup>16</sup>All proofs can be found in Appendix 10.



## 6 Private Information with Borrowing Constraints

This section analyzes an environment without perfect capital markets. The capital market imperfection studied is an extreme one: financial autarky. This is, capital markets are shut down entirely and agents have to self-finance their investments in education. Self-financing can be interpreted literally, or as the effective disposable wealth agents have to finance education after exhausting any potential way they have to obtain financing.

This section starts studying the mechanism design problem assuming that the social planner can commit to randomization. As I show below, this means that even though some agents reveal to be of high ability type to the social planner and “deserving” to attend higher education in the sense that agents with the same or even less ability attend higher education, it is optimal for the social planner *not* to let them obtain education. Then, Section 6.2 studies the case in which the social planner cannot commit to exclude from education all agents that signal to have high ability. I show that this is isomorphic to a world in which no communication is possible, and the optimal mechanism used to allocate agents to schools consist of just school fees. Finally, I allow for an exam technology to relax the commitment (or communication) problem. The exam technology gives an additional screening mechanism to the social planner and allows the planner to make the schooling allocation contingent on exam performance.

### 6.1 The Mechanism Design Problem

This subsection studies the design of the optimal educational system when agents are borrowing constrained. By the revelation principle, I restrict attention to direct-revelation mechanisms in which each type has an incentive to report private information truthfully. This constitutes a bi-dimensional screening problem, as both ability  $a$  and wealth  $\phi$  are private information. The mechanism specifies a type-contingent transfer  $t(a, \phi) \in \mathbb{R}$  and probability  $\pi(a, \phi) \in [0, 1]$  of attending higher education. Formally, a mechanism is a mapping from the type space to the transfers and probability space,  $\langle t, \pi \rangle : \mathcal{A} \times \Phi \rightarrow [-\infty, \infty] \times [0, 1]$ . Any feasible mechanism has to satisfy participation and incentive compatibility constraints for all agents, in addition agents’ transfers cannot exceed their wealth (borrowing constraint) and the social planner has to satisfy a break-even constraint.

Let  $w(a, \phi) = \pi(a, \phi)H(a, 1) + (1 - \pi(a, \phi))H(a, 0)$  denote the expected return or wage of an agent of type  $(a, \phi)$ . With this notation at hand, the participation constraint can be stated as

$$w(a, \phi) + t(a, \phi) \geq 0, \quad \forall (a, \phi). \quad (6)$$

Note that the participation constraint is equivalent to the social planner being constrained to supply at least basic education to all agents in the economy. The incentive compatibility

constraint is

$$w(a, \phi) + t(a, \phi) \geq \pi(\tilde{a}, \tilde{\phi})H(a, 1) + (1 - \pi(\tilde{a}, \tilde{\phi}))H(a, 0) + t(\tilde{a}, \tilde{\phi}), \quad \forall (a, \phi) \text{ and } (\tilde{a}, \tilde{\phi}). \quad (7)$$

The borrowing constraint implies that

$$\phi + t(a, \phi) \geq 0 \quad \forall (a, \phi). \quad (8)$$

Finally, note that if the social planner could use negative transfers, it would like to do so to subsidize education and overcome the borrowing constraints. Thus, a budget constraint condition needs to be imposed. As the planner has no access to additional resources, a natural benchmark is that no subsidization is possible. That is, transfers are restricted to be negative,<sup>17</sup>

$$t(a, \phi) \in \mathbb{R}_-. \quad (10)$$

The social planner problem consists on finding the schedule of transfers  $t$  and associated probabilities  $\pi$  that maximize the objective

$$\int_{\underline{a}}^{\bar{a}} \int_0^{\bar{\phi}} [\pi(a, \phi)a^*h(a, 1) + (1 - \pi(a, \phi))\underline{a}h(a, 0)] dG(a)dF(\phi), \quad (11)$$

subject to (6), (7), (8) and (10), where  $a^*$  is defined by the lowest ability agent attending higher education,  $a^* = \min_a \{a \in s = 1\}$ .

**Proposition 2** (*Optimal Schedule*) *The optimal mechanism featuring agents of ability  $a \geq a^*$  in higher education takes the form of a higher education fee menu  $t(\phi), \pi(\phi)$ ,*

$$t(a, \phi) = \begin{cases} -\psi & \text{for } \phi \geq \psi, a \geq a^*, \\ -\phi & \text{for } \phi < \psi, a \geq a^*, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \pi(a, \phi) = \begin{cases} 1 & \text{for } \phi \geq \psi, a \geq a^*, \\ \frac{\phi}{\psi} & \text{for } \phi < \psi, a \geq a^*, \\ 0 & \text{otherwise,} \end{cases}$$

with  $\psi = a^*h(a^*, 1) - \underline{a}h(a^*, 0)$ .

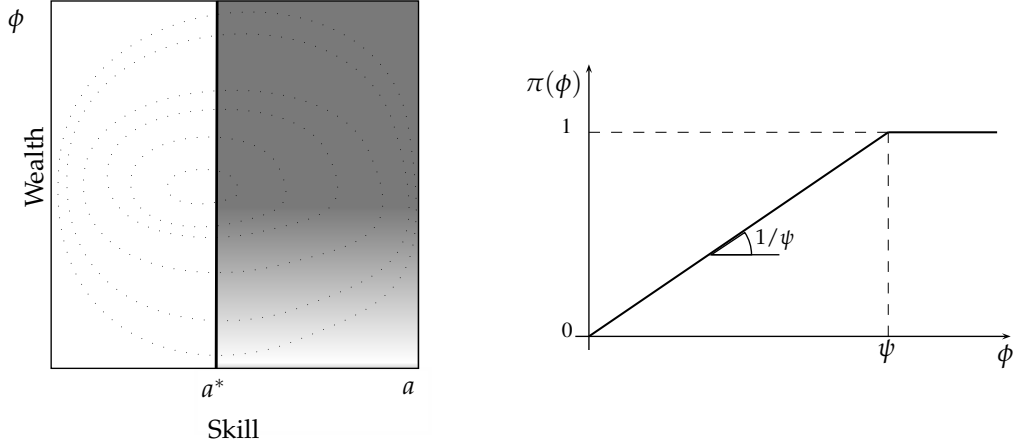
The formal proof of the result can be found in Appendix 10. Here I provide a sketch of the proof. Consider the case in which, due to spillovers in human capital production, the social planner wants to segregate agents in the two school tiers. In this case, the social planner has

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<sup>17</sup>Section 7 and Appendix 9 analyze an alternative setup in which the social planner can do “simultaneous” redistribution and has only to break even on net,

$$\int_{\underline{a}}^{\bar{a}} \int_0^{\bar{\phi}} t(a, \phi) dG(a)dF(\phi) \leq 0, \quad (9)$$

and show that the same qualitative results hold.



(a) The probability of accessing higher education is increasing in wealth.

(b) Optimal probabilities of attending higher education for different wealth levels.

Figure 4: Constrained efficient allocation

to choose the spillover level of higher education (i.e., the threshold type  $a^*$ ). For a given  $a^*$ , it would be optimal (in the first-best sense) to have all agents with  $a \geq a^*$  attending higher education. However, borrowing constraints prevent doing so because the transfer level needed to separate agents according to their willingness to pay is too high for some agents (i.e., the borrowing constraint is binding for them). Note, however, that the constrained agents cannot afford any mechanism that involves paying a fee higher than their wealth. In other words, rich agents can always claim to be poor, but not the other way around. As a result, the optimal mechanism can offer to rich, unconstrained agents the “best deal”, which is to attend higher education with probability 1, provided that they are of ability greater than  $a^*$ . This deal does not affect credit-constrained agents because they cannot afford it. For the credit-constrained agents, then, the social planner provides a lottery that offers the highest possible probability to access higher education (provided agents are of ability higher than  $a^*$ ). This lottery is decreasing in agents wealth to ensure incentive compatibility. In short, the fact that poorer agents cannot afford better lotteries compartmentalizes the problem into layers of wealth. The allocation patterns of the mechanism are represented graphically in Figure 4.

Proposition 2 allows to rewrite the original planner’s problem (11) as an essentially uni-dimensional optimization problem. For simplicity, I work with the negative of the transfers, which I refer to as fees. The objective function of the planner can be expressed now as

$$\max_{\psi} \int_{\psi}^{\bar{\phi}} \int_{a^*(\psi)}^{\bar{a}} \Delta w(a, a^*) dG(a) dF(\phi) + \int_0^{\psi} \int_{a^*(\psi)}^{\bar{a}} \pi(\phi, \psi) \Delta w(a, a^*) dG(a) dF(\phi), \quad (12)$$

subject to  $\psi = a^*h(a^*, 1) - ah(a^*, 0)$ , where I have used the notation  $\Delta w(a, a^*) = a^*h(a, 1) - ah(a, 0)$ . The case of interest is when the problem has an interior solution, i.e., it is optimal to have some agents with just basic education. In this case, the first order condition is<sup>18</sup>

$$(1 - F(\psi)) \left( -\frac{\partial a^*}{\partial \psi} \Delta w(a^*, a^*) g(a^*) + \int_{a^*}^{\bar{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) \right) + \int_0^\psi \left( -\frac{\partial a^*}{\partial \psi} \frac{\phi}{\psi} \Delta w(a^*, a^*) g(a^*) + \int_{a^*}^{\bar{a}} \frac{\partial}{\partial \psi} \left( \frac{\phi}{\psi} \Delta w \right) dG(a) dF(\phi) \right) = 0. \quad (13)$$

From this expression, it is apparent that the optimal solution balances costs and benefits of raising tuition fees. The cost of raising the tuition fee is that it reduces the number of agents attending higher education. This effect is captured in the first and third terms of (13), as the threshold type  $a^*$  moves with  $\psi$ . Moreover, this appears in the decrease in the probability of accessing higher education for borrowing-constrained agents in the last term of (13) (i.e.,  $\phi/\psi$  being decreasing in  $\psi$ ). The benefit of higher education fees comes through the spillover effect. By increasing  $\psi$ , the spillover effect increases and, thus, makes all agents that attend higher education obtain more human capital.

To investigate the effect of wealth distribution on the optimal schedule, condition (13) can be rewritten so that the effect of the wealth distribution is encapsulated in one term,

$$\frac{\frac{1}{\psi} \int_0^\psi \pi(\phi, \psi) dF(\phi)}{1 - F(\psi)} = \frac{\int_{a^*(\psi)}^{\bar{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) - \frac{\partial a^*}{\partial \psi} \Delta w(a^*) g(a^*)}{\int_{a^*(\psi)}^{\bar{a}} \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right) dG(a) + \frac{\partial a^*}{\partial \psi} \psi \Delta w(a^*) g(a^*)}. \quad (14)$$

The left hand side of equation (14) contains all the influence of the wealth distribution on the optimal solution. If there were no borrowing constraints this term would be zero, and the optimal  $\psi$  would be given by equating the numerator of the right hand side to zero (which coincides with the first best first order condition, equation 3).

To gain intuition on how the wealth distribution affects the optimal fee  $\psi$  (i.e., the left hand side of (14)), suppose that no lotteries were used. In this case, the probability of attending higher education would be given by a step function,  $\pi(\phi, \psi) = 0$  for  $\phi < \psi$  and  $\pi(\phi, \psi) = 1$  for  $\phi \geq \psi$ . This would simplify the left hand side to the hazard-rate of the wealth distribution. In this case what would only matter in making the trade-off between the mass of agents attending higher education and the spillover effect is the percent increase in the mass of agents that can afford education at the margin,  $f(\psi)$ , relative to the total mass of agents educating,  $1 - F(\psi)$ . Now, going back to the original formulation, rather than just having the density at  $\psi$ , thanks to the lottery, the mechanism can include some borrowing constrained agents with wealth  $\phi < \psi$ . Thus, the social planner considers a weighted average of the mass of agents that can attend higher education at different levels of wealth. To

<sup>18</sup>The derivation of the first order condition and the proof of its sufficiency is shown in Appendix 10.

see that, note that  $\int_0^\psi \pi(\phi, \psi) dF(\phi)$  computes the total mass of constrained agents attending higher education, while the term  $1/\psi$  in front, implies that the planner takes into account the average value of the integral relative to the total mass of unconstrained agents  $1 - F(\psi)$ .

The behavior of the left hand side term of (14) as a function of  $\psi$  can be, in general, non-monotonic. However, it is monotonically increasing for log-concave functions that have support in  $[0, \infty)$  such as the Weibull, Exponential, Gamma (with shape parameter greater than one) and for some other distributions such as the Uniform and the Pareto distribution with well defined mean and variance (i.e., with shape parameter, greater than 2). Given that the left hand side term is monotonically increasing for the most usual distributions used to model wealth distributions (except for the Log-Normal, for which is non-monotonic), I restrict my attention to wealth distributions that generate an increasing left hand side term.

The right hand side of equation (14) is the ratio of the net marginal gain in output if all agents were unconstrained relative to the net gain in output due to spillover gains evaluated at the average-constrained agent.<sup>19</sup> In Appendix 10, I show that under mild conditions on the intrinsic production function  $h(a, s)$ , the right hand side of (14) is strictly decreasing. This is intuitive, the numerator captures the net marginal gain from segregation (as in the first best), and it is decreasing in  $\psi$  because of the concavity in the human capital production function. On the contrary, the denominator is increasing in  $\psi$  because the quality of the higher education sector increases with the spillover. Thus, the right hand side of (14) is decreasing in  $\psi$ .

### 6.1.1 Comparative Statics on the Wealth Distribution

With the previous discussion at hand, I begin to study the main question of the paper: how optimal educational systems change with shifts in the wealth distribution? To do so, I introduce a one-dimensional ranking of wealth distributions that is amenable to our purposes.

**Definition** (Wealth abundance) Consider two wealth distributions,  $\tilde{F}$  and  $F$ , with associated densities  $\tilde{f}$  and  $f$ . A distribution  $\tilde{f}$  is more wealth abundant than  $f$ , denoted by  $\tilde{f} \succ_w f$ , if  $\tilde{f}(\phi_1)f(\phi_0) \geq \tilde{f}(\phi_0)f(\phi_1)$  for all  $\phi_1 > \phi_0$ .

The interpretation of the notion of wealth abundance is intuitive. For non-vanishing values of the density, the wealth abundance condition can be written as

$$\frac{\tilde{f}(\phi_1)}{f(\phi_1)} \geq \frac{\tilde{f}(\phi_0)}{f(\phi_0)}.$$

<sup>19</sup>The denominator is always positive. This follows from the concavity of  $\Delta w$ , for which Appendix 10 provides sufficient conditions. A concave function satisfies the following property (c.f. Varian (1992))  $\Delta w(0) + \Delta w'(\psi)\psi \leq \Delta w(\psi)$ . Note that if  $\psi = 0$ ,  $a^* = \underline{a} = 1$  and  $\Delta w(0) = \underline{a}(h(a, 1) - h(a, 0)) \geq 0$ . Using this result with  $\Delta w'(\psi) > 0$ , it follows that  $\Delta w'(\psi)\psi \leq \Delta w(\psi)$  for all  $a$ .

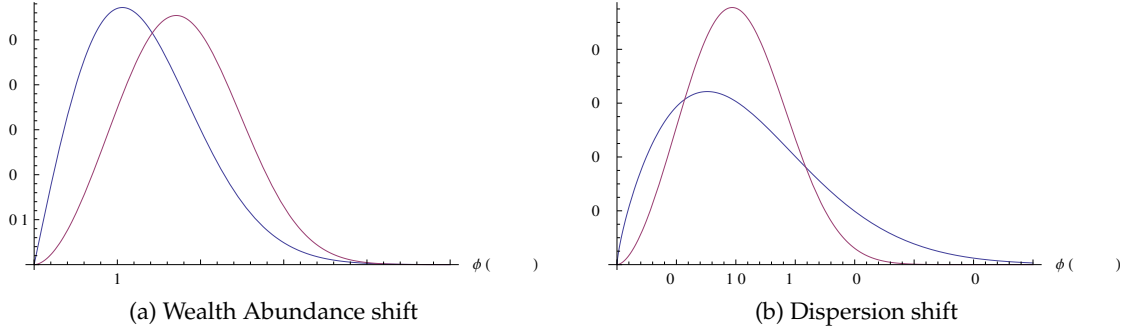


Figure 5: Wealth Abundance and Dispersion Shift of a Weibull Distribution

This means that if one economy is more wealth abundant than another, there are relatively more rich agents in this economy when comparing any arbitrary two wealth levels,  $\phi_1$  and  $\phi_0$ .<sup>20</sup> This notion of wealth abundance requires that the two distributions satisfy a Monotone Likelihood Ratio Property (MLRP). Figure 5 provides a graphical intuition. Having two distributions ranked according to MLRP implies both hazard rate and first order stochastic dominance between these two distributions.

**Remark** Consider two wealth distributions,  $\tilde{F} \succ_w F$ , then

$$\frac{\frac{1}{\psi} \int_0^\psi \phi f(\phi) d\phi}{1 - F(\psi)} \geq \frac{\frac{1}{\psi} \int_0^\psi \phi \tilde{f}(\phi) d\phi}{1 - \tilde{F}(\psi)}.$$

*Proof:* Rewrite the previous expression as  $(\psi(1 - F(\psi))(1 - \tilde{F}(\psi)))^{-1} \int_0^\psi \phi (f(\phi)(1 - \tilde{F}(\psi)) - \tilde{f}(\phi)(1 - F(\psi))) d\phi$ . Thus, a sufficient condition for the inequality condition to hold is that  $f(\phi)(1 - \tilde{F}(\psi)) - \tilde{f}(\phi)(1 - F(\psi)) \geq 0$ , where  $\phi \leq \psi$ . Now, I show that this sufficient condition is implied by the definition of wealth abundance. From the definition of wealth abundance,  $\tilde{f}(\phi_1)f(\phi_0) \geq f(\phi_1)\tilde{f}(\phi_0)$  for all  $\phi_1 > \phi_0$ . Integrating both sides of the inequality from  $\phi_1 = \psi \geq \phi_0$  up to  $\tilde{\phi}$ ,

$$\int_\psi^{\tilde{\phi}} \tilde{f}(\phi_1)f(\phi_0) d\phi_1 \geq \int_\psi^{\tilde{\phi}} f(\phi_1)\tilde{f}(\phi_0) d\phi_1,$$

this implies that  $(1 - \tilde{F}(\psi))f(\phi_0) \geq (1 - F(\psi))\tilde{f}(\phi_0)$  for all  $\phi_0 \leq \psi$ .  $\square$

With this definition at hand, I proceed to do comparative statics on the wealth distribution in terms of wealth abundance to derive the first main result of the paper.

**Proposition 3** Consider a wealth abundance shift,  $\tilde{F} \succ_w F$ . The optimal maximal fee  $\psi$  under  $\tilde{F}$  is higher than under  $F$ .

The results follows immediately from the left hand side of equation (14) being increasing, the right hand side being decreasing and the observation that a Wealth Abundance shift

<sup>20</sup>This notion of abundance is analogous to the skill abundance notion in Costinot and Vogel (2010).

only moves downwards the left hand side of equation (14).

Proposition 3 implies that more wealth abundant countries have higher ability agents in higher education because the threshold type  $a^*$  is strictly increasing in  $\psi$ . That is, agents accessing higher education in more wealth abundant economies have higher ability on average. Thus, the ability-composition of higher education is better in more wealth abundant economies. Moreover, the level of randomization at all wealth levels is smaller in the wealth abundant country  $\pi_{\bar{f}}(\phi) \leq \pi_f(\phi)$  for all  $\phi$ . This implies that it is *less likely* for borrowing constrained agents to access higher education in relatively more wealth abundant countries. Indeed, there are less agents constrained in wealth abundant economies, so the relative cost of not providing them the right type of education is relatively low.

Another consequence of this result is that the the schooling system in wealth abundant countries amplifies the dispersion in the earnings distribution compared to less wealth abundant countries. To see this, note that the equilibrium spillover level is increasing in wealth abundance. Thus, conditional on accessing higher education, wealth abundant countries generate more human capital for an agent of a given ability. However, access to higher education happens at higher levels of ability in more wealth abundant countries.

The previous discussion highlights another dimension of the effect of borrowing constraints: the mismatch of ability to schools. With borrowing constraints, there is an increasing mass of mismatched agents. That is, higher ability agents that have to attend basic education because of credit constraints. This results in a change in the ranking of earnings of agents relative to the first-best. As an economy becomes less wealth abundant, high-ability low-wealth agents tend to fall in the ranking at the expense of low-ability high-wealth agents who rise.

I now provide a comparison of the mass of agents that are being educated in environments with and without borrowing constraints.

**Proposition 4** *There is a reduction in the mass of agents obtaining higher education when the economy transitions from no agents being effectively borrowing constrained to a (small) mass of agents being borrowing constrained.*

This result is intuitive. As the ability of agents to express their valuations of attending higher education is hindered, the social planner finds better to reduce the capacity of higher education. Note however that this is only true around the transition from no borrowing constrained agents to a small mass of borrowing constrained agents. The reason why the result does not hold for all levels of borrowing constraints is that benefits from the spillover are traded-off against the mass of agents attending higher education. With two school tiers, if borrowing constraints are very prevalent, it could be the case that it is better to reduce the spillover effect to be able to admit more students. In Section 7.2, I show that when there are more school layers, tighter results are obtained. In particular, I show that the result is true



at all levels of borrowing constraints for a positive measure of school tiers that contains the highest level school.<sup>21</sup>

A related question that can be investigated is how changes in wealth dispersion affect the educational system.

**Definition** (Wealth dispersion) Let  $\phi_m$  denote the median wealth of an economy. A distribution with density  $\tilde{F}$  has more wealth dispersion, denoted by  $\tilde{F} \succ_d F$ , if and only if  $\tilde{F}(\phi) \succ_w F(\phi)$  for  $\phi > \phi_m$  and  $F(\phi) \succ_w \tilde{F}(\phi)$  for  $\phi < \phi_m$ .

This definition captures the idea that there are more agents with extreme wealth values.

Applying Proposition 3, the following result follows.

**Proposition 5** Consider a wealth dispersion shift  $\tilde{F} \succ_d F$ . If the optimal fee under  $F$  featured  $\psi > \phi_m$ , a wealth dispersion shift increases the optimal fee, while if  $\psi < \phi_m$ , a wealth dispersion shift reduces  $\psi$ .

This result shows that changes in wealth dispersion have opposite effects depending on whether the median wealth type can afford higher education with certainty. If the original equilibrium featured an allocation in which only agents above the median wealth could afford attending higher education with certainty, an increase in the wealth dispersion makes it optimal to set the education fee even higher. This makes the average type attending school 1 higher and increases the school quality. Indeed, the increase in school quality is at the expense of making it less likely for poor people to access it. This change in the educational system makes the earnings distribution more disperse than the original wealth distribution. In other words, the optimal educational system enhances inequality. The opposite is true if the original equilibrium featured agents with wealth below the mean. Upon a wealth dispersion shift, the educational system becomes more inclusive and, in fact, the optimal educational system tends to undo the increase in wealth dispersion, by making the ex-post earnings distribution less disperse under the new optimal schooling system.

### 6.1.2 Decentralization: A Fair Lottery Market on Wealth

In this subsection, I discuss how to decentralize the previous allocation.<sup>22</sup> Similar to Becker et al. (2005) and Cole and Prescott (1997), I show that with a market for lotteries over income, the optimal allocation can be decentralized. Given the working assumption of zero marginal cost of provision, consider firms (schools) providing mandatory and higher education at

<sup>21</sup>When there is a cost of provision of schooling, this result in Proposition 4 is true when there is a large mass of borrowing constrained agents even with only two schools. The reason is that the social planner cannot reduce the price below the marginal cost of provision.

<sup>22</sup>I thank Iván Werning for suggesting this discussion.

price 0.<sup>23</sup> Suppose that the social planner sets a school-contingent tax that has to be satisfied to attend school  $s$  such that

$$\tau(s) = \begin{cases} 0 & \text{for } s = 0, \\ \psi & \text{for } s = 1. \end{cases} \quad (15)$$

Now, consider a market that opens after the social planner announces the school-contingent taxes and before agents attend schools, in which fair lotteries  $l$  over wealth are traded. There is a continuum of these lotteries, indexed by  $i \in [0, \psi]$ . A lottery  $l_i$  delivers  $\psi$  with probability  $\pi_i$  and 0 with probability  $1 - \pi_i$ . There is a competitive market for each lottery.<sup>24</sup> Thus the price of lottery  $i$ ,  $p_i$ , is given by the break even constraint (or the actuarially fair lottery),  $p_i = \pi_i \psi$ .

By the discussion on the previous section, the school-contingent tax  $\tau(s)$  makes attending higher education attractive for all agents with ability  $a \geq a^*$ . Agents that are not borrowing constrained, do not derive any gain from participating in the lottery market. If they had to, they would purchase the lottery with corresponding price  $\psi$ , which returns wealth  $\psi$  with probability 1. However, agents that are borrowing constrained derive positive gains from participating in this market. Note that should they not participate, they would attend school 0 with probability one, while by participating in the lottery market they can attend school 1 with some positive probability.<sup>25</sup> Moreover, purchasing a lottery with a higher probability  $\pi_i$  and, hence, a higher price  $p_i$  is (weakly) better than purchasing a lottery with lower probability. Thus, constrained agents exhaust their initial wealth when purchasing lotteries. As a result, agents select into lotteries that have the same expected value of their initial wealth endowment. This discussion shows the following result.

**Proposition 6** (*Wealth Market*) *The optimal schooling system can be decentralized with school-contingent taxes and a market for wealth.*

## 6.2 Solution with School Fees Only

In this section I show that in environments in which the social planner has no commitment or limited communication, no lotteries can be used.

**Proposition 7** (*Credible Mechanism*) *Let the cost of reallocating agents to higher education be zero. If the social planner has no commitment, the only credible mechanism are school fees. Similarly, if the economy has limited communication such that no announcements can be made, the only feasible mechanism are school fees.*

<sup>23</sup>Section 7 shows that an analogous result holds when there is a positive marginal cost of provision.

<sup>24</sup>If each market is operated by more than one broker, the assumption is that each broker serves a positive mass of agents, so that there is no uncertainty on the returns of the lottery.

<sup>25</sup>The formal argument is analogous to equation (7),  $\pi_i \Delta w - p_i + w(a, 0) \geq w(a, 0)$ , for all  $a \geq a^*$ .

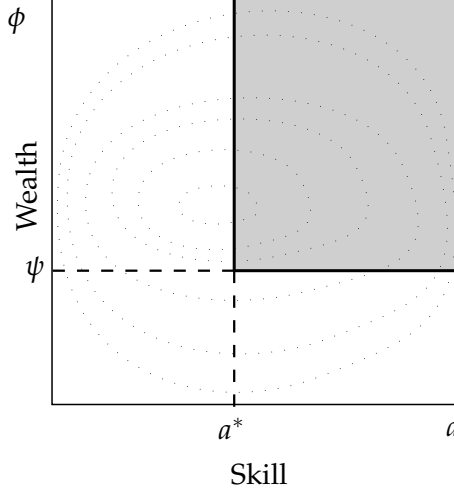


Figure 6: Allocation with school fees only.

Note that in the previous subsection, some borrowing constrained agents with  $a \geq a^*$  (that truthfully report their type) are not allocated to higher education. This would not happen in a first-best world, and it requires a commitment from the social planner of not reallocating ex-post agents once they have announced their type.<sup>26</sup> In this environment, the social planner cannot use lotteries to relax the borrowing constraints, because agents anticipate that if any randomization is announced, it is not credible. Thus, only school fees (without lotteries) can credibly be used. For the case of limited communication, the result follows purely from the constraints imposed from the limited communication in the transfer space.

The problem of the social planner reduces to decide the fee  $\psi$  that it charges to attend higher education. As in the previous section, the optimal school choice of an individual of type  $(a, \phi)$  is to choose  $s = 1$  if  $H(a, 1) - \psi \geq H(a, 0)$  and  $\phi \geq \psi$ . Otherwise, either because she does not have a high enough ability or enough wealth, she chooses  $s = 0$ . Figure 6 represents the region of agents that attend higher education in the type space.

The objective function of the Social Planner in the restricted problem is

$$\max_{\psi} \int_{\psi}^{\bar{\phi}} \int_{a^*(\psi)}^{\bar{a}} \Delta w(a, a^*(\psi)) dG(a) dF(\phi) + \int_0^{\bar{\phi}} \int_{\underline{a}}^{\bar{a}} \underline{a} h(a, 0) dG(a) dF(\phi), \quad (16)$$

where  $a^*(\psi)$  is implicitly defined by  $a^* h(a^*, 1) - \underline{a} h(a^*, 0) = \psi$ , and the notation  $\Delta w \equiv a^* h(a, 1) - \underline{a} h(a, 0)$  has been used. The interpretation of the objective function is that all agents obtain at least human capital  $\underline{a} h(a, 0)$  while the mass of agents  $(1 - F(\psi))(1 - G(a^*(\psi)))$

<sup>26</sup>Note that the zero cost of reallocation is important. If there was an investment stage in which the number of “seats” (capacity) of each tier are decided and they could not be changed ex-post, this would suffice to ensure that lotteries are credible.

attending school 1 obtain an additional amount of human capital. The goal of the Social Planner is to precisely maximize the additional gain coming from higher education.

Under regularity assumptions on the wealth distribution  $F$  to be discussed below, the FOC gives a sufficient condition to the problem,

$$f(\psi) \int_{a^*(\psi)}^{\bar{a}} \Delta w dG(a) da = (1 - F(\psi)) \left( \int_{a^*(\psi)}^{\bar{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) - \frac{\partial a^*}{\partial \psi} \Delta w(a^*, a^*) g(a^*) \right). \quad (17)$$

The left hand side term in equation (17) captures the costs that raising the tuition fee has in reducing the number of agents attending school 1. The right hand side captures the marginal benefit that increasing the tuition fee has on raising the spillover for agents in school 1.

The next proposition identifies sufficient conditions for the solutions implicitly defined by the first order condition (17) to be local maxima.

**Proposition 8** *If the hazard rate of the wealth distribution,*

$$\frac{f(\phi)}{1 - F(\phi)},$$

*is increasing, then the first order condition (17) has a unique solution that is the maximum of the planner's objective function (16).<sup>27</sup>*

Proposition (8) establishes a sufficient condition for the first order condition uniquely pinning down the optimal fee  $\psi$ . This requires the hazard rate of the wealth distribution to be increasing. Thus, any distribution with a log-concave density yields a unique solution. For the purposes of the paper, the empirically relevant distributions with log-concave density are the Beta, Weibull, Gamma and Exponential distributions.<sup>28</sup> Note, however, that there can be distributions that are not log-concave and have an increasing hazard rate.<sup>29</sup> Finally, footnote 27 identifies a relaxed sufficient condition that allows to include the Pareto distribution in the set of distributions for which the first order condition is sufficient.

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<sup>27</sup> If the wealth distribution satisfies the relaxed condition

$$-\frac{d}{dp} \frac{f(p)}{\psi(p)} < \left( \frac{f(p)}{\psi(p)} \right)^2, \quad (18)$$

then, the solution of the first order condition (17) may not be unique but contains the solution that maximizes the planner's objective function (16). The interest on the relaxed formulation is that it accommodates Pareto distributions with well defined mean, i.e., with shape parameter greater than one. This can be readily verified by checking the condition directly. The hazard rate of a Pareto distribution with index  $\alpha$  and support lower bound  $x_m$ ,  $1 - F(x) = \left(\frac{x_m}{x}\right)^\alpha$ ,  $f(x) = \alpha \frac{x_m^\alpha}{x^{\alpha+1}}$ , is equal to  $\alpha x^{-1}$ . Thus, condition (18) is satisfied if and only if  $\alpha > 1$ .

<sup>28</sup> Except for the Exponential distribution, these distributions are log-concave when they have a hump-shape. Bagnoli and Bergstrom (2005) provide a discussion of the parameter ranges in which these distributions have a hump-shape.

<sup>29</sup> Bagnoli and Bergstrom (2005) show these results and provide further examples. Note that the log-normal distribution has a non-monotonic hazard rate and proposition 8 does not apply.

### 6.2.1 Comparative Statics on the Wealth Distribution

I now go back to the study of the behavior of the first order condition of the constrained problem, (17), under changes in the wealth distribution. The discussion is brief, as this is essentially a particular case of equation (14) with a degenerate lottery. To simplify the discussion in the comparative statics exercise, I assume that the wealth distribution has increasing hazard rate. Rearranging, equation (17) can be written as

$$\frac{f(\psi)}{1 - F(\psi)} = \frac{\int_{a^*(\psi)}^{\bar{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) - \frac{\partial a^*}{\partial \psi} \Delta w(a^*, a^*) g(a^*)}{\int_{a^*(\psi)}^{\bar{a}} \Delta w dG(a)}. \quad (19)$$

The left hand side of equation (19) is the hazard rate of the wealth distribution. This is a particular case of the discussion following the general case with lotteries, equation (14). The right hand side is the ability-average marginal return of increasing segregation in higher education divided by the ability-average of people who stay in higher education. Note that the right hand side is independent of the wealth distribution. Moreover, the numerator is decreasing in  $\psi$  while the denominator is increasing in  $\psi$ . Thus, the right hand side of (19) is decreasing in  $\psi$ . This discussion yields to the following result.

**Proposition 9** *Consider two wealth distributions with  $\tilde{F} \succ_w F$ . The education fee  $\psi$  is higher under  $\tilde{F}$  than under  $F$ .*

*There is a reduction in the mass of agents obtaining higher education when the economy transitions from no agents being effectively borrowing constrained to a (small) mass of agents being borrowing constrained.*

The first part of the proposition shows that the comparative statics derived in the previous section hold when only school fees are used. However, in this case a weaker condition on the ranking of wealth distribution suffices to ensure the result: as long as there is dominance in terms of the hazard rate the result follows. The second part of Proposition 9 shows that the comparative statics on the mass of agents attending higher education is inherited as well. Similarly, changes in the wealth dispersion yield results analogous to the case with lotteries.

As a final remark, the decentralization of this schooling system is immediate. The planner sets school contingent taxes equal to the optimal fees.

### 6.3 School Fees and Exams

This section studies an environment in which the social planner has access to a signaling technology: exams. These can be used as an additional mechanism to screen agents that access higher education. The reason why the social planner may want to use them is that exams have screening power. This is, it is more costly for low ability agents to obtain a given

test score. However, exam preparation implies a waste of resources (e.g., tutoring). This introduces a trade-off between better screen capacity and wasteful spending. Obviously, without borrowing constraints it is never optimal to use exams, because incentives can be given perfectly with school fees, which do not convey wasteful spending.<sup>30</sup>

The signal technology considered is similar to [Fernández and Galí \(1999\)](#). It is represented by the mapping  $T : \mathcal{A} \times \Phi \rightarrow \mathcal{T}$  with  $t(a, c)$  measuring the score generated by an agent of type  $a$  who spends resources  $c$ , and  $\mathcal{T} \subseteq \mathbb{R}_+$ . I shall be working with the associated cost function,  $c(a, t)$ , which is defined implicitly by  $t(a, c(a, t)) = t$ , for all  $a \in \mathcal{A}$  and  $t \in \mathcal{T}$ . In this context  $t$  has the natural interpretation of a test level. The interpretation of the cost function is that an agent with ability  $a$  has to spend an amount  $c(a, t)$  to obtain a test score  $t$  in the exam. I assume that  $c_a < 0$ ,  $c_t > 0$ ,  $c_{at} \leq 0$ ,  $c(a; t = 0) = 0$  (i.e., not taking the exam has zero cost).

I consider two different exam specifications. I begin by characterizing the optimal mechanism with a simple pass/fail exam in which the difficulty of passing is exogenously given. Then, I consider an exam technology in which there is a continuum of possible test scores and, thus, the difficulty of an exam to access higher education becomes endogenous. Moreover, more than one test score can give access to higher education (with different school fees associated to different scores). I show that the qualitative results derived in the simple pass/fail exam hold in the general set-up.

### A simple Pass/Fail Exam

The main insight of this section can be obtained by looking at a simple formulation in which the government has access to a very limited technology, a pass-fail exam,  $\mathcal{T} = \{0, 1\}$ . In this environment, school fees can be indexed by whether or not an agent passes the exam (i.e., invests in the signal technology) –if the social planner decides to use the signaling technology.

When does the social planner want to use exams? Suppose only a small mass  $\varepsilon > 0$  of agents is constrained in an allocation that uses only fees. In this case, it is not optimal for the social planner to use exams in the assignment mechanism. To see this, suppose that agents can access education by either paying a fee  $\psi_0$  and not taking the exam ( $t=0$ ) or by paying a  $\psi_1$  and passing the exam ( $t = 1$ ). With this mechanism at hand, all agents with ability  $a \geq \tilde{a}$ , with  $\tilde{a}$  defined by  $\psi_0 = \psi_1 + c(\tilde{a}, t)$  prefer to take the exam. Note that the mechanism cannot give incentives along the wealth dimension and thus *all* agents with ability  $a \geq \tilde{a}$  take the

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<sup>30</sup>The assumption that the all exam preparation is wasteful spending is an extreme one. It could be the case that agents learn by preparing an exam. In this sense, the exam component that the model is capturing is the resources that are spent to do well in an exam that are orthogonal to knowledge acquisition. [Fernández \(1998\)](#) argues that this constitutes a sizable part of exam preparation. Other researchers, such as [Bishop \(1997\)](#), have argued that exams can be beneficial because they are coordination devices. I abstract from this feature as well.

exam. As a result, conditional on a  $\psi_0$  (which pins down the spillover level), the additional gain in output of using exams comes from the additional mass of poor agents with wealth between  $\psi_0$  and  $\psi_1 + c(a, t)$  that can access school by taking the exam but would otherwise be excluded,

$$\int_{\tilde{a}}^{\bar{a}} \int_{\psi_1 + c(a, t)}^{\psi_0} \Delta w dF(\phi) dG(a). \quad (20)$$

The cost of using exams is the wasteful spending incurred by all agents with  $a \geq \tilde{a}$

$$\int_{\tilde{a}}^{\bar{a}} \int_{\psi_1 + c(a, t)}^{\bar{\phi}} c(a, 1) dF(\phi) dG(a). \quad (21)$$

Thus, output gains are smaller than costs of introducing exams whenever the mass of additional agents that select into higher education relative to the original mass,  $\int_{\tilde{a}}^{\bar{a}} \int_{\psi_1 + c(a, t)}^{\psi_0} dF(\phi) / (1 - \psi_0)$ , is small. This discussion shows the following proposition.

**Proposition 10** *Consider a test technology  $\mathcal{T} = \{0, 1\}$  and a family of wealth distributions that can be ranked according to the wealth abundance criterion. Then, there exists a threshold distribution  $F^*$  such that for all  $F \succ_w F^*$  the optimal allocation mechanism does not use exams, i.e.,  $a^* = \tilde{a}$ . Moreover, if  $F_1 \succ_w F_0$ , then  $a_1^* \leq a_0^*$  and  $\tilde{a}_1 \leq \tilde{a}_0$ .*

Proposition 10 states the two key results of this section. First, sufficiently wealth abundant economies do not use exams. Second, the less wealth abundant an economy is, the more it relies on exams to allocate agents to higher education. To be more precise, the ability range in which exams are used,  $[\tilde{a}, \bar{a}]$ , increases. Moreover, the comparative statics on the threshold type attending higher education are the same as in the previous sections (propositions 3 and 9). Thus, less wealth abundant economies have a worse selection of agents into higher education in terms of ability.

These results resonate with the empirical evidence presented in the Introduction. Developing countries make relatively more extensive use of gate-keeping exams to complete basic education and access higher levels of education. This is often coupled with tutoring to prepare exams, especially in Asia, Africa, Latin America and Eastern Europe (Bray, 2000).

The pattern of selection of types that attend higher education is pictured in Figure 7. In general, it can be the case that  $\tilde{a} > a^*$ . This resonates as well with practices of access to higher education. For example, in India, access to prestigious higher education institutions can be done through two different paths. Access for the general body of students is through an exam requirement and a tuition fee. But, in addition, there is access through school fees only, known as management quotas.<sup>31</sup> A similar finding is documented for Tanzania by Al-Samarrai and Peasgood (1998).

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<sup>31</sup>I thank Abhijit Banerjee for pointing out this example to me.



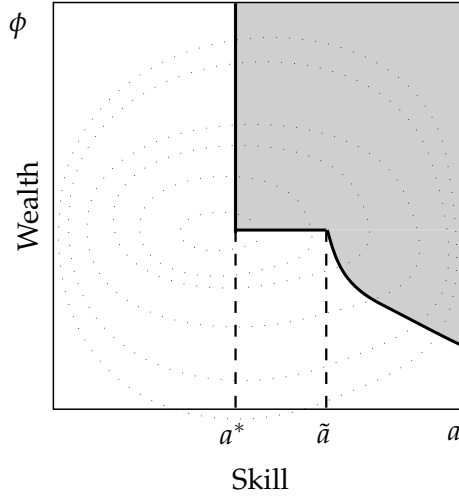


Figure 7: Allocation with school fees and a pass/fail exam.

### Full-Blown Exam Technology and Fellowship Schemes

I now consider the case in which the social planner has access to a continuum of exam technologies,  $\mathcal{T} = [0, \infty)$ . That is, the planner can ask an agent to obtain any score  $t \in \mathcal{T}$ . I show that in this environment the qualitative results of Proposition 10 hold. More specifically, it is still optimal to reduce school quality and resort more extensively in exams in less wealth abundant economies. Before proceeding, the following assumption on costs is made.

**Assumption 2** *The associated cost function to the exam technology takes the form  $c(t, a) = c_1(t)c_2(a)$ , where  $c_1$  and  $c_2$  are positive, twice-continuously differentiable functions. Moreover,  $c_2(a)$  is log-convex.*

Using the revelation principle, I look for a schedule  $\{\psi(a), t(a)\}$ . The social planner problem is

$$\max_{\psi(a), t(a)} \int_{a^*}^{\bar{a}} \int_{\psi+c(t,a)}^{\bar{\phi}} (\Delta H(a, a^*) - \kappa - c(a, t)) dG(a) dF(\phi), \quad (22)$$

where  $\Delta H(a, a^*) \equiv a^*h(a, 1) - ah(a, 0)$ , subject to

$$\Delta H(a, a^*) - \psi(a) - c(t(a), a) \geq 0 \quad \text{for all } a \in [a^*, 1], \quad (23)$$

$$\Delta H(a, a^*) - \psi(a) - c(t(a), a) \geq \Delta H(a, a^*) - \psi(\hat{a}) - c(t(\hat{a}), a), \quad (24)$$

for all  $a, \hat{a} \in [a^*, 1]$ , where (23) and (24) are the participation and incentive compatibility constraints. I adopt a first order approach to solve the problem. I first discuss the sufficient conditions for implementability and then discuss the optimization part. The set of sufficient conditions for implementability under the first order approach are as in a standard screening problem (e.g., Bolton and Dewatripont (2005)). The incentive compatibility constraints are

satisfied if and only if there is local incentive compatibility

$$\psi'(a) + c_t(t, a)t'(a) = 0, \quad (25)$$

and monotonicity,

$$t'(a) \geq 0 \quad \text{and} \quad \psi'(a) \leq 0. \quad (26)$$

Note that the implementation problem is essentially one dimensional, because incentives are provided along the ability dimension only. Calculating the difference of the second order condition with the total derivative of the local incentive compatibility constraint only involves  $t'(a)$ . However, differently than the standard screening problem, one cannot get rid of  $t$  and  $\psi$  in the objective function by integrating by parts. Finally, before proceeding to the optimization stage, using the complementarity in the intrinsic human capital technology, the participation constraint can be substituted by the condition that the marginal type accessing higher education has to be indifferent,

$$\Delta H(a^*, a^*) - \psi(a^*) - c(t(a^*), a^*) = 0. \quad (27)$$

The Social Planner problem needs to be solved in two steps because the optimization problem cannot be written exactly as an optimal control or calculus of variations problem. This comes from the complementarity between the spillover and the intrinsic human capital production function. That is, in addition to the initial boundary condition for  $a^*$  to lie on the curve defined by (27), any change in  $a^*$  affects the value at any point of the integrand of the objective function (22). Thus, first I solve an inner problem in which the spillover term  $a^*$  that enters in  $\Delta H(a, a^*)$  is kept fixed at  $\hat{a}^*$ . This problem is readily amenable to optimal control techniques. Note that at this step, I obtain an optimal value for  $a^*(\hat{a}^*)$  coming from the initial condition problem. Then, I solve an outer problem for  $\hat{a}^*$  under the constraint that  $a^* = \hat{a}^*$ .

Appendix 11 characterizes the optimal solution and the comparative statics in the wealth distribution. The results are summarized below.

**Proposition 11** *Consider a family of log-concave wealth distributions. Let  $\tilde{F} \succ_w F$ , then (i) the total cost of education  $\psi(a) + c(a, t(a))$  at all ability levels  $a \in [a^*, \bar{a}]$  is greater under  $\tilde{F}$ , (ii) the test level  $t(a)$  at all ability levels  $a \in [a^*, \bar{a}]$  is smaller under  $\tilde{F}$ , (iii) the threshold type  $a^*$  is greater under  $\tilde{F}$  and (iv) the optimal fee-test schedule does not have any bunching region.*

These results generalize the ones obtained in Proposition 10 for the simple pass/fail exam technology. Poorer economies rely more on exams to access higher education. The total cost of education holding ability constant is increasing with a wealth abundance shift. This means that the total cost of education (exam plus transfer) in poor countries is less than in

rich countries. However, point (ii) of Proposition 11 shows that the test level required to an agent of a given ability is higher in less wealth abundant countries. These two observations imply that the ratio of exam cost to total cost

$$\frac{c(a, t(a))}{c(a, t(a)) + \psi(a)}$$

is decreasing upon wealth abundance shifts. Thus, exam expenditure relative to total school expenditure is higher in poorer countries. This result relates to the stylized fact discussed in the Introduction that developing countries rely more on exams than rich countries. Finally, as in the previous comparative statics results, point (iii) shows that the ability-composition of agents selecting into higher education is worse in poorer countries.

A corollary of Proposition 11 is the following. Consider a wealth distribution such that higher education is provided to some agents with wealth below the median. Let  $\tilde{F} \succ_d F$ , then the total cost of education  $\psi(a) + c(a, t(a))$  decreases for agents below the median wealth relative to those above, but test requirements increase. The difference in test scores required to access schools increases for agents with wealth below the median relative to those above it. That is, exam requirements (i.e., test levels) increase in the range in which the mass of borrowing constrained agents rises. This comes as no surprise, exams are more intensively used in the region where inequality changes are more pronounced, which is precisely where their screening power has a higher relative benefit.

*Decentralization.* The previous mechanism gives incentives by rewarding (high ability) agents that obtain high test scores with low school fees. In a decentralized equilibrium, (as the marginal cost of provision is zero) this is implemented with a tax contingent on school and test performance, so that agents that obtain better grades pay a lower tax. Thus, this very much resembles the usage of scholarship schemes. The previous discussion implies that poorer countries rely more in scholarship-like schedules to implement the optimal solution.

## 7 Extensions

The goal of this section is to show that the results highlighted in the baseline model hold in more general environments. In Subsection 7.1, I consider a yeomen-farmer like economy, in which each agent produces a differentiated intermediate good, and the final consumption good is a CES composite of all intermediates. Then, I discuss how the results extend to more general type of spillovers.

Subsection 7.2 analyzes an environment in which, within each education tier, there is a continuum of sub-tiers, so that a school can be tailored to each ability level. Even though this extension mutes the endogenous quality degradation margin, it provides a useful benchmark to analyze how the mass of mismatched agents and school capacities change at differ-

ent wealth levels. I use this simplified environment to show how the planner uses cross-subsidization within schools to increase access of borrowing constrained agents to schools and to show how the presence of an unregulated private sector provider may hamper its ability to do so.

## 7.1 Yeomen farmers and general spillovers

Consider the following extension of the baseline model. Each agent produces a differentiated intermediate good with her human capital. The final good is produced as an aggregator of intermediates with elasticity  $\varepsilon$ ,

$$Y = \left( \int y(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1.$$

Thus, the technology specification in the baseline model is a limiting case in which all intermediates are perfectly substitutable ( $\varepsilon \rightarrow \infty$ ).

Markets are competitive, and payments to factors of production are made according to marginal productivity.<sup>32</sup> Thus, an agent with ability  $a$  that attends school  $s$ , earns

$$w(a, s) = Y^{1/\varepsilon} H(a, s)^{\frac{\varepsilon-1}{\varepsilon}} = Y^{1/\varepsilon} A(s)^{\frac{\varepsilon-1}{\varepsilon}} h(a, s)^{\frac{\varepsilon-1}{\varepsilon}}.$$

This payoff structure resembles the structure of [Bénabou \(1996b\)](#) in that there is a “local” and “global” externality.

Next, I show that the same qualitative results hold in this generalized set-up. Define  $\tilde{h}(a, s) = h(a, s)^{\varepsilon/(\varepsilon-1)}$ . Note that the complementarity between  $a$  and  $s$  is preserved as  $\varepsilon > 1$ . Analogously, define  $\tilde{A}(s) = (\min_s \{a \in s\})^{\varepsilon/(\varepsilon-1)}$ . Let  $a^* = \min_s \{a \in s\}$ . The derivative of  $\tilde{A}(s)$  with respect to  $a^*$  is positive. The argument that maximizes the value of  $Y(a^*)$  coincides with  $Y(a^*)^{(\varepsilon-1)/\varepsilon}$ . Thus, the problem of finding the argmax of  $Y(a^*)^{(\varepsilon-1)/\varepsilon}$  is isomorphic to the baseline model replacing  $\tilde{h}(a, s)$  for  $h(a, s)$ , and  $\tilde{A}(s)$  for  $A(s)$ . As a result, the same methods and results derived in the linear technology case apply to this general set-up.<sup>33</sup>

Now, I discuss how the results extend in two different alternative specifications of the spillover. The main lead theme of both specifications is to relax the “least common denominator” specification to address the concerns that (i) agents may learn even if a curriculum is tailored for higher ability agents ([Duflo et al. \(2008\)](#) offer evidence along these lines), (ii) there are other forces that can generate the complementarity in human capital production.

A simple extension that generalizes the results presented in the baseline model without

<sup>32</sup>I maintain the assumption that the social planner cannot manipulate the production of goods, it can only choose the educational system structure.

<sup>33</sup>Note, however, that in this case the sufficiency conditions derived for the baseline model need to be adjusted by the presence of the additional factor  $\varepsilon/\varepsilon - 1$ .

adding any complexity to the problem is to allow the spillover to be a convex combination of the highest and lowest ability agent in a particular school,

$$A(s) = \alpha \min_s \{a \in s\} + (1 - \alpha) \max_s \{a \in s\}, \quad \alpha \in [0, 1]. \quad (28)$$

Note that in this formulation, a reduction on the threshold type attending higher education would affect both the spillovers at the basic and higher education. However, it is immediate to check that the complementarity in the intrinsic human capital  $h$  is enough to ensure that the results derived in the baseline hold with a spillover as in (28).

One might argue that specification (28) is blind to whether most of agents are close to the maximum or the minimum, and that this is likely to matter. One can further generalize the spillover to

$$A(s) = \left( \iint a^{\frac{\sigma-1}{\sigma}} dZ(a, \phi|s) \right)^{\frac{\sigma}{\sigma-1}}, \quad (29)$$

where  $Z(a, \phi|s)$  is the joint ability-wealth distribution of types selecting into school  $s$ . This spillover spans the range from a Leontief as  $1/\sigma$  grows to infinity to a “best shot” as  $1/\sigma$  goes to minus infinity. Note that as  $\sigma \rightarrow \infty$  the spillover becomes the average type attending the school. The results derived in the baseline section can be extended to a spillover with this specification (29) if basic education is modeled as an outside option with its value normalized to zero for all agents (i.e.,  $h(a, 0) = 0$ ). In this case, it is immediate to verify that analogous results to the baseline follow. Intuitively, when the spillover is of the “best shot” no segregation is optimal, while when it is Leontief, some segregation is always optimal. An intermediate value of  $1/\sigma$ , gives an intermediate level of segregation between these two extremes.

If the outside option of basic education is not normalized to zero then the problem becomes more complicated and some additional structure to the human capital production function and the distribution of types is needed in order to have clear comparative statics. This comes from the fact that the difference in spillovers  $A(1)h(a, 1) - A(0)h(a, 0)$  may be either concave or convex on changes in the marginal type obtaining education when there are borrowing constraints.

## 7.2 Continuum of Schools

This section analyzes the optimal schooling system when within the two broad curricula of basic and higher education, there are finer curriculum options. For example, higher education can be subdivided in associate, bachelor, master and Ph.D. degrees. Even further, it may be the case that different schools can have some margin to adapt their curricula. For example, to the extent that educational institutions have some discretion in setting their own standards, curriculum requirements may vary to some extent among institutions in the same

education tier. I capture this richer environment by allowing for different sub-tiers in both mandatory and higher education. These can be interpreted as either a finer partition across different educational tiers or differences at school level. As it turns out, the most convenient formulation is to allow for a continuum of sub-tiers within each educational tier.

The goal of this exercise is two-fold. First, it allows to investigate to which extent the reduction in capacity of higher education schools and the mismatch of ability to schools persists even when there is a school tailored to each ability level. Second, it provides a natural framework to analyze competition between an education sector regulated by the social planner and an unregulated sector.<sup>34</sup>

Compared to the baseline environment, in this section I allow for a marginal cost of provision of higher education  $\kappa$ . More importantly, I allow the planner to cross-subsidize across schools. That is, the budget constraint of transfers has to break even on net, and not school by school. That is, as opposed to the baseline model, the planner has the ability to do instantaneous redistribution, as in equation (9).

### 7.2.1 First Best characterization and no Borrowing Constraints

This section shows that the result derived in the two-tier case which stated that private information alone does not prevent the social planner from implementing the first best allocation system generalizes to more school tiers. The intuition for the result is the same: given the complementarities between endogenous school quality and types, higher ability agents are willing to pay more for higher quality education. This section proceeds as follows. I begin by characterizing the first best. Then, I show how a mechanism can be designed to implement the first best educational system when private information is present.

Given the complementarity of  $a$  and  $s$ , there is full segregation in the first best. That is, the first best features one school for each type  $a$ . To see this, suppose to the contrary that two types of different ability  $a_l < a_h$  with associated mass  $m_{a_l}$  and  $m_{a_h}$  attend the same school tier. Then, by segregating them, output can be increased as  $a_h$  benefit from a higher spillover, without reducing the output of  $a_l$ . Types with  $a \leq a^*$  attend school 0, where  $a^*$  is defined by

$$a^*(h(a^*, 1) - h(a^*, 0)) = \kappa.$$

Next, I show that the first best educational system can be achieved with private information using a school fee schedule. This is the continuous case counterpart to Proposition 1.

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<sup>34</sup>As opposed to the two-tier framework, this framework allows to isolate the effect of competition. In the two-tier system, additional provision from the unregulated sector generates a mechanical force towards having gains from an additional private provider just because it allows more segregation. Moreover, a two-tier system easily runs into the problem of multiple equilibria.

The problem of the social planner is

$$\max_{\psi(a)} \int_{\underline{a}}^{a^*} H(a, 0) da + \int_{a^*}^{\bar{a}} (H(a, 1) - \kappa) da$$

subject to

$$\begin{aligned} ah(a, \mathbb{1}(a)) - \psi(a) &\geq 0 \quad \text{for all } a \in [0, 1], \\ ah(a, \mathbb{1}(a)) - \psi(a) &\geq \hat{a}h(a, \mathbb{1}(\hat{a})) - \psi(\hat{a}) \end{aligned}$$

for all  $a, \hat{a} \in [0, 1]$ . And the break even constraint,

$$\int_{\underline{a}}^{\bar{a}} (\psi(a) - \kappa) da \geq 0.$$

The fee schedule that solves the problem is,

$$\psi(a) = \begin{cases} \int_{\underline{a}}^a h(a, 0) da & \text{for } a \leq a^*, \\ \int_{a^*}^a h(a, 1) da + \int_{\underline{a}}^{a^*} h(a, 0) da & \text{for } a > a^*. \end{cases} \quad (30)$$

where it is used the normalization that  $h(a = \underline{a}, 0) = 0$ . Figure 8a provides a graphical representation of the result. Note that the solution features standard properties from screening mechanisms. “All rents” are extracted for the lowest ability type  $a = 0$  and consumption is increasing in ability,  $c'(a) = ah_a(a, \cdot)$ . Moreover, it can be verified that the break even constraint is satisfied with inequality.

Finally, this solution coincides with the First Best because there is complete segregation in skill. The following proposition summarizes the previous discussion.

**Proposition 12** *The first best educational system features full-segregation. The optimal mechanism with private information can implement the first best educational system.*

Before proceeding note that there can be other price schedules that implement the first best if  $h(a, 0) > 0$ . However, these are constrained to have the same slope as (30). The reason is simple, at any other slope, there would be bunching of some types and this cannot be optimal by the previous argument that segregation is always optimal. This is stated in the next remark.

**Remark** Any transfer that implements truth-telling revelation of ability has to have slope in ability given (almost everywhere) by either the slope of  $\int_0^a h(a, 0) da$  for  $s = 0$  or  $\int_{a^*}^a h(a, 1) da$  for  $s = 1$ .



### 7.2.2 Decentralization without borrowing constraints

In this section I show that there exists an equilibrium that decentralizes the optimal schooling system. Define a competitive schooling equilibrium as a supply of schools  $S$ , pricing function  $p : S \rightarrow \mathbb{R}_+$  and agents' choices  $c : \mathcal{A} \rightarrow S$  such that (i) agents' school choices maximize utility at the stated prices, (ii) firms maximize profits, (iii) markets clear.

First, I show that it exists an equilibrium in which prices coincide with fees set by the social planner (30). The reason why sub-tiers can exist is that, despite using the same intrinsic human capital production function, different schools offer different spillovers at different prices. Thus, school prices  $p$  are indexed by both the intrinsic technology  $s$  and the spillover level  $a \in \mathcal{A}$ . Consider the following price schedule for  $p(a, s)$

$$\begin{cases} p(a, 0) = \int_0^a h(a, 0) da & \text{for } a < \tilde{a}, \\ p(a, 1) = \int_{\tilde{a}}^a h(a, 1) da + \kappa & \text{for } a \geq \tilde{a}, \end{cases} \quad (31)$$

where  $\kappa$  is the marginal cost of school provision and  $\tilde{a}$  is implicitly defined by  $\tilde{a}h(\tilde{a}, 1) - \kappa = \tilde{a}h(\tilde{a}, 0) - p(\tilde{a}, 0)$ . Note that this price schedule, conditional on  $s$ , is convex in  $a$ . Agents utility maximization,

$$\max_{\hat{a}, s} \{ \hat{a}h(\hat{a}, s) - p(\hat{a}, s), 0 \} \quad (32)$$

can be solved sequentially. First, find the optimal demand of  $\hat{a}$  in each school tier, and then comparing utility at the optimal  $\hat{a}(s)$  from attending basic and higher education (or not educating at all). The solution to (32) is  $\hat{a} = a$ . Thus an agent with ability  $a < \tilde{a}$  chooses  $s = 0$  and an agent with  $a > \tilde{a}$  chooses  $s = 1$ , an agent with  $a = \tilde{a}$  is indifferent, and no agent decides not to educate.

Given this price schedule, education provision for any  $(a, s)$  is provided at no loss, as  $p(a, 0) \geq 0$  and  $p(a, 1) \geq \kappa$ . In fact, schools make positive profits in equilibrium and no entrant can attract agents by offering lower prices. To see this, consider an entrant that provides  $(a, s)$  at a price lower than  $p(a, s)$ . At a lower price, agents with lower ability would purchase the schooling good  $(a, s)$ , but this would result in a spillover effect of lower ability than  $a$ . Thus, no price lower than  $p(a, s)$  can be credibly offered. The question is that with positive profits, there will always be schools willing to enter the market. One possibility to discipline the model is to have a given measure of potential entrants, so that schools per se become infinitesimal. Another is to have an additional type specific school production input in fixed supply that is needed to provide education, so that as more and more firms enter a particular school market, the price of the input goes up. This would pin down the number of schools entering the market. The details of how to pin down the number of firms is inessential for purposes of this discussion. In either case, the relevant part is that at the stated prices, the demand for schooling at all ability levels is positive and is met by a supply.

Thus, there is market clearing.

**Remark** Any price schedule that perfectly separates agents in terms of ability has to have slope in ability given (almost everywhere) by either the slope of  $\int_0^a h(a,0)da$  for  $s = 0$  or  $\int_{a^*}^a h(a,1)da$  for  $s = 1$ .

The equilibrium proposed as it stands does not coincide with the first best solution. In this equilibrium there is more provision of higher education than in the first best, as the lowest ability type attending higher education is

$$ah(a,1) - \kappa = ah(a,0) - \int_0^a h(a,0)da, \quad (33)$$

while in the social planner solution is  $ah(a,1) - \kappa = ah(a,0)$ . This makes clear that a tax contingent on attending school tier  $s = 1$  of value  $\tau = \int_0^a h(a,0)da$  implements the same assignment of agents to schools as in the first best.<sup>35</sup>

### 7.2.3 Private Information with Borrowing Constraints

I study the mechanism design problem when there are borrowing constrained agents. The following result greatly simplifies the analysis.

**Remark** (No bunching) Whenever there exists an agent of ability  $a$  that can afford segregation, it is optimal to offer a school tailored for agents of ability  $a$ .

This result comes from the extreme complementarity in the spillover, which implies that segregation of high-ability, unconstrained agents does not reduce the utility of high-ability constrained agents. Segregation has the advantage of increasing output and increasing revenue of the social planner (which implies a reduction on the schooling fee across-the-board). An important corollary of this remark is that it is not optimal to use lotteries. The reason is simple: given the Leontief spillover, this would involve bunching types. Thus, announcements on the wealth dimension are not relevant to the social planner because there is no added value on randomization.

With this results at hand, the mechanism to be used can be rewritten as follows. Let  $\psi(a)$  denote the fee that agents of ability  $a$  pay if they announce to be of ability  $a$ . Then, given an announcement  $(a, \phi)$  the fee that any agent with  $\phi > \psi(a)$  has to pay is  $\psi(a)$ , while if the

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<sup>35</sup>Note that if  $h(a,0) = 0$ , this is the only equilibrium that features full segregation. If  $h(a,0) > 0$ , there could be other equilibria that decentralize the first best, which would be a translation  $p(a,s) + \beta$  for some  $0 \leq \beta \leq ah(a,0)$ .

agent is constrained the fee is just  $\phi$ . The problem of the social planner can be written as

$$\begin{aligned} \max_{\psi(a), \mathbb{1}(a)} \quad & \int_{\underline{a}}^{\bar{a}} \int_{\psi(a)}^{\bar{a}} (H(a, \mathbb{1}(a)) - \mathbb{1}(a)\kappa) dF(\phi) dG(a) + \\ & \int_{\underline{a}}^{\bar{a}} \int_{\underline{a}}^{\psi(a)} (\psi^{-1}(\phi)h(a, \mathbb{1}(\psi^{-1}(\phi))) - \mathbb{1}(\psi^{-1}(\phi))\kappa) dF(\phi) dG(a), \end{aligned}$$

subject to the participation and incentive compatibility constraints,

$$\begin{aligned} ah(a, \mathbb{1}(a)) - \psi(a) &\geq 0 \quad \text{for all } a \in [\underline{a}, \bar{a}], \\ ah(a, \mathbb{1}(a)) - \psi(a) &\geq \hat{a}h(a, \mathbb{1}(\hat{a})) - \psi(\hat{a}), \end{aligned}$$

for all  $a, \hat{a} \in [\underline{a}, \bar{a}]$  that are unconstrained, and the break-even constraint

$$\begin{aligned} \int_{\underline{a}}^{\bar{a}} \left[ (1 - F(\psi(a)))\psi(a) + \int_{\underline{a}}^{\psi(a)} \phi dF(\phi) \right] dG(a) \\ \geq \kappa(1 - F(\psi(a^*))) \int_{a^*}^{\bar{a}} dG(a). \end{aligned}$$

From the results in the previous section, a fee-schedule that achieves full segregation has to have the slope of equation (30). In this case, the presence of borrowing constraints imply that it is optimal to reduce as much as possible fees (conditional on the break even constraint being binding). Thus, the level of the fee schedule  $\psi(a)$  is going to be reduced as much as possible (up to the break-even constraint being binding) without changing the slope. More specifically, the solution of the problem is given by

$$\psi(a) = \begin{cases} \int_0^a h(a, 0) da - \mathcal{C} & \text{for } a \leq a^*, \\ \int_{a^*}^a h(a, 1) da + \int_0^{a^*} h(a, 0) da - \mathcal{C} & \text{for } a > a^*. \end{cases} \quad (34)$$

with  $a^*h(a^*, 1) - \kappa = a^*h(\tilde{a}, 0)$ , where  $\mathcal{C}$  is pinned down by the break even constraint

$$\int_{\underline{a}}^{\bar{a}} \left[ (1 - F(\psi(a)))\psi(a) + \int_0^{\psi(a)} \phi f(\phi) d\phi \right] da = \kappa(1 - F(\psi(a^*))(1 - G(a^*))). \quad (35)$$

*Remark on impossibility of decentralization.* The decentralization that can be achieved is exactly the same as in the case without borrowing constraints and it is omitted from the discussion. Note that this differs from the optimal mechanism, as the price level in the decentralized equilibrium is too high. The reason is that while the planner uses revenue from schools to reduce the price level, private providers cannot do that. Thus the social planner educational system cannot be decentralized in this case.<sup>36</sup>

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<sup>36</sup>Appendix 9 shows that the same result holds in the two school case if simultaneous redistribution is allowed.

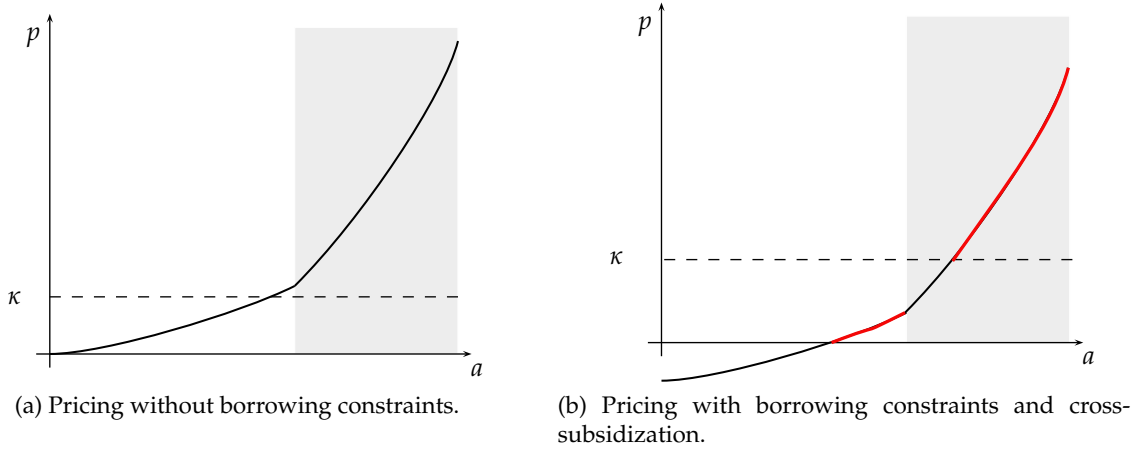


Figure 8: Optimal pricing with a continuum of schools and two technologies

#### 7.2.4 Comparative Statics on the Wealth Distribution

This section analyzes how changes in wealth abundance and dispersion affect the mismatch of agents to schools and school tier capacity. Note that the extreme assumption of the continuum of tiers washes out the endogenous deterioration of quality arising from changes in the wealth distribution.

**Proposition 13** (*Mismatch of agents to school*) Let  $\tilde{F} \succ_w F$ , the percentage of agents mismatched is higher at all levels of schooling under  $F$ . Let  $\tilde{F} \succ_d F$ , the percentage of agents with wealth above the median that are mismatched decreases relative to agents below the median.

This results are immediately interpretable, and generalize those of the baseline model. In countries that are relatively poor, there are more agents mismatched in the sense that they attend a school tailored for lower ability agents at all levels of education. In more unequal countries, the mismatch is specially aggravated at low levels of education. The logic of the proof is simple. Upon a wealth shift, planner's revenue increases and this makes the fee level to go down -this makes borrowing constraints less severe. The fraction of agents mismatched in school  $a$  is

$$\frac{f(\psi(a))}{1 - F(\psi(a))} \frac{\int_a^{\bar{a}} dG(a)}{g(a)}, \quad (36)$$

which decreases as well upon a wealth abundance shift. Thus, both effects go in the same direction and the result follows. A similar reasoning applies for changes in wealth dispersion. In this case, given the convexity of school fees, revenue of the social planner can increase. In this case, this would reduce the fee level across the board. For agents above the median wealth this amplifies the decrease in the fraction of mismatched agents. However, for agents below the median wealth the effect would be ambiguous because of the dispersion shift and

the reduction in fee levels going on opposite directions.

A related question that this model is better suited to answer than the baseline model is how does the mass of agents attending each school-tier change with changes in the wealth distribution.

**Proposition 14** (*Mass of agents attending school*) Consider  $\tilde{F} \succ_w F$ , then the school capacity of all schools with index  $a > a_m$  increases under  $\tilde{F}$ . Consider  $\tilde{F} \succ_d F$ , then the school capacity of all schools with index  $a > a_m$  increases under  $\tilde{F}$ , the effect at the bottom is ambiguous.

This result implies that top schools (those with index greater than  $a_m$ ) in poorer and unequal societies feature less capacity than in richer countries. The exact value of  $a_m$  depends on the specifics of the wealth distribution. It suffices to note that this result always holds for  $\bar{a}$ . Then, by a continuity argument, it holds in some neighborhood of  $\bar{a}$ . The results in Proposition 14 are the analogous counterparts of the message provided in the baseline model that higher education provision is reduced in poor, unequal economies. The continuum of schools case allows to identify that the reduction in capacity is localized at the top schools within the higher education tier.

### 7.2.5 Optimal Mechanism with (Unregulated) Private Schools Outside Option

Finally, I discuss the case in which the social planner has to design the optimal mechanism facing a new additional constraint. There is a mass of private agents that have access to the schooling technology and can escape from the regulation of the social planner. Thus, these private agents are free to provide education at any sub-tier that they find profitable to. This set-up is meant to be a first pass exercise in understanding possible interactions between public and private education, with the caveat that public education needs not to coincide with the social planner's optimal educational system as it may be subject to additional constraints not modeled here.

The point I want to illustrate is simple: private provision undermines the capacity of the social planner to cross-subsidize education. The reason is that private provision competes with the planner only on the profitable segments of the market, which are precisely the source the planner uses to provide subsidization. Put shortly, private firms cream-skim the market for education. To illustrate the point, suppose that the social planner tried to implement the optimal mechanism described in (34). At all school levels in which  $p(a, 0) \geq 0$  and  $p(a, 1) \geq \kappa$ , private provision occurs, because at the stated fees private firms make positive profits. These are the regions depicted in red in Figure 8b. Consider for now the extreme case in which, ceteris paribus, agents prefer to attend private schools. In this case, all the sources of positive revenue from the social planner would disappear and the social planner's budget constraint would not be satisfied. Thus, the conjectured equilibrium ceases to be an equi-

librium in the presence of private schools. In this case, the planner, would have to increase the fee level to ensure some positive revenue. However, this would backfire because all the schools yielding positive profit would be captured by the private sector. As a result, the only equilibrium that would survive is one in which there is only the private provision, as in (31). In this case, there would not be any scope for school cross-subsidization.

On the opposite extreme, one can consider the case in which, *ceteris paribus* agents prefer to attend public schools. By a similar argument, one can show that the (constrained) efficient mechanism analyzed in the previous section can be implemented in this case. Presumably, a realistic benchmark is somewhere between these two opposite poles. The point to take away is that if private providers coexist with schooling provided by the planner, then private school provision puts limits to cross school subsidization. This suggests that the cream-skimming of the sources of cross-subsidization by the private providers in poor countries may hamper the capability of the social planner to cross-subsidize education, ultimately reducing the effective level of education that can be provided to credit constrained agents.

## 8 Conclusion

Educational systems shape how human capital is produced and, thus, play a crucial role in determining the human capital of an economy. This paper developed a framework to analyze the role of the wealth distribution and borrowing constraints in molding educational systems. It showed that many of the features of educational systems in developing countries that may appear to involve inefficient rationing compared to rich countries, for example as a result of an elite capture, can be rationalized within a mechanism design framework in which the social planner maximizes aggregate welfare. Hence, the model provides a fundamental economic reason for why educational systems in developing countries need not be the same as in rich countries. The two key ingredients for this result are the existence of poor, borrowing-constrained agents and private information on ability and, thus, on the valuation of schooling. The comparative statics results show that the educational system a benevolent social planner implements in poor and unequal economies features reduced capacity and quality of higher education relative to richer or less unequal countries. Moreover, in order to improve allocative efficiency, the poorer a country is, the more the social planner relies on the usage of lotteries and gate-keeping exams to give access to higher education.

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## 9 Appendix: Model with Negative Transfers

### 9.1 The mechanism design problem

In this appendix, I show how the same qualitative results of the baseline model hold when the planner can make use of negative transfers subject to a global break-even constraint, (9). First, I state the counterpart of Proposition 2 for the optimal mechanism.

**Proposition 15** (*Optimal Schedule with negative transfers*) *If transfers are unrestricted, i.e.,  $t(a, \phi) \in \mathbb{R}$ , the optimal transfer schedule under unrestricted transfers  $t^N(a, \phi)$  is a translation of the restricted schedule  $t^N(a, \phi) = t(a, \phi) - k$  with*

$$k = \int_{\underline{a}}^{\bar{a}} \int_0^{\bar{\phi}} t(a, \phi) dG(a) dF(\phi). \quad (37)$$

*The structure of the optimal probability remains unaltered with  $\psi^N = a^*h(a^*, 1) + k - \underline{a}h(a^*, 0)$ .*

Proposition 15 highlights that the social planner effectively provides cross-subsidization between agents. This is, the social planner anticipates the revenue from transfers of rich agents to reduce the level of all transfers (so that incentives are preserved)

Moreover, Proposition 15 allows to separate the problem in two stages. First, one can define a “virtual” fee and solve for the optimal mechanism ignoring the break-even constraint, (9). Once the mechanism is obtained, there exists a one-to-one transformation to the “real” fees, provided that a sufficient condition for uniqueness in the first stage of the problem is met.

By construction, the virtual transfer is defined as

$$\psi^v = \psi - \int_{a^*(\psi)}^{\bar{a}} dG(a) \left( \int_0^{\psi} \phi dF(\phi) + 1 - F(\psi) \right). \quad (38)$$

The problem can be solved as follows. First find the solution of the restricted problem (i.e.,  $t(a, \phi) \in \mathbb{R}_+$  with virtual transfers, i.e., find the virtual transfers that maximize (12). By definition, these coincide with the solution to the restricted problem. Then, given  $\psi^v$ , find the optimal transfer  $\psi$  that solves (38). This second stage does not need to have unique solution.<sup>37</sup> This shows that the same efficient allocation can be sustained by diverse educational systems if the social planner can cross-subsidize agents. As the only relevant margin for efficient allocation is the virtual fee, which determines the marginal type attending higher education  $a^*$ , two seemingly different alternative mechanisms  $\{\psi_h(\psi^v), \psi_l(\psi^v)\}$  with  $\psi_h > \psi_l$  can coexist. In the mechanism with  $\psi_h$ , the discount from the virtual fee  $k$  is high, but the probability of attending higher education of borrowing constrained is low. On the contrary, for  $\psi_l$  the discount  $k$  is low, but the probability of attending higher education of borrowing constrained agents is relatively higher. In any event, is important to emphasize that because the same virtual fee is implemented, the allocation of agents to schools is the same. Thus, the

<sup>37</sup>This can be seen by taking the derivative

$$\frac{\partial \psi^v}{\partial \psi} = 1 + \frac{\partial a^*}{\partial \psi} g(a^*) \left( \int_0^{\psi} \phi dF(\phi) + 1 - F(\psi) \right) + \int_{a^*(\psi)}^{\bar{a}} dG(a) f(\psi) (1 - \psi).$$

For  $\psi = 0$  the derivative is unambiguously positive, while for  $\psi \rightarrow \infty$  it may be negative.

main object of interest remains to be the solution the constrained problem, i.e., the virtual transfer  $\psi^v$ .

## 9.2 Decentralization: A Fair Lottery Market on Wealth

In this subsection, I discuss how to generalize the decentralization to this relaxed environment. Following the discussion in the main text, the social planner sets a school-contingent tax that has to be satisfied to attend school  $s$  such that

$$\tau(s) = \begin{cases} -k & \text{for } s = 0, \\ \psi^v - k & \text{for } s = 1. \end{cases} \quad (39)$$

Now, consider a market that opens after the social planner announces the school-contingent taxes and before agents attend schools, in which fair lotteries  $l$  over wealth are traded. There is a continuum of these lotteries, indexed by  $i \in [0, \psi^v - k]$ . A lottery  $l_i$  delivers  $\psi^v - k$  with probability  $\pi_i$  and 0 with probability  $1 - \pi_i$ . There is a competitive market for each lottery. Thus the price of lottery  $i$ ,  $p_i$  is given by the break even constraint (or the actuarially fair lottery),  $p_i = \pi_i(\psi^v - k)$ .

The rest of the discussion to show how the decentralization is achieve mimics the main text and is omitted.

## 9.3 School fees

The environment where the only credible mechanism are school fees can be solved in a similar fashion as the full mechanism design problem. First, derive the solution of the restricted problem, in which transfers being restricted to be negative  $t(a, \phi) \in \mathbb{R}_-$ . Then, characterize the unrestricted problem, in which  $t(a, \phi) \in \mathbb{R}$ .

Define the virtual fee as

$$\psi^v = \psi - (1 - F(\psi)) \int_{a^*(\psi)}^{\bar{a}} dG(a). \quad (40)$$

Solve the restricted planner's problem (16) for the virtual price. Once it is found, use the one-to-one positive relationship defined by (40) to determine the optimal fee. Note that this result allows us to focus on  $\psi^v$  for the comparative statics with the wealth distribution, as  $\psi$  inherits the shifts in  $\psi^v$ .

## 10 Appendix: Proofs

**Proof of Proposition 1** The proof is relies on the implicit function theorem. Define  $I(a, \psi) = ah(a, 1) - \underline{a}h(a, 0) - \psi$ . Denoting derivatives with subindexs, it can be verified that  $I_a = h(a, 1) + ah_a(a, 1) - \underline{a}h_a(a, 0) > 0$  (because of the complementarity of the intrinsic human capital production function and the fact that  $a \geq \underline{a}$ ) and  $I_\psi = -1$ . The implicit function theorem states that  $da/d\psi = -I_\psi/I_a > 0$ .  $\square$

**Proof of Proposition 2 and 15** Start considering the environment in which transfers are restricted to be negative,  $t(a, \phi) \in \mathbb{R}_-$ . There are two cases to distinguish. The first case is

the no-segregation, in which all agents attend higher education. In this case  $t(a, \phi) = 0$  and  $\phi(a, \phi) = 1$  for all  $a \in [\underline{a}, \bar{a}]$  and  $\phi \in [0, \bar{\phi}]$  implements the desired allocation. The second case is the segregation case, in which some agents are excluded from higher education. In what follows, let  $\tilde{a} = \min_a \{a \in s = 1\}$ . The proof is presented in a series of lemmas.

**Lemma 1** *Conditional on a given  $\tilde{a}$  and wealth level  $\phi$ , it is optimal to maximize the amount of agents with  $a \geq \tilde{a}$  that attend  $s = 1$ .*

**Proof** By contradiction. Suppose that there exists a mechanism  $\langle \hat{t}, \hat{\pi} \rangle$  that implements the same allocation (respecting constraints (6), (7) and (8)) as the original mechanism  $\langle t, \pi \rangle$ , except for agents with wealth  $\tilde{\phi}$ , in which

$$\begin{aligned} & \int_{\tilde{a}}^{\bar{a}} \hat{\pi}(a, \tilde{\phi}) \tilde{a} h(a, 1) dG(a) + \int_{\underline{a}}^{\tilde{a}} (1 - \hat{\pi}(a, \tilde{\phi})) \underline{a} h(a, 0) dG(a) \\ & > \int_{\tilde{a}}^{\bar{a}} \pi(a, \tilde{\phi}) \tilde{a} h(a, 1) dG(a) + \int_{\underline{a}}^{\tilde{a}} (1 - \pi(a, \tilde{\phi})) \underline{a} h(a, 0) dG(a). \end{aligned}$$

If this is true, then  $\langle t, \pi \rangle$  does not maximize the objective function (11), a contradiction.  $\square$

**Lemma 2** *Consider a mechanism that achieves truth-telling in the wealth dimension. The transfer schedule as a function of the ability reported for a given wealth level  $\phi$  is a step function with a jump at  $\tilde{a}$ .*

**Proof** Consider first agents with  $a < \tilde{a}$ . They are allocated in  $s = 0$  with probability one. Otherwise, the spillover effect for  $s = 1$  would not be  $\tilde{a}$ . Thus, conditional on being allocated in school  $s = 0$  the agent reports the ability that conditional on its wealth maximizes his utility, this is, maximizes the transfer  $t(a, \phi)$ . Thus, for  $a \leq \tilde{a}$ ,  $t(a, \phi) = t(\phi)$ . Moreover, given the single-crossing property of the intrinsic human capital production function, for this allocation to be incentive compatible it has to be the case that

$$w(\tilde{a}, 0) + t(\phi) = \pi(\tilde{a}, \phi) w(\tilde{a}, 1) + (1 - \pi(\tilde{a}, \phi)) w(\tilde{a}, 0) + t(\tilde{a}, \phi), \quad (41)$$

because otherwise agents with  $a < \tilde{a}$  would choose to report ability  $\tilde{a}$ .

Next consider the case of agents with  $\tilde{a} \geq a$ . Given  $(t(\tilde{a}, \phi), \pi(\tilde{a}, \phi))$  and the single-crossing property, it is clear that they choose to report, at least, to be of type  $\tilde{a}$ . If  $t(\tilde{a}, \phi) \leq t(a, \phi)$  and  $\pi(\tilde{a}, \phi) = \pi(a, \phi)$  for  $a > \tilde{a}$ , agents weakly prefer to report type  $\tilde{a}$ , as the payoff from attending  $s = 1$  remains constant but the transfer schedule may not. From Lemma 1, it follows that,  $\pi(\tilde{a}, \phi) = \pi(a, \phi)$  for  $a > \tilde{a}$ , as otherwise output can be increased. To see this, suppose that  $\pi(\tilde{a}, \phi) \leq \pi(a, \phi)$  and  $t(\tilde{a}, \phi) \leq t(a, \phi)$  for  $a > \tilde{a}$ . Then, consider the alternative mechanism such that the highest probability and the lowest transfer are preserved,  $\tilde{\pi}(\tilde{a}, \phi) = \pi(a, \phi)$  and  $\tilde{t}(\tilde{a}, \phi) = \tilde{t}(a, \phi)$ . This alternative mechanism increases the value of the objective function and implements the same allocation of agents to schools. Thus, this analysis shows that it is optimal to set  $(t(a, \phi), \pi(a, \phi)) = (t(\tilde{a}, \phi), \pi(\tilde{a}, \phi))$  for all  $a > \tilde{a}$ .  $\square$

**Lemma 3** *All agents with  $a < \tilde{a}$  attend school  $s = 0$  with probability one, the associated transfer is at most zero irrespective of agents' wealth and ability.*

**Proof** The first claim is already shown in the proof of Lemma 2. For the second claim, consider the poorest, lowest ability agent in the economy,  $(\underline{a}, \phi = 0)$ . Given that he is borrowing constrained, the maximal transfers he can afford is  $t = 0$ . Thus, to satisfy the participation constraint (6) the maximal transfer for an agent of type  $t(\underline{a}, 0) = 0$ . Note that this can be negative. But, given that it is optimal to set  $\pi(a, \phi) = 0$  for all  $a < \tilde{a}$  irrespective of the wealth level, agents with  $a < \tilde{a}$  can always report being of type  $(a, \phi = 0)$  and thus, only, one level of transfers is implemented in equilibrium.  $\square$

**Lemma 4** *Conditional on  $a$ , agents' payoff is maximal for agents with  $(a, \bar{\phi})$ , in particular for  $a \geq \tilde{a}$ . This is implemented by a mechanism in which these agents pay the highest transfers and receive the highest probability of attending  $s = 1$ .*

**Proof** The intuition for the result is as follows. Agents with  $\phi = \bar{\phi}$  that are the “least” borrowing constrained in the economy. Thus, from the incentive compatibility constraint, (7), it follows that these agents can select the highest return announcement. As a result, the truth-telling mechanism that induces agents to report their true wealth has to be “expensive” enough for poorer agents not to be able to imitate them, and has to offer an attractive enough reward, for rich agents being willing to self-select, hence the high probability. The next lines make this intuitive reasoning more precise. Consider the announcement made by a type  $(a, \bar{\phi})$  with  $a \geq \tilde{a}$ , which has associated transfers and probabilities,  $(t(a, \bar{\phi}), \pi(a, \bar{\phi}))$ . By construction, from the incentive compatibility constraint (7), denoting the expected wage of reporting truthfully by  $w(a, \phi)$ , it is the case that  $w(a, \phi) + t(a, \phi) \geq w(a, \hat{a}, \hat{\phi}) - t(\hat{a}, \hat{\phi})$ . Note however that for types with  $\phi < \bar{\phi}$ , if  $t(a, \phi) > \phi$  it is not possible to pretend that they are richer than they actually are. Thus, given the operating assumption that it is optimal to have some segregation, it has to be the case that the fee paid by agents with  $(a, \bar{\phi})$  and  $a \geq \tilde{a}$  is weakly higher than for agents with  $\phi < \bar{\phi}$  and  $a \geq \tilde{a}$ . As a result, given the incentive compatibility constraint, wealthy agents have to be compensated to report a higher wealth by a higher expected wage, which can only be achieved by a (weakly) higher probability  $\pi$  of accessing school 1.  $\square$

**Lemma 5** *Given a threshold  $\tilde{a}$ , it is optimal to allocate agents with  $\phi \geq \hat{\phi}$  and  $a \geq \tilde{a}$ , where  $\tilde{a}h(a, 1) - \underline{a}h(a, 0) = \hat{\phi}$  to school 1 with probability 1.*

**Proof** Note first that  $\hat{\phi}$  is to be readily interpreted as  $t(a, \phi) = -\hat{\phi}$  for all  $\phi \geq \hat{\phi}$  and  $a \geq \tilde{a}$ . From Lemma (1) it is immediate to check that if this allocation is implementable it is optimal provided that it does not distort the inframarginal allocation of types with  $\phi < \hat{\phi}$ . But, by construction, agents with  $\phi < \hat{\phi}$  cannot afford signaling themselves as having wealth  $\hat{\phi}$ . To satisfy the incentive compatibility, it has to be ensured that the expected return of reporting  $(a, \phi)$  with  $\phi < \hat{\phi}$  and  $a \geq \tilde{a}$ ,  $w(a, \phi) + t(a, \phi)$  is less or equal to  $H(a, 1) + \hat{\phi}$ . Note that this restriction does not impose additional constraints. The reason is simple:  $t(a, \phi) = \hat{\phi}$  is the minimal transfer consistent with threshold  $\tilde{a}$ . (And note again that by Lemma 1 it would not be optimal to set a probability lower than one).  $\square$

**Lemma 6** *Given a threshold  $\tilde{a}$ , the optimal mechanism for agents with  $\phi < \hat{\phi}$  and  $a \geq \tilde{a}$  is*

$$t(a, \phi) = -\phi, \quad \pi(a, \phi) = \frac{\phi}{\hat{\phi}} \quad (42)$$

**Proof** The optimal schedule comes from maximizing the mass of agents with  $a \geq \tilde{a}$  of a particular wealth level that attends  $s = 1$ . Once this is done, it remains to be checked that truth-telling is optimal.

Consider agents with wealth  $\phi < \hat{\phi}$ . Note that these agents are borrowing constrained, and thus cannot announce to be of type  $(a, \hat{\phi})$  for  $a \geq \tilde{a}$ . Using Lemma 2, the transfer-probability pair that maximizes attendance of agents above  $\tilde{a}$  to school 1 has to satisfy the condition at the boundary  $\tilde{a}$ ,

$$\pi(t(\phi), \hat{\phi})\tilde{a}h(\tilde{a}, 1) + (1 - \pi(\phi, \hat{\phi}))\underline{a}h(\tilde{a}, 0) + t(\phi) = \underline{a}h(\tilde{a}, 0) \quad (43)$$

First, note that it is suboptimal to set  $t(\phi) > -\phi$ . Suppose, to the contrary, that the optimal school fee is less than  $\phi$ . As  $\pi(t(\phi), \hat{\phi})$  is a strictly increasing function of  $t$ , this implies that by setting  $t(\phi) < \phi$  the mass of agents attending school 1 can be increased by setting  $t(\phi) = \phi$ . Second, setting  $t(\phi) = \phi$  does not affect the incentives of agents with wealth strictly lower than  $\phi$ . For agents with wealth above  $\phi$ , from equation (43) it can be verified that they are indifferent (or strictly prefer if they have  $\phi > \hat{\phi}$ ) between the transfer-probability designed for them and this alternative. From this analysis, equation (42) follows.  $\square$

The previous series of lemmas show that the optimal mechanism takes the form of a menu of prices, as stated in the main proposition. I now discuss the case of unrestricted transfers,  $t(a, \phi) \in \mathbb{R}$ . In this case, it is clear that it is always weakly better to set the transfers so that the borrowing constraint of the social planner breaks-even. (If there are agents borrowing constrained is strictly better). From the incentive compatibility conditions, equation (7), it is clear that the only relevant object as far transfers are concerned for agents when considering to deviate from truth-telling is the difference between transfers,  $t(a, \phi) - t(a^\dagger, \phi^\dagger)$ . Thus, letting  $k$  denote the revenue (in negative terms) from the transfers when they were constrained to be negative,

$$k = \int_{\underline{a}}^{\tilde{a}} \int_0^{\hat{\phi}} t(a, \phi) dG(a) dF(\phi), \quad (44)$$

it is immediate to verify that a lump-sum decrease in the transfer schedule of the type  $t(a, \phi) - k$ , does not modify any of agents' decisions. So, the threshold  $\tilde{a}$  is still implemented. Yet, now agents are effectively less borrowing constrained, and thus the mass of agents that effectively can attend higher education increases.  $\square$

**Derivation of equation 14 and proof of sufficiency of FOC** The derivation of expression (14) comes from taking the derivative of the objective function with respect to  $\psi$ . Note, moreover, that for the first term in (14) the integration with respect to wealth is independent of ability. Using the Leibniz rule, this can be written as

$$\begin{aligned} & -f(\psi) \int_{a^*}^{\tilde{a}} \Delta w dG(a) + (1 - F(\psi)) \int_{a^*}^{\tilde{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) - (1 - F(\psi)) \frac{\partial a^*}{\partial \psi} (\Delta w|_{a=a^*}) g(a^*) + \\ & + f(\psi) \int_{a^*}^{\tilde{a}} \Delta w dG(a) + \int_0^\psi \int_{a^*}^{\tilde{a}} \frac{\partial}{\partial \psi} \left( \frac{\phi}{\psi} \Delta w \right) dG(a) dF(\phi) - \int_0^\psi \frac{\partial a^*}{\partial \psi} \left( \frac{\phi}{\psi} \Delta w|_{a=a^*} \right) g(a^*) dF(\phi). \end{aligned}$$

The terms appearing in the first line come from the derivative of the first term of the objective function, and the terms on the second line, correspond to the derivative of the second term.

Note that by the incentive compatibility constraints,  $\frac{\phi}{\psi}\Delta w|_{a=a^*} - \phi = 0$  for all  $\phi \leq \psi$ . Next, take the derivative of the integrand of the second term of the second line

$$\frac{\partial}{\partial \psi} \left( \frac{\phi}{\psi} \Delta w \right) = -\frac{\pi(\phi, \psi)}{\psi} \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right).$$

This allows to rewrite the integrand that contains this derivative as

$$\int_0^\psi \int_{a^*}^{\bar{a}} \frac{\pi(\phi, \psi)}{\psi} \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right) dG(a) dF(\phi) = 0.$$

Noting that the second integral can be expressed as

$$\int_0^\psi \frac{\pi(\phi, \psi)}{\psi} dF(\phi) \int_{a^*}^{\bar{a}} \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right) dG(a),$$

the result stated in the main text, equation (14), follows.

For the concavity of the objective function, I provide a sufficient condition for concavity. Instead of showing that the first order condition is decreasing, I analyze the stronger condition that the first order condition multiplied by an increasing function is still decreasing. The increasing function chosen is  $1/(1 - F(\phi))$ . Thus, I want to show that the following function is decreasing in  $\psi$ :

$$\begin{aligned} & \int_{a^*(\psi)}^{\bar{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) - \frac{\partial a^*}{\partial \psi} \Delta w(a^*) g(a^*) \\ & - \frac{\frac{1}{\psi} \int_0^\psi \pi(\phi, \psi) dF(\phi)}{1 - F(\psi)} \left( \frac{\partial a^*}{\partial \psi} \psi \Delta w(a^*) g(a^*) + \int_{a^*(\psi)}^{\bar{a}} \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right) dG(a) \right) \quad (45) \end{aligned}$$

In fact, as the next proof shows, this stronger condition ensures uniqueness of the solution as well. Before proceeding, there is a need to introduce an intermediate result regarding the concavity of  $a^*(\psi)$ .  $\square$

**Proposition 16** (Concavity of  $a^*(\psi)$ ) *The marginal type obtaining education  $\tilde{a}(\psi)$  is concave in  $\psi \in \Psi$  if and only if  $2\partial h(a, 1)/\partial a + \underline{a}|\partial^2 h(a, 0)/\partial a^2| \geq a|\partial^2 h(a, 1)/\partial a^2|$  for all  $a \in [\underline{a}, \bar{a}]$ .*

**Proof** The proof relies on the implicit function theorem. Define  $I(a, \psi) = ah(a, 1) - \underline{a}h(a, 0) - \psi$ . Denoting derivatives with subindexes, it can be verified that  $I_a = h(a, 1) + ah_a(a, 1) - \underline{a}h_a(a, 0) > 0$  (because of the complementarity of the intrinsic human capital production function and the fact that  $a \geq \underline{a}$ ,  $I_\psi = -1$ ,  $I_{a\psi} = 0$  and  $I_{aa} = 2h_a(a, 1) + ah_{aa}(a, 1) - \underline{a}h_{aa}(a, 0)$ ). The implicit function theorem implies the second derivative that  $d^2a/d\psi^2 = -(I_{\psi\psi} + 2I_{a\psi}da/d\psi + I_{aa}(da/d\psi)^2)/I_a < 0$ , if and only if  $I_{aa} > 0$ . The sufficient condition mentioned in the next paragraph is obtained from ignoring the term  $2h_a(a, 1)$  in the expression of  $I_{aa}$ . In this case, the condition for  $I_{aa} \geq 0$  is

$$\frac{a}{\underline{a}} h_{aa}(a^*, 1) \geq h_{aa}(a^*, 0),$$

as  $a \geq \underline{a}$ , the sufficient condition stated in the text follows.  $\square$



Proposition 16 identifies the conditions under which the threshold type  $\bar{a}$  is concave in school fee  $\psi$ . This condition is trivially satisfied by intrinsic production functions of the type  $h(a, s) = a\hat{h}(s)$ , as in this case  $h_{aa}(a, s) = 0$ . For more general human capital functions whether this condition is satisfied depends on the shape of the production function and the support of the ability distribution. For example if  $|\partial^2 h(a, 1)/\partial a^2|/|\partial^2 h(a, 0)/\partial a^2| \leq \underline{a}/\bar{a}$ . I proceed making the assumption that  $a^*(\psi)$  is concave.

**Assumption 3**  $a^*(\psi)$  is concave for  $\psi \in \Psi$ . This is,

$$2\partial h(a, 1)/\partial a + \underline{a}|\partial^2 h(a, 0)/\partial a^2| \geq a|\partial^2 h(a, 1)/\partial a^2| \text{ for all } a \in [\underline{a}, \bar{a}].$$

Consider terms in the first line of (45), direct differentiation shows that it is decreasing in  $\psi$ ,

$$\begin{aligned} \frac{d}{d\psi} \left( \int_{a^*(\psi)}^{\bar{a}} \frac{\partial \Delta w}{\partial \psi} dG(a) - \frac{\partial a^*}{\partial \psi} \Delta w(a^*) g(a^*) \right) &= \int_{a^*(\psi)}^{\bar{a}} g(a) \left( \frac{\partial^2 \Delta w}{\partial \psi^2} \right) da - \frac{\partial a^*}{\partial \psi} \left( \frac{\partial \Delta w}{\partial \psi} \right) \Big|_{a=a^*} \\ &\quad - \frac{\partial^2 a^*}{\partial \psi^2} \Delta w(a^*) g(a^*) \\ &\quad - \frac{\partial a^*}{\partial \psi} \frac{\partial \Delta w(a^*)}{\partial \psi} g(a^*) < 0. \end{aligned} \quad (46)$$

To obtain the result that the derivative is decreasing, note that the integrand in the first term of right hand side of (46) is equal to  $\frac{\partial^2 a^*}{\partial \psi^2} h(a, 1)$ . Thus the difference between the first and third terms is negative because  $h(a, 1)$  is weakly increasing in  $a$ ,  $h(a, 1) \geq ah(a, 1) - \underline{a}h(a, 0)$  (recall that  $\bar{a} \leq 1$ ) and the ability distribution is uniform. Note, moreover, that  $\frac{\partial g(a^*)}{\partial \psi} = 0$  because of the assumption that the ability distribution is uniform.

Consider the term in parenthesis in the second line of (45). Applying the Leibniz rule,

$$\begin{aligned} \frac{d}{d\psi} \left( \int_{a^*(\psi)}^{\bar{a}} g(a) \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right) dG(a) + \frac{\partial a^*}{\partial \psi} \psi \Delta w(a^*) g(a^*) \right) &= \\ - \int_{a^*(\psi)}^{\bar{a}} \frac{\partial^2 \Delta w}{\partial \psi^2} g(a) da - a^{*'}(\psi) \left( \Delta w - \psi \frac{\partial \Delta w}{\partial \psi} \right) \Big|_{a=a^*} g(a^*) &\quad (47) \\ + \frac{\partial^2 a^*}{\partial \psi^2} \psi \Delta w(a^*) g(a^*) + \frac{\partial a^*}{\partial \psi} \Delta w(a^*) g(a^*) + \frac{\partial a^*}{\partial \psi} \psi \frac{\partial \Delta w(a^*)}{\partial \psi} g(a^*) &> 0. \end{aligned}$$

The result that the derivative is increasing follows from an analogous argument to the one used in the previous derivative, equation (46).

Finally, note that the reminder term, in (45),

$$\frac{\frac{1}{\psi} \int_0^\psi \pi(\phi, \psi) dF(\phi)}{1 - F(\psi)},$$

is increasing by assumption in the text. This analysis shows that (45) is decreasing, and, hence, the objective function is globally concave.  $\square$

**Analysis of equation 14** Consider the right hand side of (14). From previous proof, equations (46) and (47) show that the numerator of the right hand side is decreasing and the denominator is increasing. Hence, the right hand side is decreasing.

For the left hand side, the main text already discusses under which conditions it is increasing, which is the working assumption in any case. As the LHS is increasing and the RHS is decreasing, they can cross almost once. Indeed, this is the case if the solution is assumed to be interior, which is the environment of interest discussed in the main text.  $\square$

**Proof of Proposition 3** The results follows immediately for the virtual fee  $\psi^v$ . The LHS of equation (12) being increasing, the RHS, decreasing and the property that a Wealth Abundance shift the whole LHS curve downwards. The RHS remains unaffected. As both the LHS and RHS are continuous and monotone the result follows.  $\square$

**Proof of Proposition 4** The mass of agents attending higher education is given by

$$\int_{a^*(\psi)}^{\bar{a}} dG(a) \left( \int_0^\psi dF(\phi) + 1 - F(\psi) \right).$$

In the limiting case in which no agent is borrowing constrained, the mass of agents attending school reduces to

$$\int_{a^*(\psi)}^{\bar{a}} dG(a),$$

as  $f(\phi) < \varepsilon$  for a small  $\varepsilon > 0$  in the range  $\phi \in [0, \psi]$ . The first order condition of the objective function, (4), for the case without borrowing constraints can be written as

$$\frac{d}{d\psi} \ln \left( \int_{\bar{a}(\psi)}^{\bar{a}} \Delta w dG(a) \right) = 0. \quad (48)$$

Let  $\psi^{unc}$  denote the solution to the problem without borrowing constraints, (48). When borrowing constraints start to bind, assuming

$$\frac{\frac{1}{\psi} \int_0^\psi \pi(\phi, \psi) dF(\phi)}{1 - F(\psi)} \leq \varepsilon$$

for some “small”  $\varepsilon > 0$  in the relevant range, the first order condition becomes approximately

$$\frac{d}{d\psi} \ln \left( \int_{a^*(\psi)}^{\bar{a}} \Delta w dG(a) \right) = \varepsilon. \quad (49)$$

Denoting the solution to the problem by  $\psi^{bc}$ , it follows that  $\psi^{bc} \simeq \psi^{unc} - \delta(\varepsilon)$  for some  $\delta(\varepsilon) > 0$ , where the dependence on  $\varepsilon$  is carried over to emphasize the dependence in the approximation of the solution. Now, approximating the integral

$$\int_{a^*(\psi^{bc})}^{\bar{a}} dG(a) \simeq \int_{\bar{a}(\psi^{unc})}^{\bar{a}} dG(a) - \frac{\partial \bar{a}}{\partial \psi} \delta(\varepsilon) g(\bar{a}), \quad (50)$$

the difference in the mass of agents attending higher education with borrowing constraints

minus the mass without reduces to

$$(1 - F(\psi^{unc} - \delta)) \left( \int_{\tilde{a}(\psi^{unc})}^{\tilde{a}} dG(a) - \frac{\partial \tilde{a}}{\partial \psi^{unc}} \delta(\varepsilon) g(\tilde{a}) - \int_{\tilde{a}(\psi^{unc})}^{\tilde{a}} dG(a) \right) - F(\psi^{unc} - \delta) \int_{\tilde{a}(\psi^{unc})}^{\tilde{a}} dG(a), \quad (51)$$

where I have used the fact that borrowing constraints “start to bind” and thus  $\int_0^{\psi^{unc} - \delta} \phi f(\phi) \ll (1 - F(\psi^{unc} - \delta))$ . Expression (51) is unambiguously negative. This shows the result stated in the proposition.  $\square$

**Proof of Proposition 8** I first show that the relaxed condition (which implies increasing hazard rate) implies local concavity at the solution of the first order condition. Take the derivative of (17) with respect to  $\psi$ . Before analyzing the sign of the LHS of the FOC (17), it is convenient to take the derivative of the first term RHS of the FOC. Its derivative with respect to  $\psi$  is unambiguously negative,

$$-f'(\psi) \int_{a^*(\psi)}^{\tilde{a}} \frac{d\Delta w}{d\psi} dG(a) - (1 - F(\psi)) g(a^*) \frac{da^*}{d\psi} \Big|_{a=a^*} + (1 - F(\psi)) \int_{a^*(\psi)}^{\tilde{a}} \frac{d^2 \Delta w}{d\psi^2} dG(a) da < 0.$$

The analysis of the derivative of the second term is analogous to the previous proposition and is omitted. The derivative of the LHS in the FOC (17) is

$$-f'(\psi) \int_{a^*(\psi)}^{\tilde{a}} \Delta w dG(a) - f(\psi) \int_{a^*(\psi)}^{\tilde{a}} \frac{dw}{d\psi} dG(a) + f(\psi) \frac{da^*}{d\psi} \frac{dw}{d\psi} g(a^*). \quad (52)$$

The sum of the second term and third terms is negative for an analogous reason as in the previous proposition. This first term has an ambiguous sign, as  $f'()$  can be either positive or negative. Take the expression of the derivative of the first term of the FOC, equation (52) and use that, at the optimum  $\psi$ , the FOC (17) is satisfied to rewrite (the ambiguous part of) equation (52) as

$$\left( -f'(\psi) - 2 \frac{f'(\psi)^2}{1 - F(\psi)} \right) \int_{a^*(\psi)}^{\tilde{a}} \Delta w dG(a).$$

As the integrand is always positive, the sign of this expression is the sign of the term multiplying the integral,  $\left( -f'(\psi) - 2 \frac{f'(\psi)^2}{1 - F(\psi)} \right)$ . The sign of this first term coincides with  $(1 - F(\psi))''(1 - F(\psi)) - 2f'(p)^2$ , which in turn implies the relaxed condition.

To ensure uniqueness of the solution with an increasing the hazard rate, divide through the first order condition by  $1 - F(\psi)$  and proceed exactly as in the previous proposition. Namely show that the without the hazard rate is decreasing in  $\psi$  and the term that is multiplying the hazard rate is increasing. Then a sufficient condition for uniqueness is an increasing hazard rate.  $\square$

**Proof of Proposition 9** The proof is analogous to the proof of Proposition 4 and it is omitted.

**Proof of Proposition 10** The analysis in the main text identifies that the exams may not be used in wealth abundant countries. The problem of interest is the comparative statics

when they are used, which can be written as

$$\max_{a^*, \tilde{a}} (1 - F(\psi_0)) \int_{a^*}^{\tilde{a}} \Delta w dG(a) + \int_{\tilde{a}}^{\bar{a}} dG(a) \int_{\psi_0 - c(\tilde{a}, 1) + c(a, 1)}^{\bar{\phi}} dF(\phi) (\Delta w - c(a, 1)) \quad (53)$$

The problem can be thought as being solved in a “telescopic” way. That is, solve for the optimal  $\tilde{a}(a^*)$  and then solve for the optimal  $a^*$ . The FOC for  $\tilde{a}$  is

$$\frac{c(\tilde{a}, 1)}{c_a(\tilde{a}, 1)} = \int_{\tilde{a}}^{\bar{a}} \frac{f(\psi_0)}{1 - F(\psi_0)} (\Delta w - c(t, a)) dG(a). \quad (54)$$

Note that the left hand side is increasing in  $a$  because of the concavity assumption  $c_{aa} \leq 0$ . The right hand side is decreasing in  $\tilde{a}$ , increasing in  $a^*$  and a wealth abundance shift. To see this last point, note that one can rewrite  $\int_{\tilde{a}}^{\bar{a}} f(\psi_0 - c(\tilde{a}, 1) + c(a, 1)) / (1 - F(\psi_0))$  as  $\int_{\phi_1}^{\phi_2} f(\phi) d\phi / (1 - F(\phi_2))$ , and thus one can apply directly the result from the remark. Thus,  $\tilde{a}$  is increasing in  $a^*$  and decreasing in a hazard rate shift. Rewrite the objective function with  $\psi_0(a^*)$  and  $\tilde{a}(a^*)$ . Then, the first order condition can be written as,

$$\begin{aligned} 0 = & -\frac{f(\psi_0)}{1 - F(\psi_0)} \psi'_0(a^*) \int_{a^*}^{\tilde{a}} \Delta w dG(a) + \int_{a^*}^{\tilde{a}} \Delta w dG(a) \\ & - \Delta w(a^*) - \tilde{a}'(a^*) \int_{\psi_0 - c(\tilde{a}, 1)}^{\bar{\phi}} (\Delta w(\tilde{a}) - c(\tilde{a}, 1)) \frac{dF(\phi)}{1 - F(\psi_0)} \\ & \int_{\tilde{a}(a^*)}^{\bar{a}} \frac{dG(a)}{1 - F(\psi_0)} \left( -(\psi'_0(a^*) - c_a(\tilde{a}, 1) \tilde{a}'(a^*)) f(\psi_1 + c(a, 1)) (\Delta w - c(a, 1)) + \int_{\psi_1 + c(a, t)}^{\bar{a}} \frac{\partial \Delta w}{\partial a^*} dF(\phi) \right) \end{aligned} \quad (55)$$

One can use an analysis analogous to Proposition 3 to show that the first order condition is decreasing provided that the same regularity condition on  $\int_{\phi_1}^{\phi_2} f(\phi) d\phi / (1 - F(\phi_2))$  of the mechanism design problem applies here to show that the first order condition is decreasing. Using the MLRP property of the wealth abundance definition as in the Remark in page 20 it follows that  $a^*$  is decreasing in a wealth abundance shift that reduces  $\int_a^b f(\phi) d\phi / (1 - F(\phi))$ . Thus,  $\tilde{a}$  decreases with a wealth abundance shift as both the direct effect in (54) and the effect through  $a^*$  in (55) go in the same direction.  $\square$

The omitted proofs and those corresponding to section 7.2 are discussed in the main text and the formal proof is omitted.

## 11 Appendix: Optimal Test-Fee Schedule Problem

This appendix presents a general solution to the optimal test-fee schedule that encompasses the results in Section 6 and an analogous exam problem with a continuum of schools, as in Subsection 7.2. (The latter is not discussed in the main text.)

Consider a payoff structure in the objective function of the type

$$\int_{a^*}^{\bar{a}} (w(a, a^*) - \kappa - c(t, a)) (1 - F(p(a) + c(t, a))) dG(a), \quad (56)$$

subject to three constraints. (Note the use of  $p$  instead of  $\psi$ ). First, the incentive compatibility constraint

$$\zeta(a) - \dot{p}(a) - c_t(t, a)\dot{t} = 0, \quad (57)$$

where the operator dot stands for the total derivative with respect to  $a$ , and the subindex  $t$ , for the partial derivative with respect to  $t$ . Second, the possible levels for  $a^*$  have to belong to the family of curves of the type

$$g(a^*) \equiv w(a^*, a^*) - p(a^*) - c(a^*, t(a^*)) = 0. \quad (58)$$

Third, the monotonicity constraints,

$$p'(a) \leq 0, \quad t'(a) \geq 0 \quad \forall a \in [a^*, \bar{a}]. \quad (59)$$

The problem at hand is to

$$\max_{\{a^*, p(a), t(a)\}} \int_{a^*}^{\bar{a}} (w(a, a^*) - \kappa - c(t, a))(1 - F(p(a) + c(t, a)))dG(a) \quad (60)$$

subject to (57), (58) and (59). This problem almost fits the standard formulation of optimal control/calculus of variation. The only difference is that the initial condition  $a^*$  enters directly through  $w(\cdot, a^*)$  the objective. In order to solve the problem fully, I proceed in two steps. First, conditional on a threshold  $\hat{a}^*$  on  $w(\cdot, \hat{a}^*)$ , I solve an inner optimization problem and find the optimal  $p$ ,  $t$  (and boundary conditions) conditional on  $\hat{a}^*$ . This inner problem is formulated as an optimal control problem in subsection 11.1 (and subsection 11.2 shows the equivalence with a more intuitive formulation using calculus of variations). Then, the outer problem simply consists on a pointwise maximization of the objective with respect to  $\hat{a}^*$  subject to  $\hat{a}^* = a^*$ .

The following two lemmas simplify the analysis.

**Lemma 7** *Any optimal solution features  $t(a^*) = 0$ .*

**Proof** By contradiction. Suppose the opposite,  $(p(a^*), t(a^*))$  with  $t(a^*) > 0$ . Now, consider an alternative plan with  $\tilde{t}(a^*)$  and  $\tilde{p}(a^*) = p(a^*) + c(a^*, t(a^*))$  (note that by assumption  $c(a, 0) = 0$ ). By construction, constraint (58) is satisfied. Yet, the objective function (56) increases under this alternative plan. A contradiction.  $\square$

**Lemma 8** *It is not optimal to set  $t(a) = 0$  for  $a \in (a^*, a^* + \varepsilon)$  with  $\varepsilon > 0$ .*

**Proof** I show that for every  $a$  within a radius  $\varepsilon$  exists a positive exam level that improves upon a zero test level. Suppose that the optimal solution features  $t(a) = 0$ . Consider the alternative policy of  $t(a) = \delta > 0$ . Use a first order approximation to write  $p(a) \simeq p(a^*) + \dot{p}(a^*)\varepsilon$  and  $c(a, t) \simeq c_t(a, 0)\eta$ . From equation (20), the difference in output from this change in policy is proportional to

$$\varepsilon f(p(a^*))w(a^*, a^*)(-\dot{p}(a^*)\varepsilon - c_t(a^*, 0)\eta). \quad (61)$$

From equation (21), the wasteful spending is

$$\varepsilon f(p(a^*))c_t(a^*, 0)\eta. \quad (62)$$

Thus, selecting a  $\eta$  such that

$$\eta < \frac{(-\dot{p}(a^*)\varepsilon w(a^*, a^*))}{1 + w(a^*, a^*)}, \quad (63)$$

increases the objective function without violating any constraint.  $\square$

### 11.1 Optimal Control Formulation of the Inner Problem

Define the state variable  $x(a) = p(a) + c(a, t)$ , and the control variable as  $t(a)$ . The incentive compatibility condition (57) and the boundary condition can be written as

$$\dot{x} = \xi(a) + c_a(t, a), \quad (64)$$

$$0 = w(\hat{a}^*) - x(a^*) + p_0 \equiv g(\hat{a}^*, x(a^*), p_0) \quad (65)$$

Note that at this inner stage of the problem  $\hat{a}^*$  is taken as given, and it will be optimized over in the outer problem (subject to  $a^* = \hat{a}^*$ ). As it is usually done in this types of problems, I proceed by ignoring the monotonicity constraints (59) and verifying that they hold ex-post. This allows to express the problem in a simpler manner and avoid discussing ironing and bunching procedures. Moreover, as it will become apparent, the same properties of the solution emphasized in the text arise when using monotonicity constraints in an optimal control problem.

Define the following Hamiltonian,

$$\mathcal{H} = \int_{a^*}^{\bar{a}} [(w(a, \hat{a}^*) - \kappa - c(t, a))(1 - F(x)) + \lambda_1(a) (\xi(a) + c_a(t, a))] dG(a) + \lambda_2 g(x(a^*), \hat{a}^*, p_0).$$

The necessary conditions for an optimum are<sup>38</sup>

$$\dot{x}(a) = \xi(a) + c_a(t, a), \quad (66)$$

$$\dot{\lambda}(a) = f(x)(w(a, a^*) - \kappa - c(t, a)), \quad (67)$$

$$0 = -c_t(t, a)(1 - F(x)) + \lambda_1(a)c_{at}(t, a), \quad (68)$$

and the boundary conditions

$$\lambda_1(a^*) = -\lambda_2, \quad (69)$$

$$\lambda_1(\bar{a}) = 0. \quad (70)$$

Equations (66) and (67) form a system of differential equations in  $x$  and  $\lambda$ , intermediated through the control  $t$ , (68). This system is somewhat complicated by the fact that boundary conditions are given at opposite ends. In any case, the system of equations (66) to (68), the boundary conditions (69) and (70), and the constraint (65) characterize the solution of the problem (there are 2 differential equations with two boundary conditions to pin down  $\lambda_1$  and  $x$ , and 2 additional equations to pin down  $t$  and  $\lambda_2$ ).

I now proceed to manipulate the system of differential equations in order to investigate how the optimal solution depends on the wealth distribution. Similar to the cases analyzed in the main text, the key element is the hazard ratio of the wealth distribution. Rearranging

<sup>38</sup>These can be found, for example, in Chachuat (2007) or Luenberger (1969)

(68),

$$\lambda_1(a) = \frac{c_t(t, a)}{c_{at}(t, a)}(1 - F(x)),$$

and taking the total derivative with respect to  $a$ ,

$$\dot{\lambda}(a) = \frac{d}{da} \left( \frac{c_t(t, a)}{c_{at}(t, a)} \right) (1 - F(x)) - \frac{c_t(t, a)}{c_{at}(t, a)} f(x) \dot{x}. \quad (71)$$

This expression makes clear how the assumption that  $c(a, t) \equiv c_1(t)c_2(a)$  greatly simplifies the analysis: in this case  $c/c_a$  is independent of  $t$ . For example, equation (67) can be written now as

$$c_1(t) = \frac{1}{c_2(a)} \left( w(a) - \kappa - \frac{\dot{\lambda}(a)}{f(x)} \right). \quad (72)$$

Using (72) and (71) in (66), omitting dependence from  $a$ , and denoting  $c_2$  by just  $c$ ,

$$\dot{x} = \xi + \frac{\dot{c}}{c}(w - k) - \frac{\dot{c}}{cf(x)} \left[ \frac{d}{da} \left( \frac{c}{\dot{c}} \right) (1 - F(x)) - \dot{x} f(x) \frac{\dot{c}}{c} \right], \quad (73)$$

which results into

$$\frac{1 - F(x)}{f(x)} = \frac{1}{\frac{\dot{c}(a)}{c(a)} \frac{d}{da} \left( \frac{c(a)}{\dot{c}(a)} \right)} \left( \xi(a) + \frac{\dot{c}(a)}{c(a)} (w(a) - k) \right). \quad (74)$$

It can be verified by direct derivation that log-convexity of  $c$  is sufficient to guarantee that

$$\frac{\dot{c}(a)}{c(a)} \frac{d}{da} \left( \frac{c(a)}{\dot{c}(a)} \right) > 0.$$

Moreover, log-convexity implies that the terms in brackets in (72) is strictly increasing in  $a$ . The assumptions made in the main text guarantee that the right hand side is increasing in  $a$  and the left hand side decreasing. For the two school case, Section 6,  $\xi(a) = 0$ , and then it follows from log-convexity that the right hand side is increasing. For the case in Section 7.2, a sufficient condition for the left hand side to be increasing is that  $\xi(a)$  grows at a faster rate than  $\dot{c}/c$ . Thus,  $x$  is an increasing function of  $a$ .

Consider a family of wealth distributions  $f(x; s)$  that can be ranked in their hazard-rate according to an index  $s \in \mathbb{R}$ , so that the hazard rate is decreasing in  $s$ , (i.e., high  $s$  are relatively wealth abundant countries). It is immediate to verify that

$$\frac{\partial x}{\partial s} > 0. \quad (75)$$

That is, more wealth abundant societies use higher  $x$  at each level of  $a$ . An analogous argument can be done for changes in the wealth dispersion. Now, integrating equation (66)

$$x = \int \dot{x} da = \int_{a^*}^a \xi(a) da + \int_{a^*}^a \dot{c}_2(a) c_1(t(a)) da, \quad (76)$$

it is immediate to verify that any increase in  $x(a)$  has to be accompanied with a decrease in

$t(a)$ , as  $\dot{c}_2 < 0$ .

For the terminal condition, combine equation (69) and (68) to find that

$$(1 - F(x(a^*))) = -\lambda_2 \frac{\dot{c}_2(a^*)}{c_2(a^*)}. \quad (77)$$

The left hand side is decreasing in  $a$  while the right hand side is increasing. This makes apparent that a more wealth abundant country chooses a higher  $a^*$ .

## 11.2 Calculus of Variations Formulation of the Inner problem

In this subsection, I briefly show how a calculus of variations approach in which explicitly the two functions over which the problem is optimized are  $p$  and  $t$  yields the same set of necessary conditions. Construct the following lagrangean

$$\mathcal{L} = \int_{a^*}^{\bar{a}} [(w(a, a^*) - \kappa - c(t, a))(1 - F(p(a) + c(t, a))) - \lambda_1(a)(\zeta(a) - \dot{p}(a) - c_t(t, a)\dot{t})] dG(a)$$

Now, the problem under consideration is

$$\max_{t(a), p(a), \lambda_1(a)} \mathcal{L} \quad \text{s.t.} \quad w(a^*) - p(a^*) = 0, \quad (78)$$

where I have used the result of Lemma 7 to simplify constraint (58). The necessary conditions for the solution are the Euler-Lagrange equations

$$\frac{d}{da} \mathcal{L}_{\dot{x}} = \mathcal{L}_x, \quad (79)$$

where  $x = \{t(a), p(a), \lambda_1(a)\}$ . These are

$$\dot{\lambda}(a) = f(p(a) + c(a, t))(w(a, a^*) - \kappa - c(a, t)) \quad (80)$$

$$\dot{\lambda}(a) + \frac{c_{at}(t, a)}{c_t(t, a)} \lambda(a) = 1 - F(p(a) + c(a, t)) + f(p(a) + c(a, t))(w(a, a^*) - \kappa - c) \quad (81)$$

$$0 = \zeta(a) - \dot{p}(a) - c_t(t, a)\dot{t}. \quad (82)$$

Note that (80) is equivalent to (67). Combining (80) and (81) one obtains (68), while (82) is merely the incentive compatibility constraint as (66). Thus the set of equations are equivalent.

## 11.3 Formulation of the Outer Problem

Once the solution of the inner problem has been found, it remains to ensure that the optimal  $a^*$  has been selected. This can be done by point-wise optimization

$$\max_{\hat{a}^*} \int_{a^*}^{\bar{a}} [(w(a, \hat{a}^*) - \kappa - c(t(a, \hat{a}^*), a))(1 - F(x(a, \hat{a}^*)))] dG(a) \quad \text{s.t.} \quad a^* = \hat{a}^*. \quad (83)$$



The dependence of  $x$  and  $t$  with respect to  $a^*$  is through  $w$  (see equation (74), for instance). The first order condition is

$$\begin{aligned} \int_{a^*}^{\bar{a}} \frac{\partial(w(a, \hat{a}^*) - c(t(a, \hat{a}^*), a))}{\partial \hat{a}^*} (1 - F(x)) dG(a) + \lambda = \\ \int_{a^*}^{\bar{a}} (w(a, a^*) - \kappa - c(t, a)) f(x) \frac{\partial x(a, a^*)}{\partial a^*} dG(a), \end{aligned} \quad (84)$$

where  $\lambda$  is the Lagrange multiplier on the constraint. Using the specifics of the model of interest (note that the problem with the continuum of schools only has the inner problem to be solved),  $w(a, a^*) = a^* h(a, 1) - \underline{h}(a, 0)$ , it follows that

$$\frac{\partial^2 w(a, a^*)}{\partial a^{*2}} = 0.$$

If the hazard rate of the wealth distribution is concave, the first term in the first line of equation (84) is decreasing in  $a^*$ . To ensure that the first order yields a maximum, it is necessary to impose more structure on the wealth distribution. A sufficient condition is that the wealth distribution is log-concave, see [Bagnoli and Bergstrom \(2005\)](#). In this case, the the second line of equation (84) is increasing in  $\hat{a}^*$ , as  $1 - F(x)$  and the hazard rates are ensured to be concave. (Note that  $x$  depends on the inverse hazard rate, from equation 74).