A Worker's Backpack as an alternative to PAYG pension systems

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Abstract

With ageing population and historical trends of low employment rates, pay-as-you-go (PAYG) pension systems, currently in place in several European countries, imply very large economic and welfare costs in the coming decades, threatening the sustainability of these systems. In an overlapping generations economy with incomplete insurance markets and frictional labour markets, an employment fund, which can be used while unemployed or retired can enhance production efficiency and social welfare. With an appropriate design, the sustainable Backpack employment fund (BP) can greatly outperform – measured by average social welfare in the economy – existing pay-as-you go systems and also Pareto dominate a full privatization of the pension system, as well as a standard fully funded defined contribution pension system. We show this in a calibrated model of the Spanish economy, by comparing steady-state economies after the ongoing demographic transition under these different pension systems and by showing how a front-loaded transition from the PAYG to the BP system, ahead of the 'ageing transition', can be Pareto improving (i.e. without losers), while minimizing the cost of the reform.

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1 Introduction

Advanced economies in the 21st Century are characterized by their ageing population and relatively low employment rates, threatened by automation and in some cases prevailing rigidities in labour markets. Furthermore, the financial and euro debt crisis first and then the COVID pandemic crisis, have put under extreme financial stress unfunded social insurance systems such as pay-as-you-go (PAYG) retirement pensions. For these economies, 'unfunded' can only mean 'bankrupt' that is, partially default in promised pension payments, or 'disruptive'; that is, high, and highly distorting, payroll taxes to finance these promises. Governments in these countries can either face this latter choice or change their PAYG system. Other, so-called, social security reforms that do not face these choices, are bound to face a major social security crisis.

Since the pioneer work of Auerbach and Kotlikoff (1987a,b) there has been an extensive, theoretical, quantitative and empirical, literature comparing social security systems. Early examples of studies on the economic and welfare implications of Social Security reform, in particular transitions from unfunded to private or fully funded (FF) retirement finance systems are Kotlikoff et al. (1998, 1999) and De Nardi et al. (1999). Our paper contributes to this literature in five dimensions. First, by building a quantitative framework – an overlapping generations model with rich intra and inter-generational heterogeneity and labour market frictions – suited for quantitative evaluation of labor market policies (pension systems, as in this paper, and unemployment insurance, minimum income programs, etc.).[1] The framework allows us to study the interaction between optimal life-cycle consumption, savings, labor market and retirement decisions, and government tax and transfer programs, in particular the retirement pension system. Second, by considering different alternatives to existing unfunded PAYG systems, accounting for the economic and welfare implications of such reforms across and within generations alive during the transition. Third, by focusing on an employment fund that can be used during unemployment and after retirement, which according to our analysis outperforms alternative systems often studied in the literature, such as fully funded, defined contribution systems, or private savings economies. Fourth, in calibrating our model to the Spanish economy which is an extreme case, but not an exception, in terms of historically low employment rates and its fast ageing process. Finally, by considering the full transitional path, in showing quantitatively that the timing of the reform is relevant during a demographic shift and can have a significant impact on its final cost and ultimately on welfare. We find that the long-run welfare gains of replacing the PAYG by a Backpack system are large, which allows for a Pareto improving transition financed with debt (the 'entitlement debt', as in Huang et al. (1997) and Conesa and Garriga (2008), generated by implicit promises in the PAYG system), and that the transition should be fast in order to minimize its overlap with the incoming demographic transition.

The basic features of a 'Backpack' (BP) employment fund that we study are: it is a fund contract

[1] We use the same framework in related work Brogueira de Sousa et al. (2018)

with the employee which accumulates the individual savings of a basic payroll tax (BP tax), while working; it is transferable across jobs and can be used during periods of unemployment and finally as a pension fund; it earns a market interest rate (i.e. it can be privately managed), but there are restrictions in its use (e.g. additional individual contributions are restricted and the worker may only be able to use it if he or she is unemployed or retired). While different forms of private employment funds are not a novelty in some countries such funds are not common as part of the public insurance policy.[2] Austria in 2003 is a recent example where a (small) Backpack-type employment fund was introduced during a reform of the tenure based severance pay system to improve flexibility in the labor market.[3] One of the main features that distinguishes the BP system from a standard defined contribution public pension system is its additional flexibility in allowing for withdrawals during unemployment spells. While existing many retirement plans and individual retirement accounts in different countries allow for some forms of early distribution, these often come with penalties or unfavorable tax treatment. Recently, in the wake of the coronavirus pandemic, several countries implemented temporary measures in order to make mandatory retirement savings more flexible, by expanding distribution options with favorable tax treatment and increasing borrowing possibilities for workers facing pandemic related financial consequences.[4]

Our work builds directly on two models: the model of Díaz-Giménez and Díaz-Saavedra (2009) and Díaz-Giménez and Díaz-Saavedra (2017), developed to study pension system reforms in Spain using overlapping generations general equilibrium models, and the model with job creation and destruction with search frictions and three employment states (employed, unemployed and inactive) of Krusell et al. (2011), further developed in Ábrahám et al. (2019) to study unemployment insurance reforms in Europe. Our benchmark model economy allows for a detailed description of the Social Security system: there are transfers for low-income households, a public unemployment insurance and a pay-as-you go pension system, financed with payroll taxes. Agents find jobs in a stochastic search environment and, while working, face idiosyncratic productivity shocks, as well as layoff shocks. After a certain age, a worker can choose to retire. These exogenous factors and their optimal work and search decisions generate a labour market distribution of households, into employed, unemployed, inactive and retired. In addition to payroll taxes, there are income, consumption and capital taxes. An aggregate production function and a government that must balance the budget to close the model. The model is calibrated to the Spanish economy with its public policies in 2018, as an initial steady state. We simulate the economy in the following decades, accounting for the projected demographic changes in the age and education distributions[5].

[2] One example of a private funding scheme is the TIAA-CREF (Teachers Insurance and Annuity Association-College Retirement Equities Fund), which is a non-profit employment fund founded by Andrew Carnegie in 1918 and nowadays serving over 5 million active and retired employees; it has played, and plays, an important role in enhancing mobility among university professors across US universities. However, it is a retirement fund not designed to provide unemployment insurance, while the BP provides both forms of insurance.

[3] See Kettemann et al. (2017) for the details of the reform.

[4] Two examples are the CARES Act in the United States and the legislative package by the Australian government, both enacted in March 2020.

[5] Unfortunately, the most recent and reliable long-term demographic forecasts do not incorporate the effect of the

Spain is a particularly interesting economy to study. Unemployment is high, and highly volatile, population is ageing and the PAYG system, which had a separate budget and fund, has seen its social security fund being depleted in the aftermath of the euro-debt crisis . If one assumes that the current system prevails in the next decades, given the expected fall of the employees/retired ratio, fulfilling the unemployment insurance and pensions promises will be extremely costly and distorting, as Figure 1 obtained with our calibrated model shows: doubling the dependency ratio implies that to fulfil unemployment benefits and pension promises the distorting payroll tax needs to be doubled too.[6]

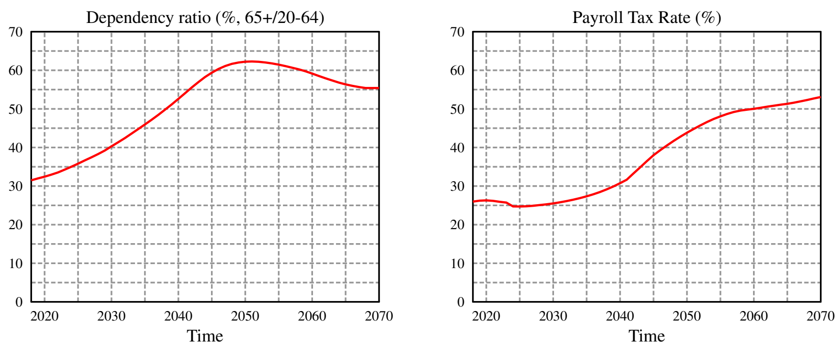


Figure 1: The expected evolution of the dependency ratio and payroll tax rate in Spain.

Figure 1 is also behind the results of other studies of the Spanish pension system namely that, with the ongoing ageing process of the population, its sustainability is under immense pressure.[7] Many advanced economies are, or will be, going through similar demographic transitions and the concern about the sustainability of the unfunded PAYG system with ageing populations and the difficulties to replace it with a funded system is neither unique nor new.[8] In a 21st Century perspective, the main problem is not the efficiency or sustainability of the PAYG system in itself, but the large and perverse effect of the system with an ageing population: it deters late retirements when life expectancy is high and its financing, with distortionary taxes, may further depresses labour supply (see Erosa et al. (2012) and Cooley et al. (2020)).

We assume that after the demographic transition the economy reaches a new steady-state. We then compare the PAYG steady-state with three alternative steady-states for the same economy

COVID-19 crisis. Nevertheless, this may not substantially change our results since there has been a reduction in the number of retired, but also of employed and, looking further ahead, births

[6]Some reforms, or more appropriately, parametric changes to the PAYG system have had a positive, but almost negligible, effect in reducing social security liabilities. The small reduction of the payroll tax in the 2020's captures the effect of two reforms in 2011: increasing the number of years of labor income used to compute the pension, from the last 15 to the last 25, and increasing the legal retirement ages in one more year (see Appendix E for a description of the Spanish PAYG).

[7]The already mentioned Díaz-Giménez and Díaz-Saavedra (2009) and Díaz-Giménez and Díaz-Saavedra (2017), as well as Rojas (2005), De la Fuente et al. (2019), de Cos et al. (2017) and García-Gómez et al. (2020).

[8]Early warnings, stressing the general dynamic equilibrium effects, are Conesa and Krueger (1999) and De Nardi et al. (1999); see Aubuchon et al. (2011) for an introduction to the replacement problem.

with the same policies and institutions, except for the PAYG system which is replaced by a: *i*) private savings (PS), an economy without public pensions in which households' retirement is fully financed by the proceeds of their private savings at the risk-less interest rate; *ii*) a fully funded pension fund (FF), financed with a defined contribution rate (a payroll tax), and, upon retirement, an actuarially fair annuity, and *iii*) the Backpack (BP) fund, as already described. In the latter two economies, households can complement the retirement (forced) savings with private savings, and to determine the retirement fund savings rate we search for the welfare maximizing contribution rate, which is 19% for the FF fund and 26% for the BP fund.

Both the aggregate and individual level effects of eliminating (PS) or replacing (FF & BP) the PAYG system are very large. There are differences among the three alternatives but, in relation to the benchmark PAYG economy, they are very similar. Overall these alternative economies are more productive (working hours per worker are lower but aggregate labour supply is higher) and agents benefit from higher consumption; as a result, the average welfare increase – measured as consumption equivalent variation – of eliminating PAYG by PS is 35.4%, replacing it by FF 35.1% and by BP 41.7%; furthermore, all different groups of households gain from these reforms of the PAYG system. Behind these huge welfare gains there is a factor that partially explains them: while the steady-state effective labour tax[9] is 66.8% in the benchmark PAYG economy, in the alternative economies is: 32.1% PS, 43.8% FF and 48.2 BP. Even in the alternative system with the largest social insurance coverage (the Backpack), the effective labour tax is substantially lower. In fact, the reduction of labour supply distortions is a feature of optimal reform designs (see Conesa and Garriga (2008)).

Our work also helps to elucidate the non-obvious welfare differences among the three alternatives to PAYG. The economies with an employees' fund Pareto dominate the *laissez-faire* private savings economy (PS) mainly for two reasons. First, as it is common practice with social security funds, (forced) savings into the fund are not part of taxable income, only if there are capital gains these are taxed as other capital gains. Second, when a worker decides to retire, the accumulated assets in the fund account can be used as private savings or, as we have seen, converted into an actuarially fair annuity. As existing employees' funds, FF and BP funds can be managed privately (with proper regulations), therefore one can argue that the possibility to transform assets into actuarially fair annuities could also exists in the PS economy, which would increase its estimated welfare gains. However, while these contracts exists in advanced economies, these markets are thin and having them as part of a large public pension programme can change their relevance and fairness; in particular, guarantees that the reform preserves a valued feature of a sustainable PAYG system: a worker upon retirement can have a stable source of income.[10] The Backpack is the winner of the

[9]The effective labour tax, τ_e, is given by $(1-\tau_e) = (1-\tau_y)(1-(\tau_p+\tau_x))/(1+\tau_c)$, where τ_y is the income tax, τ_p the payroll tax, τ_c the consumption tax, and τ_x is the fund tax; i.e. $x = f, b$ in FF and BP, respectively, and $\tau_x = 0$ in PAYG and PS.

[10]In well designed credible funds, such as in the Swedish social security system, the retirement promises are conditional on the fund's returns; in our simulated economies there is no aggregate uncertainty at the steady-state

race among the four social insurance systems because, in contrast with the other three systems, it provides additional unemployment insurance. As we show, with the BP households can better manage the loss of income due to the loss of employment, as well as their life insurance profile. This individual gain translates into better employment choices, which in turn aggregate into the general equilibrium effects that make the BP economy the most (constrained) efficient among the four we analyse.

More importantly, we provide an analysis of a Pareto improving transition from the PAYG to the Backpack social insurance system. In an economy which will go through an ageing demographic transition in the incoming decades, the well known problem of how to design a transition without losers is aggravated (e.g. Aubuchon et al. (2011)). Our transition relies on the long-term large welfare gains of having the BP instead of the PAYG, which provides fiscal space to compensate the initial cohorts who are entitled to PAYG promises. Uncovered PAYG liabilities, due to workers moving from the PAYG to the Backpack, are funded with public debt. We take as a benchmark a low interest rate envirenment, in line with recent conditions in the Euro area and globally. The fast ageing process (see Figure 1) dictates the need to go antecipate the social security reform, before the demographic transition takes place. We show that this is possible, in the case of Spain, with a front-loaded transition, in which backpack asset transfers make the Backpack system (weakly) preferred to the PAYG by the working age population, already in the first year of the reform, limiting PAYG claims to those of the retirees that year. Public debt finances the backpack asset transfers and these PAYG liabilities. In our calibrated Spanish economy this level of debt is relatively high: 160% of GDP in the first year, 2019, increasing to 310% at the end of the transition (late 2050s) when there are no more PAYG liabilities. To put this figure in perspective, Conesa and Garriga (2008) calculates that the implicit debt in the US Social Security system in 2005 at 2.2 times GDP, in a steady state without accouting for the demographic evolution. Alternatively, one could also front-load these debt liabilities, transforming PAYG pensions into (Pareto improving) BP life annuities, front-loading the full amount of the debt needed.

Our paper is closely related to the literature on the sustainability of Social Security systems in economies with ageing populations. Kitao (2014) considers four options to make the U.S. Social Security system sustainable taking into account the ongoing ageing process that increases the dependency ratio in the population. These options involve increasing taxes or decreasing benefits (by increasing the minimum retirement age, decreasing the pensions replacement rate, or make the system means-tested and let benefits decline with income). Despite all options making the system sustainable, the paper finds significant differences among alternatives in the aggregate and individual level responses, as well as heterogeneous distribution of welfare costs during the transition. As in Kitao (2014), individuals in our model make decisions on consumption, savings, labor force participation and hours work over a life cycle. In contrast, our analysis considers

and, therefore annuities are constant.

the replacement of an unfunded PAYG system with alternative funded systems (by construction, sustainable regardless of the demographic structure) and constructs Pareto improving transitions in which no generation is worse off with the reform.

McGrattan and Prescott (2017) engineers a Pareto improving transition for the U.S. economy, where the dependency ratio increases from 25% to 41% (i.e. from 4 workers per retiree to 2.4) without debt financing. Aside from the fact that the U.S. is a milder demographic transition from a better initial position than the Spanish one, there are important differences in their work with the transition we analyse. Their main transition is not from a PAYG to a Fund (in our case, the Backpack), but from transfers to the retirees paid by current payroll taxes to a system where the transfers are paid from the general federal budget, which is subject to a timely overhaul of the tax system. We compare different social security systems, without resourcing to a major overhaul of the tax system and, in contrast with them, the transition of the PAYG to the Backpack system is financed with public debt, taking advantage of the long-term gains of the reform and the existing low interest rates. In this dimension our work is closer to Conesa and Garriga (2008) who study optimal reform of Social Security as a Ramsey problem. In their baseline policy exercise, the planner is constrained to a single tax instrument (labor income tax), individual asset transfers and government debt. We follow this approach and study the transition into a Backpack system with debt issuance and initial BP asset transfers to the young cohorts. Other related papers in the quantitative literature on Social Security reform are Imrohoroğlu and Kitao (2012), Díaz-Giménez and Díaz-Saavedra (2009) and Díaz-Giménez and Díaz-Saavedra (2017). In addition to the labor supply decision (intensive and extensive margins), our model includes job search effort over the life-cycle, which produces three possible labor market states: employment, unemployment or inactive. Our analysis shows how labor supply and job search, in particular at older ages, respond significantly to changes in taxes and retirement pension rules.[11]

The related finance literature focuses on the portfolio choice over the life cycle within a partial equilibrium framework (Cocco et al. (2005)) and how accounting for this choice can help the design of social security systems. Agents in our model make a limited portfolio choice in private assets and backpack assets during unemployment and at retirement taking into account the annuity value of backpack asset after retirement (Larsen and Munk (2020)). Similarly, recent work emphasises the possible gains of having age-dependent taxes or flexible defined contributions plans (Schlafmann et al. (2020)). These are improvements that could be added to the BP design that we consider, but on this we also have followed a parsimonious approach.

[11] In related work, de la Croix et al. (2013) studies the consequences of ageing and pension system reform in a model with search and matching frictions, and shows that changes in participation rates of older workers is an important margin of adjustment. The paper compares predictions about pension system reforms in a model with labor market frictions to the competitive economy benchmark, and concludes that labor market frictions are important. In our model, labor market frictions also help explaining participation over the life cycle, and use it to study alternative reform scenarios and their fiscal and welfare implications.

The next section presents our model economy, Section 3 describes our calibration, Sections 4 and 5 the steady-state results, Section 6 the transition from the PAYG to the Backpack security system and Section 7 concludes.

2 The Model Economy

This section presents the model economy. We study an overlapping generations economy with heterogeneous households, a representative firm, and a government. The framework is based on Díaz-Giménez and Díaz-Saavedra (2009), with job creation and destruction and dynamic work and search decisions as in Ábrahám et al. (2019).

Time is discrete and runs forever, and each time period represents one calendar year. All model quantities depend on calendar time t, but we ommit this dependence where it does not lead to ambiguity.[12] We begin with a description of household heterogeneity.

2.1 The Households

Households in our economy are heterogeneous and differ in their age, $j \in J$; in their education, $h \in H$; in their productivity level $z \in \mathcal{Z}$; in their labor market status $s \in S$; in their private assets, $a \in A$; and in their backpack savings, $b \in B$. Sets J, H, $\mathcal{Z}$, S, A, and B are all finite sets and we use $\mu_{j,h,z,s,a,b}$ to denote the measure of households of type (j, h, z, s, a, b). They also differ in their claims to different social insurance systems: unemployment benefits UB, retirement pensions P, and government transfers TR. We think of a household in our model as a single individual, even though we use the two terms interchangeably. To calibrate the model, we use individual data of persons older than 20 in the Spanish economy.

Age. Individuals enter the economy at age 20, the duration of their lifetimes is random, and they exit the economy at age $T = 100$ at the latest. Therefore $J = \{20, 21, ..., 100\}$. The parameter ψ_j denotes the conditional probability of surviving from age j to age $j+1$. The notation makes explicit that the exogenous probabilities depend on age j, but not on education or other factors.

Education. Households can either be high school dropouts with $h = 1$, high school graduates who have not completed college $h = 2$, or college graduates denoted $h = 3$. Therefore $H = \{1, 2, 3\}$. A household's education level is exogenous and determined forever at the age of 20.

Labor market productivity. Individuals receive an endowment of efficiency labor units every period. This endowment has two components: a deterministic component, denoted $\epsilon_{h,j}$ and a stochastic

[12] In the steady state, defined below, all aggregate variables are constant and the exogenous age-education distribution is also constant. During transitional dynamics, the dependence on t comes from the ageing transition that is modelled as a change, over time, of the age-specific survival probabilities and the share of age groups and education levels as new cohorts enter the economy.

component, denoted by z. The deterministic component depends on the household's age and education, and we use it to characterize the life-cycle profiles of earnings. The stochastic component is independently and identically distributed across the households, and we use it to generate earnings and income dispersion in the economy. This component does not depend on the age or the education of the households, and we assume that it follows a first order, finite state, Markov chain with conditional transition probabilities given by Γ:

$$\Gamma\left[z'|z\right] = \Pr\left\{z_{j+1} = z'|z_j = z\right\}, \text{ with } z, z' \in Z. \tag{1}$$

Every period agents receive a new realization of z. Total labor productivity is then given by $\epsilon_{h,j}z$. A worker who supplies l hours of labor has gross labor earnings y given by:

$$y = \omega \epsilon_{h,j} z l, \tag{2}$$

where the economy-wide wage rate ω.

Labor market status. In the model, an agent is either employed, unemployed, non-active or retired. Among the unemployed, there are individuals who are eligible to receive unemployment benefits and access their backpack savings (workers who have recently been laid off), and others who are not eligible (either because eligibility expired, or because they quit work). Worker decide when to retire, leaving the labor force permanently once they do. Upon entering the economy, individuals randomly draw a job opportunity and then decide to work or not during the first period. Similarly, in subsequent years the labor market status evolves according to both optimal work and job search decisions (described below), and exogenous job separation and job finding probabilities.

Employed. An individual with a job at hand in the beginning of the period, and who decides to work, is employed in that period and his labor market status is denoted by $s = e$. An employed worker provides labor services and receives a salary that depends on his efficiency labor units and hours worked. Workers face a probability of loosing their job at the end of the period, denoted σ_j. This probability is age dependent, and we use it to generate the observed labor market flows between employment and non-employment states within age cohorts.

Unemployed. An agent may not have a job opportunity at the beginning of a period, because he lost his job last period, because he quit his job, or because he was unemployed last period and did not find (or did not accept) a new job offer. Without a job, agents may actively search for a job offer next period. If they do actively search we label them as unemployed. Unemployed agents who have lost a job are eligible for unemployment benefits (we refer to them as *unemployed eligible*, with $s = ue$). A formal description of eligibility criteria is given below. Agents who have quit work are not eligible for unemployment compensation (we often refer to this group as *unemployed non-eligible*, $s = un$). Active job searchers receive a job offer at the end of the period with probability $\lambda_j^{\mathbf{u}}$. The probabilities are again age dependent, and we use it to generate the observed labor market flows between unemployment and employment.

Non-Active. Agents without a job and who do not actively search for a new one are labeled non-active, with $s = n$. Those agents are not eligible for unemployment benefits, and receive a job offer for next period with a lower probability than an unemployed agent, $\lambda_j^{\mathbf{n}} < \lambda_j^{\mathbf{u}}$. This probability is also age dependent, and we use it to generate the observed labor market flows between non-activity and employment.

Retirees. In our model, workers optimally decide whether to retire and leave the labor force. They take this decision after observing their current labor productivity. If they decide to retire, $s = r$, they loose the endowment of labor efficiency units for ever and exit the labor market. Depending on the pension system in place, they may receive retirement pension payments after retirement.

Private Assets. Households in our model economy endogenously differ in their asset holdings, which are constrained to being non-negative. The absence of insurance markets give the households a precautionary motive to save. They do so by accumulating real assets which take the form of productive capital, denoted $a \in A$. Different retirement pension systems affect, among others, the agents' private savings decisions.

Backpack Assets. Workers accumulate backpack savings while they work. These savings result from a mandatory contribution out of workers' salaries, and are invested in productive capital and earn the real rate of return in the international capital market. When workers loose a job, they can access their accumulated savings and decide how much to keep in their individual accounts or how much to use, while out of work, to finance consumption. A formal description of the decision problem is given below. At retirement, backpack assets are converted into retirement pension payments (an actuarily fair life annuity).

Households derive utility from consumption, and disutility from labor and the search effort. Labor is decided both at the extensive and intensive margins, while search is a discrete choice. The period utility is described by a utility flow from consumption and the utility cost of time allocated to market work and to job search. Non-active and retired agents dedicate all the time endowment to leisure consumption. Accordingly, lifetime utility is given by

$$\mathbb{E}\sum_{j=20}^{100}\beta^{j-20}\psi_j\Big[u(c,l)-\gamma e\Big], \tag{3}$$

where β is a time discount factor, u satisfies standard assumptions, c is consumption and l is labor supply, and γ representes a job search utility cost. l can take values between 0 and 1, while e equals 1 in periods of active job search and is zero otherwise. Survival probabilities ψ_j determine average life expectancy in the economy, a central object in our analysis.[13]

At this point it is useful to clarify the timing of events within a period. At the begining of each period, z, households' stochastic productivity component, is realized. When entering the economy

[13] Fertility and immigration flows are exogenous.

(at age 20) agents additionaly learn their education level and draw a job oportunity, that they can either accept of reject. For older households, if they start a period with a job oportunity, they decide whether to work and if so, by how much. If they lost job or decided not to work in the previous period, they choose whether to search for a new job or not. Depending on these decisions, individuals then spend the period working, unemplyoed or inactive. Wages and unemployment benefits are received, and decisions on consumption and savings are taken. At the end of the period, workers observe the job separation shock, and unemployed or inactive learn if the found a job for next period. Households can choose to retire at the beginning of the period, and once they do they leave the labor market permanently.

2.2 The Firm

In our model economy there is a representative firm. Aggregate output depends on aggregate capital, K, and on the aggregate labor input, L, through a constant returns to scale, Cobb-Douglas, aggregate production function of the form

$$Y = K^{\theta}(HL)^{1-\theta} \tag{4}$$

where H denotes labor-augmenting productivity factor. Factor and product markets are perfectly competitive and the capital stock depreciates geometrically at a constant rate, δ. The firm rents capital in the international capital market at an interest rate r, and hires workers in the domestic market at a wage rate ω per efficiency unit of labor.

2.3 Backpack System

The BP economy features a fully funded pension system, funded by individual worker contributions. Workers may choose to use all or a fraction of the BP savings duing periods of involuntary unemployment. Every individual enters the economy without backpack claims. For every period of employment, a worker sees a fraction τ_b of his gross labor earnings deducted and invested into a personal employment-linked savings account, which is remunerated at the capital market rate of return, r. If b_t is the level of backpack assets at the beginning of an employment period, then next period's backpack evolves according to:

$$b_{t+1} = \tau_b y + (1 + r(1 - \tau_k))b_t, \tag{5}$$

with τ_k being the capital income tax rate. When a worker looses his job, his backpack assets can be allocated to finance consumption (present or future, as he can choose to save the backpack assets). Next period's backpack assets become a choice variable for the involuntary unemployed. In contrast, if a worker chooses to quit his job while still in the labor force, he keeps the backpack but cannot withdraw. In that period, the backpack evolves according to

$$b_{t+1} = (1 + r(1 - \tau_k))b_t. \tag{6}$$

Upon retirement, backpack assets can be used to buy a lifetime annuity or added to private savings. If the worker decides retire at age R and allocate b amount of BP savings to the purchase of the annuity contract, he receives in return:

$$p^B(b) = \frac{(1+r)^{R-T}}{\sum_{j=R}^{T} \psi_j} b. \tag{7}$$

The aggregate amount of backpack assets is invested in the capital market and adds to the stock of productive capital available in the economy. Since this is an individual, fully funded system, the aggregate amount of BP assets used to purchase annuity contracts equals the total amount of annuity payments received by retirees. Hence we do not include it in the Social Security budget equation, shown below.

2.4 The Government

Before we specify the government budget constraint, we describe the government programs other than retirement pensions discuss above.

Unemployment Benefits. The governmet taxes workers and provides unemployment benefits to the unemployed. Eligibility for unemployment benefits – denoted $\mathbb{1}_{UB} = 1$, below – is conditional on: i) having lost a job (i.e. a job separation) and not having started a new job yet, ii) on actively searching for a job, and iii) having been unemployed for less than a given number of periods, $\bar{d}$. Eligibility expires when one of the conditions is not met, and non-eligibility is an absorbing state. Eligible agents receive unemployment benefits given by $u_b = b_0 \bar{y}_h$, where $b_0 \in (0,1)$ is a replacement rate and $\bar{y}_h$ is the average labor earnings of workers with education h. Unemployment benefits are financed with payroll taxes, described below.

Other transfers. Households below an income level $y < \overline{t_r}$ receive a transfer from the government, denoted TR. Eligibility for transfers is conditional on income only and denoted by $\mathbb{1}_{TR} = 1$. Eligible households receive an amount $t_r = b_1 \overline{t_r}$.

We model the government budget restriction with two separate identities. Unemployment benefits and unfunded pension systems, in the case of the *Baseline* and *PAYG* economy, are financed with payroll taxes and form the social security budget. Other government expenditures and revenues form the overall government budget. In the *BP* economy presented here, retirement pensions are fully funded and therefore are not a government liability.

The government taxes capital income, household income and consumption, and it confiscates (part of the) unintentional bequests. It uses its revenues to finance an exogenous flow of public consumption and to service debt, and to make transfers to poor households. In addition, the government provides unemployment benefits and, in the economy with PAYG pension system, runs a pension system.

The government budget constraint is then:[14]

$$G_t + T_{r,t} + D_{t+1} = T_{k,t} + T_{y,t} + T_{c,t} + E_t + (1+r)D_t, \tag{8}$$

$$U_{b,t} = T_{p,t}, \tag{9}$$

where G_t denotes government consumption, $T_{r,t}$ denotes government transfers, $T_{k,t}$, $T_{y,t}$, and $T_{c,t}$, denote the revenues collected with the capital income tax, the household income tax, and the consumption tax, and E_t denotes unintentional bequests. $U_{b,t}$ denotes unemployment benefits, and $T_{p,t}$ denotes revenues collected with the payroll tax. In the remaining of the paper we assume that the level of public debt is fixed at the baseline calibration year level, $D_{t+1} = D_t$.

Capital income taxes. Capital income taxes are given by $\tau_k y_k$, where τ_k is the tax rate on gross capital income $y_k = ra$. a denotes capital holdings, and r the economy rate of return on capital.

Payroll taxes. Payroll taxes are proportional to before-tax labor earnings: $\tau_p y$.

Backpack taxes. Similarly, taxes to accumulate assets in the individual Backpack Fund account are given by: $\tau_b y$.

Consumption taxes. Similarly, consumption taxes are simply $\tau_c c$, where τ_c is the consumption tax rate and c is consumption.

Income taxes. We assume a simplified income tax formula according to which the income tax is proportional to the income level: $\tau_y \hat{y}$, where τ_y is a tax rate parameter and $\hat{y}$ is the tax base. The income tax base depends on the employment status. If a household is employed

$$\hat{y} = (1 - (\tau_p + \tau_b))y + r(1 - \tau_k)a. \tag{10}$$

For the unemployed and non-active agents,

$$\hat{y} = r(1 - \tau_k)a, \tag{11}$$

and for a retired household:

$$\hat{y} = r(1 - \tau_k)a + p^B. \tag{12}$$

In the last expression, p^B is the retirement pension.[15]

Insurance Markets. An important feature of the model is that there are no insurance markets for the stochastic component of the endowment shock, for unemployment risk, or survival risk. We model different public insurance systems that help agents in the economy smooth consumption in face of these shocks.

[14] In the *Baseline* and *PAYG* economies, the second equation is replaced with: $P_t + U_{b,t} = T_{p,t}$, where P_t denotes pension payments in period t.

[15] With the PAYG system, pension payments are given by p_h^S. In the Private Savings economy, there are no pension payments.

2.5 Individual Decision Problem

As noted before, here we describe only the problem in the *BP* economy.[16] The households' problem is described recursively. To simplify the notation, we omit in the main text the dependence of the value functions on the state variables age, education, private savings, backpack savings, and unemployment duration.

We first state the decision problem of a worker at the beginning of the period after the job acceptance was taken. Only after all the value functions are introduced we define the job acceptance and retirement decisions. An individual who is currently employed decides how much to consume c, save a', and work $l \in [0, 1]$, according to the following optimization problem:

$$W = \max_{c,l,a'} \left\{ u(c, l) + \beta \mathbb{E}\Big[(1 - \sigma_j)J + \sigma_j U\Big] \right\} \tag{13}$$

subject to:

$$(1 + \tau_c)c + a' + \tau_y \hat{y} + (\tau_p + \tau_b)y \leq (1 + r(1 - \tau_k))a + y + TR(y), \tag{14}$$

the backpack law of motion,

$$b' = \tau_b y + (1 + r(1 - \tau_k))b, \tag{15}$$

and a no-borrowing constraint:

$$a' \geq 0. \tag{16}$$

Gross labor income is $y = \omega\epsilon z l$, income tax base $\hat{y} = (1 - \tau_p - \tau_b)y + r(1 - \tau_k)a$ and government transfers for low income households are denoted by $TR(y) = t_r \mathbb{1}_{TR}(y)$, where $\mathbb{1}_{TR}(y) = 1$ if $y < \bar{t_r}$ and zero otherwise, as explained above.

Equation (13) above reads in the following way: the first term inside the curly brackets represents the utility flow from consumption and labor. The expected continuation value, discounted by β, takes into account the survival probability, all possible continuation histories of the realization of the stochastic component $z' \in \mathcal{Z}$, and two distinct labor market outcomes that are explicitly in the notation. With probability $1 - \sigma_j$, the worker keeps the job in the next period (and therefore is not eligible to claim unemployment benefits), with value denoted J that depends on next period's private and backpack assets, respectively a' and b', and the new realization of idiosyncratic productivity z'. Alternatively, with probability σ_j, the job is destroyed and the worker starts next period without a job, with value U. This value depends on the number of periods after an involuntary job separation (relevant to determine eligibility for unemployment benefits), d. In the first period after a layoff, $d = 0$. z' follows the Markov chain described in (1).

[16]The problem in the other economies can be found in the Appendix B.

Workers can start the period without a job. In the BP economy, a job searcher who faced a job separation shock and has yet to start a new job has access to his backpack savings and, depending on low long he has been without working, may be eligible to receive unemployment benefits from the government. He therefore solves a consumption-savings problem, a job-search problem, and a portfolio problem for the allocation of his private and backpack savings. At the beginning of the period, the state vector for the agent is given by private asset holdings a, backpack savings b, stochastic productivity z, and layoff duration d. Given the current state, the agent chooses consumption, future asset holdings and the search effort $e \in \{0, 1\}$ according to:

$$U = \max_{c,a',b',e} \left\{ u(c) - \gamma e + \beta \mathbb{E}\Big[e\Big(\lambda_j^{\mathbf{u}} J + (1-\lambda_j^{\mathbf{u}})U\Big) + (1-e)\Big(\lambda_j^{\mathbf{n}} J + (1-\lambda_j^{\mathbf{n}})N\Big)\Big] \right\} \tag{17}$$

subject to

$$(1+\tau_c)c + a' + b'(e) + \tau_y \hat{y} \leq (1 + r(1-\tau_k))(a+b) + UB(d,e) + TR(y), \tag{18}$$

and

$$a', b'(e) \geq 0. \tag{19}$$

Equation (17) can be read as follows. The first term inside the curly brackets is the flow utility from consumption and the utility cost of search, given by γe. The expected continuation value takes into account the survival probability and the evolution of the stochastic productivity component, z. Higher search effort ($e = 1$) translates into higher probability of finding a job: $\lambda_j^{\mathbf{u}} > \lambda_j^{\mathbf{n}}$. The tradeoff in the job-search problem is made explicit inside the expectation operator. With high search effort during the current period, with utility cost γ, the agent finds a job next period with probability $\lambda_j^{\mathbf{u}}$. With low search effort ($e = 0$), a job arrives with lower probability, $\lambda_j^{\mathbf{n}}$. In the event the worker finds a job, he decides in the beginning of next period whether to work or not, with associated option value J which depends on beginning of period assets and labor productivity. If search is not sucessful the worker continues unemployed next period with probability $(1 - \lambda_j^{\mathbf{u}})$, with value U which again depends on assets, productivity and unemployment duration d' which increases deterministically by one. If the unemployed worker decides not to search, $e = 0$, and does not find a job, he becomes non-eligible for unemployment insurance benefits and may again search for a job next period, with associated value N.

Equation (18) represents the budget constraint. Total income is used to finance the left had side of (18) composed of consumption expenditures, next period assets and income taxes, with the income tax base given by $\hat{y} = r(1 - \tau_k)a$. The right hand side is the sum of beginning of period private and backpack assets, plus after-tax return, unemployment benefits $UB(d, e)$ and government transfers for low-income households, $TR(y)$. The laid off worker may be entitled to unemployment benefits: $UB(d, e) = u_b \mathbb{1}_{UB}(d, e)$, with $\mathbb{1}_{UB}(d, e) = 1$ indicating eligibility for unemployment benefits. Formally:

$$\mathbb{1}_{UB}(d, e) = \begin{cases} 1 & \text{if } e = 1 \text{ and } d \leq \bar{d}, \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

The state variable d evolves deterministically according to $d' = d + 1$ if the worker continues unemployed in the following period, and $d = 0$ in the period immediately after a separation shock. We make two important simplifying assumptions here. The search effort is a dichotumous control: the agent can either actively search for a job ($e = 1$), or he doesn't search ($e = 0$). Additionaly, the possibility of using backpack assets while unemployed and searching for a job is represented by $b'(e)$ in the constraint (18): the laid-off worker can use his backpack savings to finance present (or future) consumption if he searches for a new job, but cannot increase backpack holdings other than through wage contributions (i.e. while working). Formally:

$$b'(e) \begin{cases} \leq & \tau_b y + (1 + r(1 - \tau_k))b, \text{if } e = 1, \\ = & \tau_b y + (1 + r(1 - \tau_k))b, \text{if } e = 0. \end{cases} \tag{21}$$

As before, there is a no-borrowing constraint given by (19).

Finally, an agent may start the period without a job because he has decided not to work or not to search in previous periods, not having found a new job yet. In this scenario, he solves the following problem:

$$N = \max_{c,a',e} \left\{ u(c) - \gamma e + \beta \mathbb{E}\Big[e\Big(\lambda_j^{\mathbf{u}} J + (1 - \lambda_j^{\mathbf{u}})N\Big) + (1 - e)\Big(\lambda_j^{\mathbf{n}} J + (1 - \lambda_j^{\mathbf{n}})N\Big)\Big] \right\}, \tag{22}$$

subject to

$$(1 + \tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1 - \tau_k))a + TR(y), \tag{23}$$

and

$$a' \geq 0, \tag{24}$$

$$b' = (1 + r(1 - \tau_k))b. \tag{25}$$

As above, $\hat{y} = r(1 - \tau_k)a$. The decision problem is similar to (17), with key differences related to eligibility to unemployment benefits and access to BP savings. Specifically, in this case the unemployed worker is not eligible for unemployment benefits, and he also cannot use backpack assets. Accordingly, the evolution of BP assets is given by (25).

We consider now the the problem of the retiree after the retirement decision. Retired individuals are not in the labor market and have no endowment of efficiency units of labor. They finance consumption with past private savings, and pension payments. The problem is a standard consumption-savings decision, with survival risk and a certain maximum attainable age, assumed to be $j = 100$. At age $j = 99$, the continuation value is zero because the agent exist the economy next period with probability one. During retirement, the retired household solves a standard consumption-savings problem taking into account survival probabilities and pension payments:

$$V(a) = \max_{c,a'} \left\{ u(c) + \beta \mathbb{E}\Big[V(a') \Big] \right\}, \tag{26}$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + p^B(b) + TR(y). \tag{27}$$

Pension payments are part of the income side of the budget constraint. In this case, $\hat{y} = r(1-\tau_k)a + p^B(b)$. After retirement, labor market productivity is always zero and hence expectations take into account only the survival risk.

To close the description of the household's problem, we define the job acceptance and retirement decisions. These jointly pin down the value of having a job offer at the beginning of a period:

$$J = \max \Big\{ V, \max\{W, N\} \Big\}. \tag{28}$$

The outermost max operator represents the retirement decision, while the inner operator is the job acceptance decision.

2.6 Definition of Stationary Equilibrium

The definition of a stationary equilibrium can be found in Appendix C.

2.7 Steady-state dynamics

The steady-state dynamics of the economies under study have the following characterisation. Given a distribution of households entering the economy ($j = 20$ and $a = 0$; say, at T) they all receive a job opportunity and make their consumption, asset and employment decisions. These households' decisions together with their survival probabilities define the distribution of this cohort the following year ($T+1$) at $j = 21$, but it also the distribution of households of $j = 21$ at T. Similarly, for $j = 22, ..., 100$; that is, the different cohorts coexisting at T mirror the evolution of the distribution of households entering the economy at T up to the end of their potential survival $j = 100$. In other words, the decisions that agents of generation T make through their live are already made in the year they enter the labour market by older agents if they have the same state. By construction, this is a steady-state distribution, which is our benchmark distribution. Different economies simply expose the T cohort distribution to different public insurance systems and, therefore, all the cohorts coexisting at T behave as if the given system was in place when they entered the economy.

3 Calibration

In order to calibrate the model parameters using Spanish data, we need to modify the environment described in Section 2 to take into account the pay-as-you-go pension system which is part of the Social Security system in Spain. These modifications are however restricted to the pension system itself, and therefore the decision problem facing households, described above, is almost unchanged. In this economy there is no Backpack fund, backpack assets (and contributions) are zero and claims on future consumption take only two forms: private savings and government retirement pensions. Henceforth we use the following designation:

Baseline economy. The status quo economy, calibrated to the Spanish data in 2018, which includes a pay-as-you-go retirement pension system (see Appendix E for details about the PAYG system). There is no Backpack system: $\tau_b = 0$.

The full description of the Baseline economy is included in Appendix B; the description of the pay-as-you-go system is given below.

3.1 Pay-as-you-go System

The PAYG system is an unfunded defined contribution pension system, where pension payments mostly depend on individual workers history of salaries, among other factors. In the model, pension payments depend on average earnings during the N_b years prior to retirement. In Spain, as in many other countries were a PAYG system exists, there is a minimum retirement age after which worker can decide to retire. We denote it by R_0. In order to capture the heterogeneity in pension payments that arises from different lifetime earnings histories, but at the same time reduce the dimensionality of the problem, we model pension payments that differ for each educational group (instead of each individual). Specifically, pension payments for retirees of educational group h are:

$$p_h^S = p_r \bar{y}_h^S, \tag{29}$$

where $\bar{y}_h^S$ is the average earnings of households in educational group h during the last N_b years before the retirement age, R_0, and p_r is a replacement rate. $\bar{y}_h^S$ is computed as:

$$\bar{y}_h^S = \frac{1}{N_b} \sum_{j=R_0-N_b}^{R_0-1} \bar{y}_{j,h} \tag{30}$$

where $\bar{y}_{j,h}$ is the average gross labor earnings of workers aged j and with education h. We assume that there are no early retirement penalties, nor minimum or maximum pensions. As mentioned before, this system is an unfunded system, financed through taxes. We model it as part of the Social Security budget, separatly from the general governmnet budget (8):

$$U_{b,t} + P_t = T_{p,t}, \tag{31}$$

where, as above, $U_{b,t}$ are aggregate unemployment benefit expenditures and $T_{p,t}$ are payroll tax collections, and now P_t are aggregate retirement pension expenditures. These are a liability of the Social Security system (and a claim on pension payments for households) and therefore are in the expenditure side of equation (31). The consumption tax rate clears the government budget (8), and the payroll tax rate τ_p clears condition (31).

To calibrate our model economy we do the following: First, we choose a calibration target country – Spain in this article – and a calibration target year – 2018. We then choose the initial conditions and the parameter values that allow our model economy to replicate as closely as possible selected macroeconomic aggregates and ratios, distributional statistics, and the institutional details of our chosen country in the target year. More specifically, to characterize our model economy fully, we must choose the values of 4 initial conditions and 38 parameters. To choose the values of these 38 parameters, we need 38 equations or calibration targets.

An important assumption we maintain throught the paper is that we treat Spain as a small open economy. This means that the interest rate (and therefore, from the representative firm optimization conditions, the capital-labor ratio and the wage rate) is constant. We follow this assumption in the main text in order to isolate the direct effects of population ageing on pension system sustainability, but present all the closed economy (i.e. general equilibrium) results in Appendix A. We describe the calibration process in all detail, including the data sources, in Appendix D.

The next section presents the most relevant calibration targets and model statistics. We also present the government expenditure and tax revenue ratios, which are an important ingredient in the analysis of the reforms of retirement pension systems presented below.

3.2 Baseline Economy

The tables presented below summarize the calibration exercise. The values shown in bold are data targets.

Table 1: Macroeconomic Aggregates and Ratios in Spain and in the model, in 2018.

	K/Y^*	C/Y^*	I^a/Y^*	h
Spain	2.94	50.70	26.95	34.59
Model	3.06	41.77	34.90	33.11

Variable Y^* denotes GDP at market prices.
I^a denotes investment.
h denotes average share of disposable time allocated to the market.
Data source: Fundación BBVA and Spanish National Institute of Statistics (INE).

Table 2: Macroeconomic Aggregates and Ratios in Spain and in the model, in 2018.

	P/Y^{*a}	U/Y^{*b}	T_r/Y^*	GW^c	W^d	I^e
Spain	**10.47**	**1.32**	**0.83**	**0.67**	**59.59**	**5.16**
Model	10.53	1.18	0.88	0.67	58.45	5.02

Y^* denotes GDP at market prices.
U/Y^* is unemployment benefits as a share of output.
GW is the Gini Index of wealth.
W is the share of workers in the Spanish population with 20+ years old.
I is the share of inactive in the Spanish population with 20+ years old.
Data source: Spanish National Institute of Statistics (INE), Spanish Social Security, Cañón et al. (2016), Anghel et al. (2018).

The model is able to capture the main output ratios in the calibrated year, shown in Table 2. As shown in Table 3, we target government expeditures and revenue ratios in order to determine the simplified tax system in the model. The payroll tax rate finances pension and unemployment benefit expenditures. Capital income and household income tax rates are chosen to collect 2.24% and 7.05% of GDP (at market prices), as it is the case in Spain in 2018. Finally, the consumption tax rate clears the government budget. Some Spanish regions feature a proportional tax on bequests. We use the aggregate revenue of this tax in 2018 as the data point for E (0.20% of output). In the model aggregate accidental bequests as a fraction of output is significantly higher (2.63). In the results shown below we assume that the portion of the accidental bequests that is not taxed by the government is wasted (thrown to the sea).

Table 3: Government Budget in Spain and in the model, in 2018 (% of output, Y, at market prices).

	Public Expenditure				Public Revenues				
	G	T_r	P	U	T_c	T_k	T_y	T_p	E
Spain	**17.40**	**0.83**	**10.47**	**1.32**	9.07	**2.24**	**7.05**	9.47	**0.20**
Model	17.40	0.88	10.53	1.18	8.68	2.33	7.05	11.72	0.20

G: government consumption, T_r: welfare transfers, P: pension payments, U: unemployment benefits expenditures; T_c: consumption tax collections, T_k: capital income taxes, T_y: household income tax revenue, T_p: payroll tax revenue, E: accidental bequests revenue.
Data source: Spanish Social Security (Resumen de Ejecución del Prespuesto) and Spanish National Institute of Statistics (Cuentas Nacionales).

The tax rates implied by the calibration are shown in Table 4.

Table 4: Policy Parameters in the model economy, in 2018.

	Tax rates (%)			
	τ_c	τ_y	τ_k	τ_p
Model	26.2	14.2	25.0	26.1

τ_c: consumption tax rate, τ_y: household income tax rate, τ_k: capital income tax rate, τ_p: payroll tax.

The model also does a good job in replicating the aggregate labor market stocks (share of workers and inactive targeted in the calibration), and the age-distribution of workers, unemployed, inactive and retirees – which is not part of the calibration targets.

Table 5: Labor Market Shares in 2018 (% of population).

	W	U	I	R
Spain	**59.59**	10.72	**5.16**	24.51
Model	58.45	11.87	5.02	24.65

W: workers, U: unemployed, I: inactive, R: retirees.
Data source: Encuesta de Población Activa (INE).

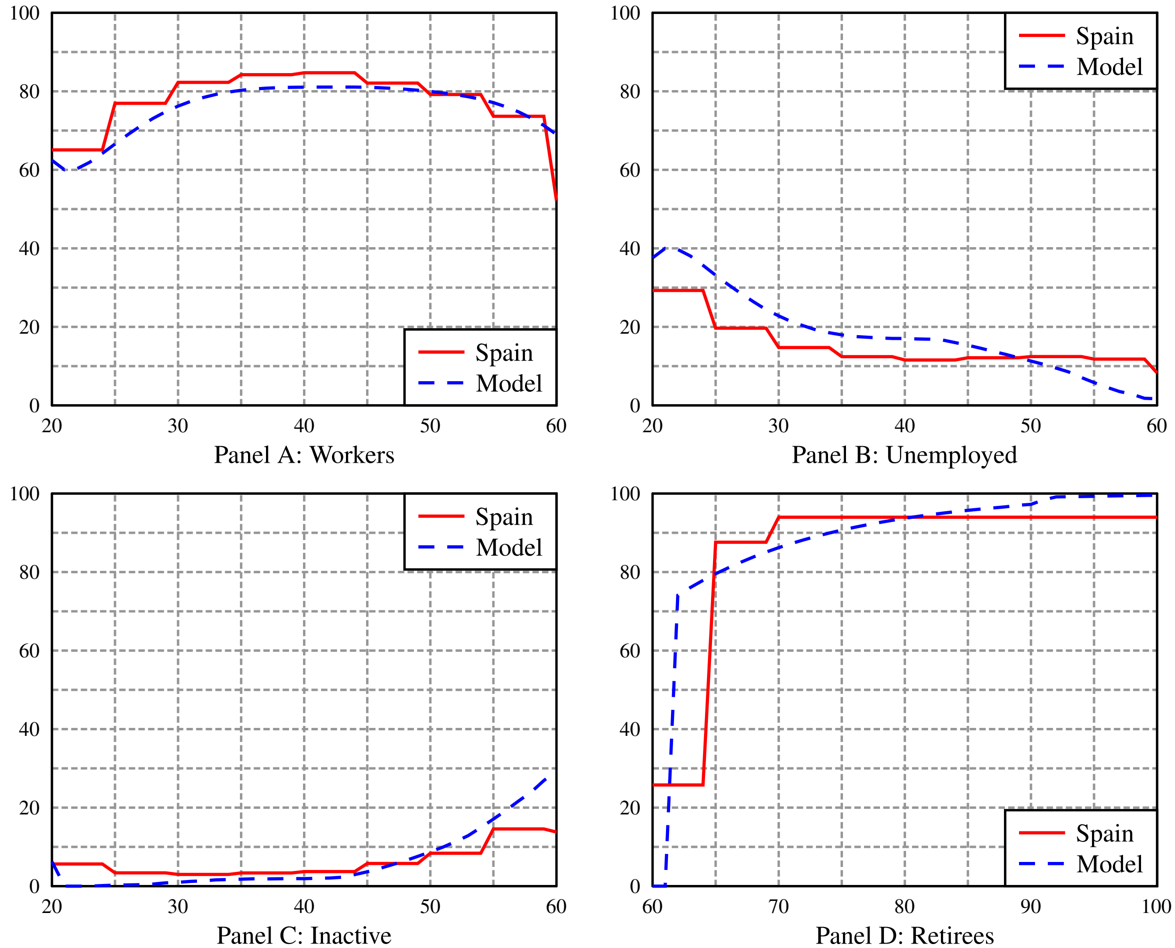


Figure 2: Labor market stocks by age in the data and in the model. Data source is the survey is Encuesta de Población Activa.

Standard heterogeneous agent models with idiosyncratic earnings risk fail to replicate earnings and wealth inequality found in most developed economies (Castaneda et al. (2003)). Our overlapping generations model with labor market frictions and a detailed description of government tax and transfer systems is able the capture the inequality in after-tax earnings, income and net wealth in the Spanish economy, as summarized in Table 6.

Table 6: Inequality in Spain and in the model in 2018*.

	GE	GI	GW
Spain	0.34	0.33	**0.67**
Model	0.34	0.36	0.67

GE: Gini Index of net earnings, *GI*: Gini Index of net income, *GWI*: Gini Index of net wealth.
*The source for the Spanish data of earnings and income are the Spanish National Institute of Statistics (INE) and the OECD. The source for the Spanish data of wealth is BDE (2018).

4 Demographic transition with a PAYG pension system.

We solve a demographic transition with the PAYG pension system in the following way. We solve for the long-run steady state equilibrium with the age and education distribution of the Spanish population which is forecasted for the year 2068. The age profile of survival probabilities and the corresponding age distribution in 2018 and in the 2068 forecast, shown in Figure 3, are taken from INE.[17] We also update the share of households in each education level that is expected for Spain in 2068.[18] Next, we solve the equilibrium transition path between the initial steady-state (the 2018 economy) and the final steady state, which corresponds to the economy under the 2068 age-education distribution. Along the transition, the survival probabilities and education levels of each cohort are updated each year according to the INE forecast, and PAYG system rules are updated according to the Spanish law.[19] Because the model variables may take more than 50 years to converge, we guess (and verify) that convergence in all aggregate variables is achieved after 80 years after the last demographic change (see Auerbach and Kotlikoff (1987b) for a detailed explanation). The final steady state, achieved after this transition period, at which point the demographic distribution and all aggregate variables are constant, is what we loosely refer to below as the "2068 economy". As a consequence of the ageing process, the share of households older than 65 increases from 24% in 2018 to 36% in 2068. Despite the changes in pension rules, the increase in the share of persons older than 65 implies that pension payments increase over time. We assume that the payroll tax rate τ_p increases to balance the increase in pension payments every period. Additionaly, we assume that the consumption tax rate τ_c adjusts to clear the government budget constraint.

[17]Instituto Nacional de Estadística, 2018-2068 series: https://ine.es/dynt3/inebase/es/index.htm?padre=4749

[18]To update the distribution of education levels, we assume that from 2018 onwards, 7.33 percent of the 20 year-old entrants have not completed their secondary education, that 62.62 percent have completed their secondary education, and that 30.05 percent have completed college. This was the educational distribution of Spanish households born between 1980 and 1984, which was the most educated cohort in 2018 data. We assume that the probabilities to find/lose a job do not change.

[19]We increase the number of years used to compute the pension, N_b, from the 21 to 25 years in 2022. Also, the minimum retirement age is increased by one year to 63 years old in 2024. We do not account for the Sustainability Factor, because its implementation has been suspended. These changes follow from the 2011 and 2013 pension reforms in Spain.

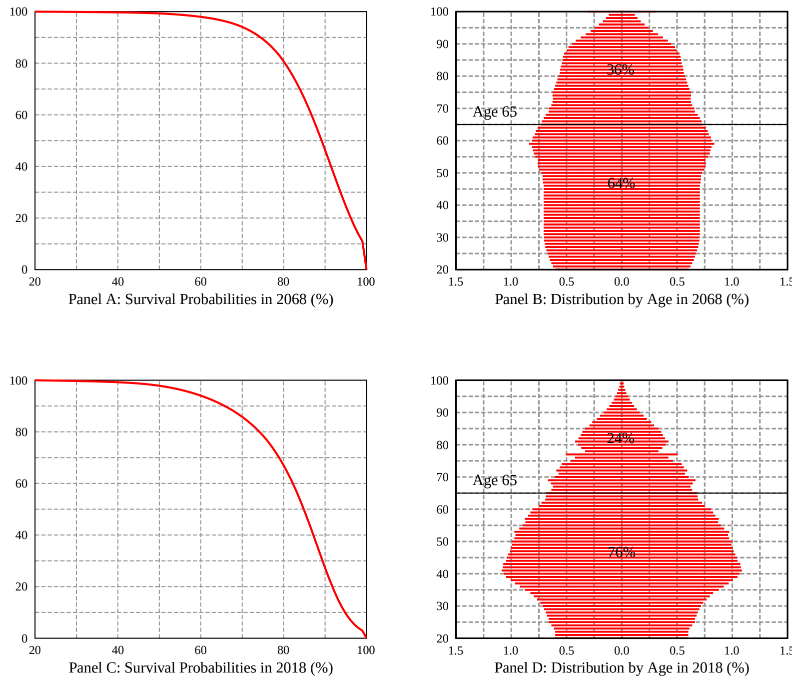


Figure 3: Survival Probabilities and Age distribution in Spain in 2018 and the 2068 forecast. Source: Instituto Nacional de Estadística, 2018-2068 series.

We start by comparing the initial economy in 2018 and the long-run 2068 steady state. We fix the following notation for the results shown below:

PAYG. A long-run economy, with a demographic structure as predicted for Spain in 2068, assuming the pay-as-you-go (PAYG) pension system is in place (with small parametric changes to minimum retirement age and pension payments formula, as described above).

The increase in the share of households aged 65 years old and older leads to a significant increase in the share of retirees in the 2068 population. As Table 7 shows, this group represents almost 35% of the population in 2068. All the other labor market groups decrease their share, with the largest fall in the stock of employed, 8 percentage points. Workers decide to retire later, with the average retirement age increasing from 63.7 in 2018 to 65.1 in 2068. The increase in the retirement age is not enough to compensate for the increase in life expectancy. Consequently, the increase in the share of retirees increases pension payments. Pension payments as a share of output more than double: from 11.5% in 2018 to 23% in 2068 (Table 10). The payroll tax rate reaches 51% in 2068 (Table 22), and total payroll tax collection increases from 12.8% to 24.4% of output in 2068.

Table 7: Labor Market Shares in the baseline 2018 model economy, and in the PAYG 2068 simulation (% of population).

	W	U	I	R
Baseline (2018)	58.45	11.87	5.02	24.65
PAYG (2068)	50.68	10.77	3.75	34.72

W: workers, U: unemployed, I: inactive, R: retirees.

The results are line with previous papers, for example Díaz-Giménez and Díaz-Saavedra (2017), De la Fuente et al. (2019) and Díaz-Saavedra (2020). Specifically, Díaz-Giménez and Díaz-Saavedra (2017) and De la Fuente et al. (2019) find that pension payments may reach around 21 percent of output at market prices in 2050. Díaz-Saavedra (2020) finds that, with the Sustainability Factor (abandoned by the Spanish legislator), this number would reach 16 percent of output that same year.

The decrease in the share of households who work and in average hours worked reduces labor and capital in the economy: output is 5.4% lower. Among the workers, hours worked decrease due to a increase in the effective marginal labor tax rate. Private savings decrease due to a large distortion of earnings from high payroll taxes. Capital and income tax collections fall. Lower lifetime disposable income and savings reduce aggregate consumption.

Table 8: Consumption and Payroll tax rates in 2018 and 2068 under the PAYG pension system.

	Tax Rates (%)	
	2018	2068
τ_c	26.2	25.4
τ_p	26.1	50.9
τ_e	48.1	65.3

τ_c: consumption tax rate, τ_p: payroll tax rate. τ_e effective labour tax rate (see Footnote 9).

Table 9: Main Macroeconomic Aggregates in the baseline 2018 economy, and in the PAYG 2068 simulation*.

	Y	L	A	C	h^a
Model (2018)	2.16	0.69	3.98	0.78	0.1935
PAYG (2068)	2.03	0.65	2.38	0.76	0.1706

*In this table, variable Y is output at factor cost, and variable A is total assets.
[a]Variable h denotes total hours of work in the economy.

Table 10: Government Budget in the 2018 model economy and in the PAYG simulation of 2068 (% of output, Y, at factor cost).

	Public Expenditure			Public Revenues			
	T_r	P	U	T_c	T_k	T_y	T_p
Model (2018)	0.97	11.54	1.29	9.51	2.55	7.72	12.83
PAYG (2068)	0.83	23.03	1.38	9.62	2.55	7.45	24.41

T_r: minimum income, P: pension payments, U: unemployment benefits expenditures; T_c: consumption tax collections, T_k: capital income taxes, T_y: household income tax revenue, T_p: payroll tax revenue.

We present additional results on changes along the demographic, income and wealth distributions when we compare the PAYG 2068 economy with alternative reformed economies, below.

In the period between 2018 and 2068, the economy undergoes a demographic transition with the dependency ratio (share of households older than 65 relative to 20 to 64 years old) increases during the first three decades, peaking above 60% around 2050, and stabilizing at 55% in 2068. Figure 4 shows this evolution. As a consequence, pension payments increase which in turn, according to our balanced budget assumption, lead to an increase in payroll taxes. The initial hump after 2018, shown in the right hand side panel in Figure 4, comes from the parametric reform to the PAYG system according to existing Spanish pension rules. As becomes clear from the figure, the demographic evolution quickly undoes its small initial effect, and the payroll tax rate steadily increase to above 50%.

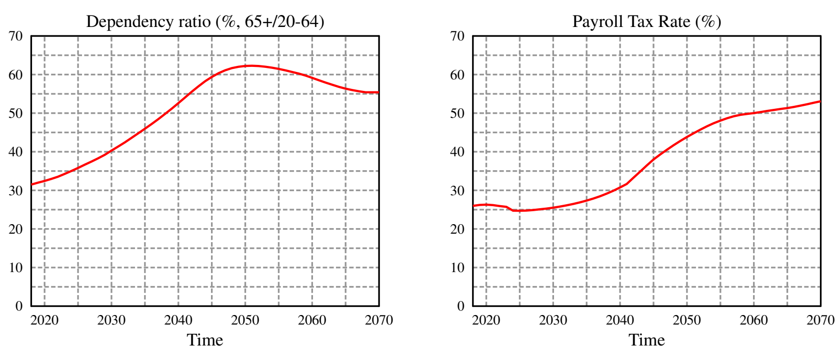
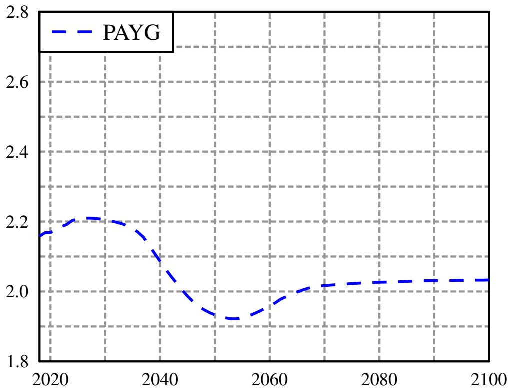
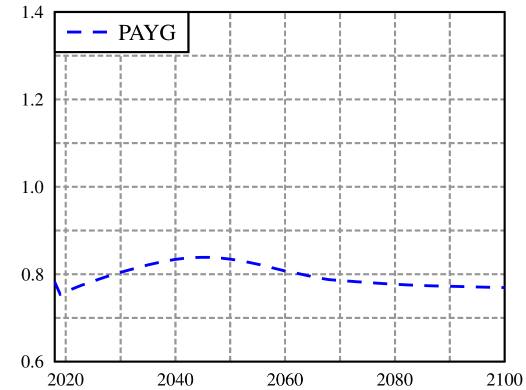


Figure 4: The expected evolution of the dependency ratio and payroll tax rate in Spain.

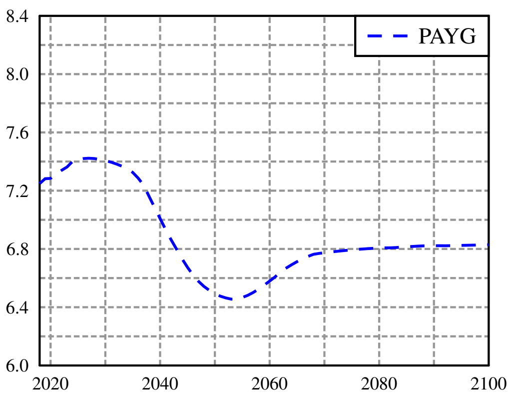
The evolution of the demographic structure between 2018 and 2068, summarized by the dependency ratio shown in 4, shapes the transition of the main aggregates, pension payments and the payroll tax rate. Recall that, under the small open economy assumption, prices are constant, the only exogenous varitation introduced in the economy comes from the evolution of the survival probabilities, that in turn indirectly induce changes in the government and social security budget and hence taxes. The demographic transition in the decades up to 2060 doubles the ratio of retirees per worker, which in order to finance pension payments in the PAYG system requires increasing the payroll tax rate under our balanced budget assumption. The effective labor tax increases from 48.1 to 65.3%. Aggregate labor supply responds inversely as payroll taxes increase. Since the capital labor ratio is constant during the transition, capital decreases in the same proportion and consequently output. The increase in the payroll tax is partially offset by a percentage point decrease in the consumption tax rate (Table 8), and aggregate consumption is almost constant, but households who derive most of their income from wages see a large increase in payroll tax and are forced to reduce consumption.



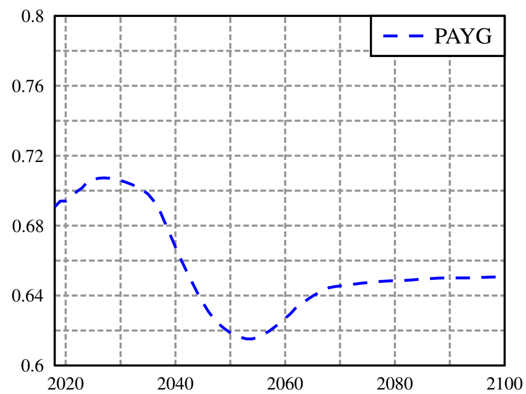
(a) Output



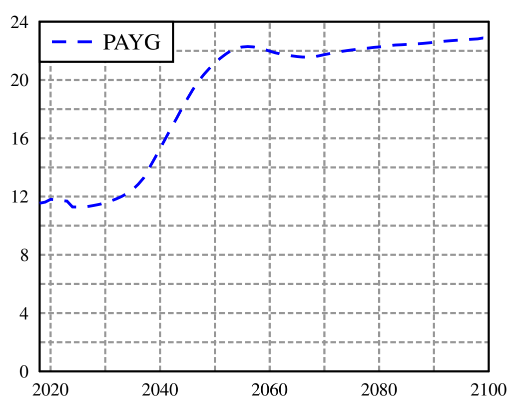
(b) Consumption



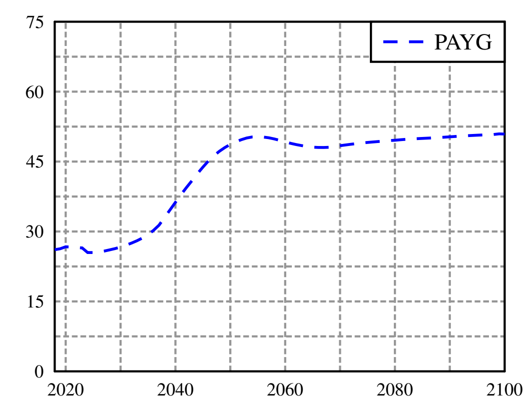
(c) Capital



(d) Labor



(e) Pension Payments (percentage of Output)



(f) Payroll tax rate (percentage)

Figure 5: Main aggregates, PAYG pensions and payroll tax rate during the transition.

In the next two sections we present the main results of the paper. We study the Backpack employment fund, and the reform of the baseline 2018 economy PAYG system taking into account the demographic transition between 2018 and 2068.

5 Demographic transition and the reform of the pension system

In this and the following sections, we solve and compare alternative equilibrium transition paths and final steady states that feature alternative funded pension systems.

In this section we study the reform of the Spanish pay-as-you-go pension system, by its replacement with a Backpack pension plan. We are interested in a reform which implements a Backpack with a tax rate that is welfare maximizing in the long-run and without losers in the transition – with respect to the status-quo of maintaining the PAYG pensions system as described in the previous section. Along the transition, the demographic distribution is updated according to the projected ageing and education process, as in the previous section. To implement a Pareto-improving reform the government respects all PAYG promises and pensions, in the following sense: first, all workers who have paid payroll taxes in the past – and, therefore, contributed to finance the PAYG system – are offered a Backpack with enough assets to guarantee that they (weakly) prefer to move to the BP system, and second, all retirees with PAYG pensions receive the pension payments they are entitled to until they die.

We consider reforms which are financed as follows: first, the cost of 'respecting all PAYG promises and pensions' is debt financed, and second, only two tax rates change: the payroll tax for those moving to the BP system (only finances UI benefits), and the consumption tax to guarantee that the government budget is balanced every year.[20] In sum, we restrict the reform problem to the choice of: *a*) a long-run welfare maximizing BP tax rate, τ_b^*, *b*) which cohort is the last to collect PAYG pensions, and *c*) which cohort is the first to enter the BP system. We further assume that the first (and all subsequent) cohort(s) to enter the BP system face τ_b^*.[21]

Different choices of *a*-*c* imply different final debt levels. We assume that debt is raised in the international capital market and is costly, with interest payments included in the expenditure side of the government budget constraint. Consumption taxes must increase to cover interest payments.[22] As a benchmark, in line with the current low interest rates paradigm, we assume that the real interest rate on public debt financing the reform is one percent per year. If debt is costly, the

[20] This exercise follows the baseline optimal policy exercise in Conesa and Garriga (2008), that restricts the planner's instruments to the labor tax, asset transfers to individuals and government debt. We let the payroll tax change, use BP asset transfers and government debt to fund the reform. In contrast, McGrattan and Prescott (2017) consider an overall reform of the income tax and transfers schedule simultaneously with the privatization of the pension system.

[21] In other words, the time path of BP tax rates considered in the reform problem are step functions: $\tau_b = 0$ before and $\tau_b = \tau_b^*$ once a worker enters the BP system.

[22] Alternatively, if we assume that the cost of debt is part of the social security budget, this raises payroll taxes. Our results are qualitatively not affected by this choice.

higher is the final post-reform debt level, the higher are taxes necessary to service it and the lower is aggregate welfare in the τ_b^* long run reformed economy. We consider two canonical choices of $(b\&c)$, corresponding to a *slow* and a *fast* transition. Generally, given $(b\&c)$ and an interest rate on the debt r^*, the algorithm to compute the transition is as follows:

Denote the optimal long-run BP tax rate in iteration i by $\tau_b^*(i)$ and final debt level by $B(i)$. In the first iteration $i = 1$ and we set $B(i) = 0$, then:

i. Perform a grid search for the welfare maximizing BP contribution rate $\tau_b^*(i)$ in the long-run steady state economy, given $B(i)$;

ii. Given $\tau_b^*(i)$ found in the previous step, solve the transition from the initial PAYG economy to the final BP steady state, computing the debt issuance every period necessary to fund this transition.

iii. Compute the cumulative stock of debt at the end of the transition from the previous step, denote it $B(i+1)$;

iv. Calculate aggregate welfare in a final steady state with a $\tau_b^*(i)$ BP system and the cost of debt calculated in the previous step included in the government budget, $r^*B(i+1)$ with r^* the real interest rate on government debt;

v. If aggregate welfare calculated in the previous step is close enough to the value found in the first step, *stop*; if not, start a new iteration, updating the value $B(i)$ to $B(i+1)$ and, if needed, of $\tau_b^*(i)$ to $\tau_b^*(i+1)$.

Following this procedure, we find that for two canonical choices of $(b\&c)$, the long-run welfare maximizing τ_b^* is relatively stable in a range close to $\tau_b = 26\%$. The reason for this is that for households who populate the long-run steady state economy, the impact of different policies regarding the transition come only through different debt levels. Insofar as the transition period does not generate *unreasonably* high levels of debt, rolling over the final debt level in perpetuity at a low interest rate has a small cost, compared to the large gains of removing a 51% payroll tax rate (and low PAYG pensions). In the next subsection, we consider the two extreme cases of *reasonable* transition periods to a $\tau_b^* = 26\%$ long-run economy: a gradual transition from the PAYG to the BP system, and a fast transition in which all workers move to the BP system immediatly in the first period of the reform.

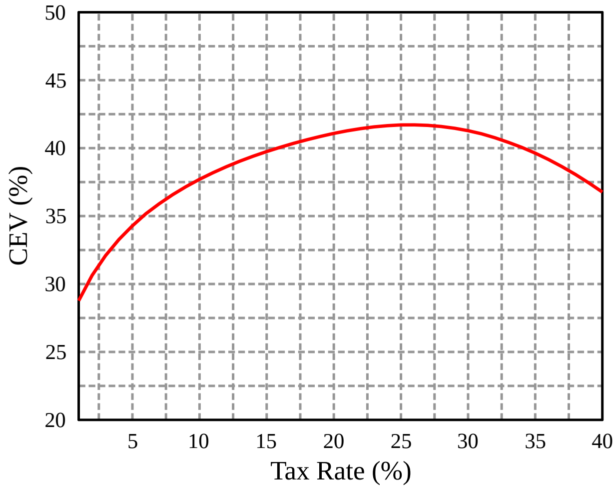


Figure 6: Average increase in welfare (CEV) at age 20 as a function of BP contribution rate (τ_b) in economies with a Backpack system relative to the PAYG economy, assuming a final debt level of 3.1 times output and $r^* = 1\%$.

5.1 Slow vs. fast reforms

The difficult political-economy of implementing deep structural reforms often calls for introducing them slowly. A slow debt-financed Pareto improving transition could be implemented, starting in 2019, following five principles:

1. everyone who is 21 or older in 2019 remains in the PAYG system which keeps operating as long as they are retirees or workers with PAYG claims;
2. those entering the labour market in 2019 (age 20) and in subsequent years, enter the Backpack system;
3. if we split the payroll tax into its unemployment and pension system components; $\tau_{p,t} = \tau_{pu,t} + \tau_{pp,t}$, then we let $\tau_{pu,t}$ such that $T_{pu,t} = U_{p,t}$ every year, while $\tau_{pp,t}$ is as in the PAYG transition (the transition without reform) in year $t > 2018$ and applies to workers older than 20 in 2019;
4. households who enter the economy in 2019 and subsequent years pay unemployment insurance contributions $\tau_{pu,t}$, and enter the backpack pension plan with $\tau_b = 26\%$;
5. all the PAYG claims that are not covered with payroll tax revenues are financed with public debt (i.e. the PAYG retiree benefits not financed by workers in the backpack system).

In a stationary demographic environment, with a constant population – in particular, a constant dependency ratio – absent general equilibrium effects through prices or tax changes due to the introduction of the Backpack, this slow reform is equivalent to a fast reform in which all new and old workers transition into the BP system immediatly (the stock of reform debt would increase sharply immediatly in the latter, as opposed to gradually over time). In contrast, the expected ageing transition implies that the dependency ratio is increasing rapidly in the decades after 2018 which, in turn, and assuming that PAYG pension claims are paid in full, implies that the cost per retiree of delivering on such claims is increasing rapidly as the population ages.

If we solve the described 'slow Pareto improving transition' following the five principles above, the final debt needed to finance it amounts to more than 7 times Spain's 2019 GDP. The reason for this large debt level is that the slow transition fails to anticipate the fast increase in the ratio of retirees per worker. In other words, the slow transition implies a long period after 2019 during which PAYG claims are still being paid for (over 50 years), with a population where the ratio of retirees per worker has doubled.

Intuitively, a *fast* transition would avoid this scenario by moving all active workers at the time of the implementation to the BP system and hence go through the ageing process with a much lower stock of PAYG claims (for less than 40 years, and only for existing retirees in 2019). The faster are PAYG claims phased out, the lower is the post-reform debt and the lower are taxes to cover interest payments in perpetuity. On the other hand, phasing out PAYG pensions without default requires some sort of (debt financed) government transfers, to compensate workers with (implicit) PAYG promises.

A front-loaded transition

A front-loaded transition can be implemented, starting in 2019, following five simple principles:

1. all retirees in 2019 remain in the PAYG system, collecting their pensions according to the PAYG pension rules;

2. all the working-age population in 2019 enters into the Backpack system, as well as those above the minimum retirement age who are still working in 2019;

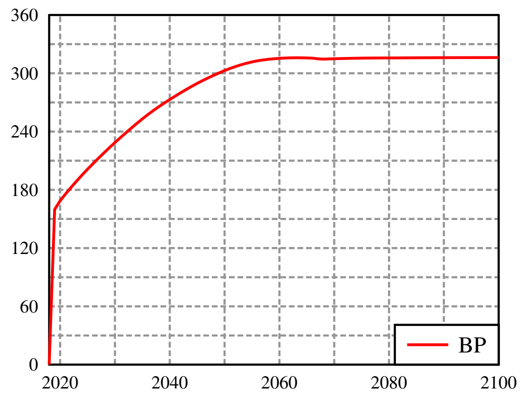
3. those that enter the Backpack system in year $t \geq 2019$ receive an initial amount of backpack assets $b_{t,h,j,}(a) \geq 0$ (a government subsidy paid into the BP account) that makes them weakly prefer entering the Backpack system than to remain in the PAYG system; in particular, at $t \geq 2019$ those with $j = 20$ receive $b_{t,h,j,}(a) = 0$, and in 2019 those with $j \geq 20$, receive $b_{t,h,j,}(a) \geq 0$ as to make them (weakly) prefer the BP reform to their PAYG retirement plan;

4. the initial backpack assets transfered to vintage workers in 2019, as well as all PAYG pensions

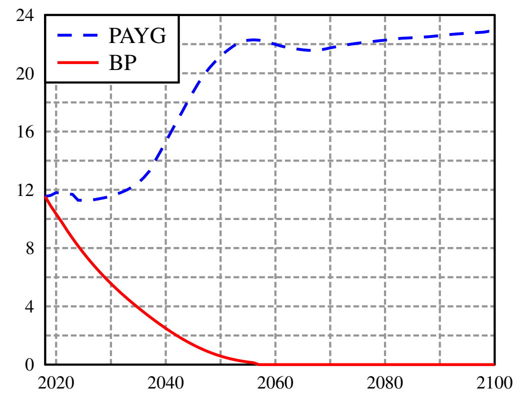
(from 2019 until the year the last retiree with PAYG claims dies) are financed with public debt.

As Figure 7 (a) shows the initial level of debt – financing the initial backpack asset transfers – increases the level of public debt by 150% of GDP and the payment of PAYG pensions in the following years increases this level of debt until it reaches circa 300% at the end of the 2050s, when PAYG claims disappear.[23] A high level of debt, but only 28.5% of what would had been with the slow transition discussed above. Note that this debt reflects the elimination of the PAYG pension system: initial BP subsidies to vintage workers in 2019, shown in Figure 8, and funding of PAYG pensions after 2018, Figure 7 (b), but not the BP system, which is fully funded by individual contributions. After the last PAYG pensions are paid, the stock of debt is constant (Figure 7 (a)). As Figure 7 (c) shows, aggregate BP assets jumpt in 2019 due to the initial asset transfer to vintage workers (the 2019 cohort starts with zero assets). Some of the initial assets are converted to liquid private assets by unemployed or retirees in 2020, at which point the stock of BP savings starts to steadily increase to the final steady state level. Figure 7 also shows how the payroll tax is not affected by the demographic transition during the BP reform, because it is only determined by unemployment insurance expenditures (stable during the transition). Accounting for the additional 26% backpack tax, the total payroll wedge is at least twenty percentage points less than in the PAYG economy by the end of the demographic transition (Figure 7 (c)).

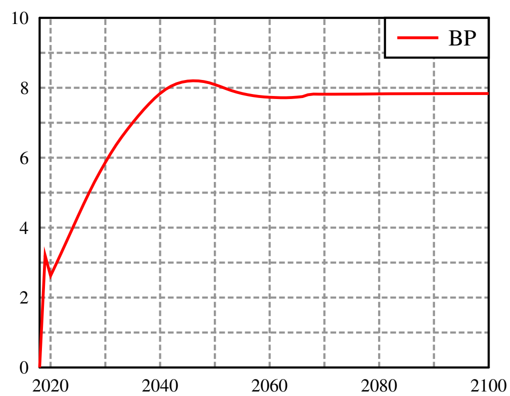
[23]Note that we have not included the existing level of debt, which in 2019 was 95.5% of GDP (AMECO) and, consistently, we have excluded debt payments from government expenditures, as well as the corresponding taxes. That is, starting the reform requires doubling the level of public debt of Spain in 2019.



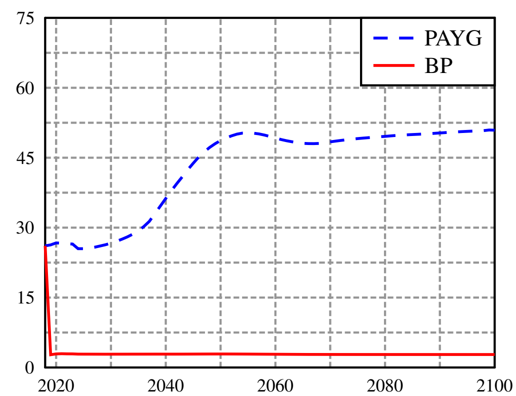
(a) Reform Debt



(b) PAYG pension payments



(c) Aggregate BP Assets



(d) Payroll tax rate

Figure 7: Reform Debt and PAYG pension payments (% of output), Aggregate BP Assets and and Payroll tax rate (percentage) during the transition.

Figure 8 shows that the BP asset transfers to vintage workers in 2019 (working age individuals who are 21 or older in 2019) increases with age and is zero up to the cohort who is 30 on that year. The younger cohorts require no BP contributions because they value positively the change in future tax payments that the pension system reform implies. First, payroll taxes decrease immediatly and permanently in the reformed economy. Aditionally, the consumption tax increases during the first decade, to compensate for the decrease in tax revenues from capital income (i.e.private savings) and increase in interest payments, but declines steadily from 2030 on (see Figure 9). The former dominates and young workers require no BP subsidy to vote in favor of the BP reform.

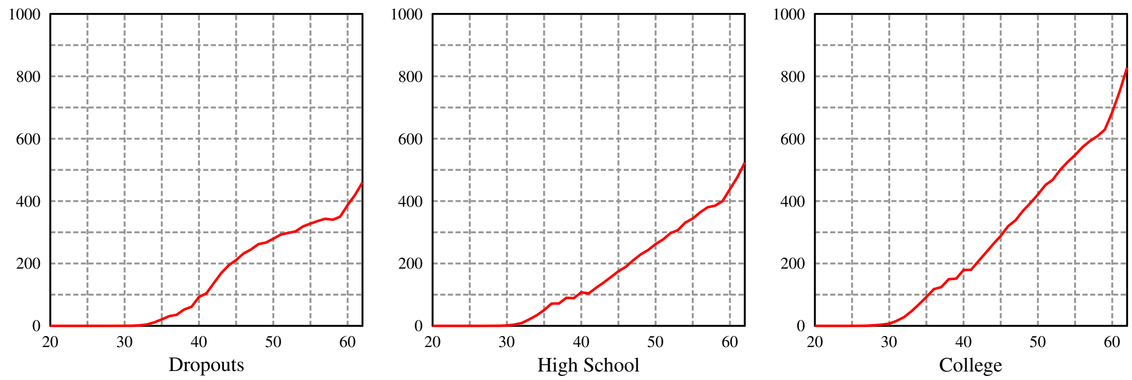


Figure 8: Average BP asset subsidies by age, for vintage workers in 2019 (% of per capita output).

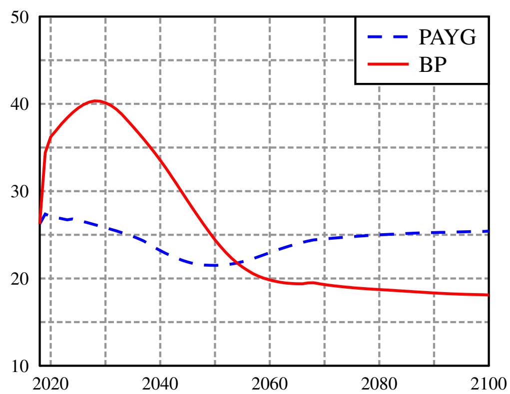


Figure 9: Consumption tax rate in the baseline scenario and during the BP reform (%).

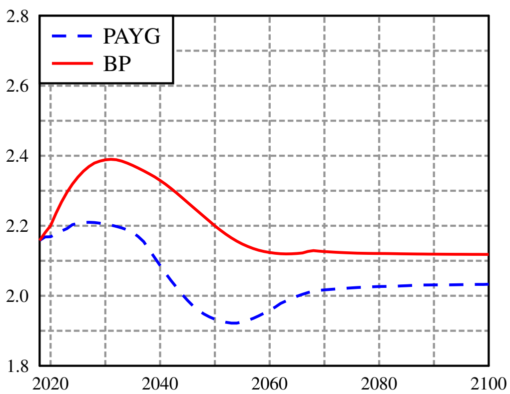
Figure 10 shows the transitional gains on output, capital, labour and aggregate consumption in the BP transition when compared to the PAYG baseline. With the decline in payroll taxes and increase in consumption tax in 2019, hours worked increases and with the elimination of PAYG pension rules for all workers, the retirement decision is delayed. Consequently, labor supply increases initially. With the ageing transition, aggregate labor supply declines to converge to the final steady state after 2030. The share of workers older than 65 increases, but average productivity declines.[24] An older population, compared to 2018, implies a larger share of retirees and lower share of workers in the aggregate. Nevertheless, aggregate labor is higher than in the PAYG transition and in the long run. Consumption declines in the aggregate in the first two years due to the initial increase in consumption taxes. The group who is most affected by the reform are 2019 retirees, who are not affected by the decrease in payroll taxes but bear the increase in the consumption tax.

[24]Recall that we assume that the age profile of productivity is fixed during the transition.

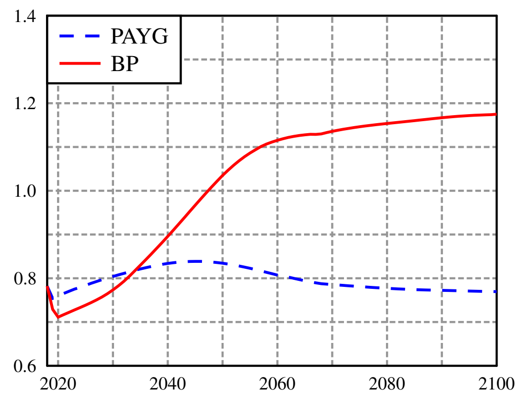
After the initial impact, consumption steadily increases after the increase in lifetime income, in particular after retirement (see Table 11).

Table 11: Average Retirement Income as a percentage of per capita output in the PAYG and BP economies in 2068.

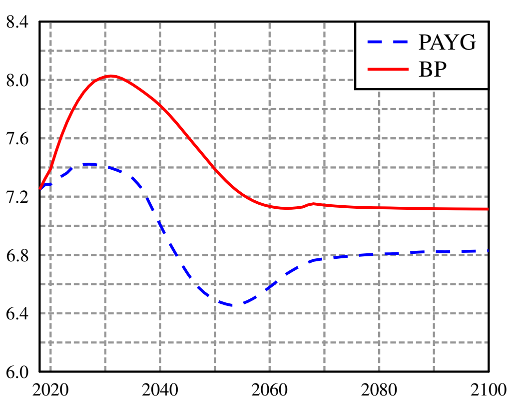
	Dropouts	High School	College
PAYG	44.07	57.90	89.54
BP	83.88	107.80	148.42



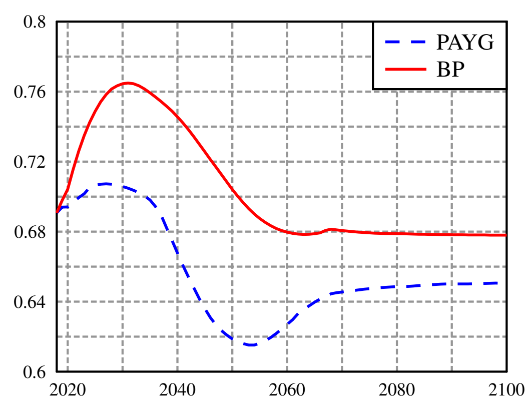
(a) Output



(b) Consumption



(c) Capital



(d) Labor

Figure 10: Main aggregates during the reform.

Figure 11 shows the change in average lifetime utility in the BP transition for each cohort who enters the economy in 2019 and after, relative to lifetime utility of the same cohort in the alternative transition under the PAYG pension system. By construction, during the reform all those who enter the Backpack system are better or equal as they would have been in the PAYG, with increasing welfare gains as cohorts reach, in the second half of the 21st Century, the steady-state welfare gains. The large long run gains can make the reform a Pareto improvement (i.e. without any losers) and robust to other specifications (such as, higher cost of debt or general equilibrium effects not accounted for in the current analysis).[25] To be more precise, in the solution described in the previous section ('front-loaded' transition), the reform is almost but not a full Pareto improvement for all households alive and unborn as of 2019, since PAYG retirees alive in 2019, while they receive their full pension, they also face higher consumption taxes that increase in the first decade. However, there is fiscal room to compensate the losses of the PAYG retirees and implement a Pareto improving reform of the social security system, e.g.with a small initial transfer to 2019 retirees that compensates for the increase in taxes.

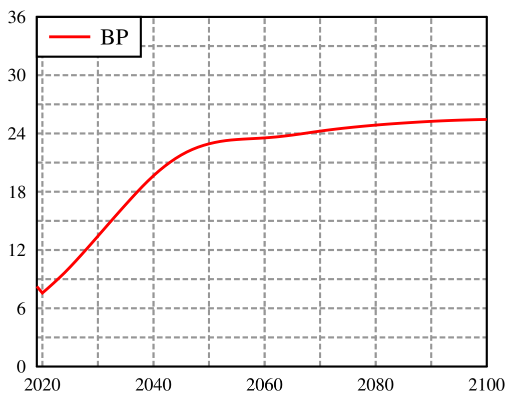


Figure 11: Average increase in lifetime utility for cohorts entering the economy in the 'front-loaded' BP transition, relative to the baseline PAYG status quo.

In the next subsection we compare the two long run scenarios: the PAYG and the reform Backpack economy.

5.2 BP and PAYG pension systems in the long run

In the tables below, the *BP* economy refers to the long run, reformed economy, with a Backpack (BP) fund as described in Section 2. This steady state assumes that the age distribution is as in the 2068 forecast and that the PAYG system was eliminated and replaced with a Backpack system with a contribution rate $\tau_b = 26\%$. This economy is reached after the transition shown above, with

[25] We find that general equilibrium effects increase the welfare gains of the reform and lead to a small final debt level, see Appendix A.

a stock of reform debt equal to 3.1 times (2068) GDP, which with a one per-cent interest is 3.1 of GDP in 2068 (permanent if, as assumed here, there is no growth).

The following tables compare the BP economy with the status quo PAYG economy in 2068.

Table 12: Aggregates in the PAYG and BP simulations of 2068.

	Y	L	A	K	BP	C	h^a
PAYG	2.04	0.65	2.23	6.85	–	0.76	0.17
BP	2.12	0.68	9.95	7.11	7.84	1.18	0.18

*In this table, variable Y is output at factor cost, and variable A is total assets.
[a]Variable h denotes total hours of time allocated to the market.

The first order effect of a mandatory retirement savings system is on private savings behaviour before and after retirement. The retirement pension system in the PAYG economy, by taxing a large fraction of workers' wages that are then paid back after retirement, discourages private savings, since workers expect pension payments during retirement. Eliminating PAYG pensions provides a strong incentive to save during working years, in order to finance consumption after retirement. On the other hand, the BP system features a fixed 26% contribution rate out of gross labor income, which is capitalized and available for consumption during involuntary unemployment and after retirement. Additionaly, workers can convert backpack savings into a life annuity at retirement, which eliminates a precautionary motive to save for the event of an above average life time. While these features of the BP system reduce incentives to save, BP contributions are invested in productive capital in the international capital market (in contrast to the PAYG pension system, which transfers resources from workers to retirees within any given year, via the Social Security budget), earning the market interest rate. Table 12 shows that while private savings are slightly lower, total private assets (private savings together with backpack savings) in the economy more than double. The stock of capitalized BP contributions is more than 3 times output. As explained above, these capitalized contribution are then converted in annuities after retirement, contributing to a large gain in post-retirement income in the BP economy and consequently large increase in consumption.

Another direct effect of the reform is on the timing of the retirement decision. Since there is no minimum nor maximum retirement age in the BP economy, workers decide when to retire according to the earnings-leisure tradeoff, taking into account labor productivity and job finding prospects in the last years of life. Table 13 shows that this drives the share of retirees substantially down in the BP economy, by 8 percentage points, and the share of workers higher by almost the same share. In 2068, the average retirement age in the PAYG economy is 65.1 and in the BP economy is 69.9.[26] The effective labor tax is higher in the PAYG economy (reducing work incentives), and cost

[26]Recall that life expectancy, education, the age profile of labor market productivity, job destruction and job finding rates are the same in the two economies.

of delaying retirement relative to wage salaries tend to increase with age (as productivity starts to decline), after the minimum retirement age. Therefore the PAYG system provides a strong incentive to retire and leave the labor force close to the minimum retirement age. In contrast, the effective labor tax is lower in the BP economy and annuity payments increase with BP savings, which accumulate by working. This provides an incentive to work until later. The share of workers older than 65 in the PAYG economy is 9% while it is 18% in the BP economy. Higher work incentives increase job search and hence unemployment is higher (and inactivity lower) in the BP economy.

Table 13: Labor Market Shares in the PAYG and BP simulations of 2068 (% of population).

	W	U	I	R
PAYG	50.77	10.77	3.75	34.70
BP	55.48	12.40	3.37	28.73

W: workers, U: unemployed, I: inactive, R: retirees.

Table 14: Average Retirement Age and the percentage of workers older than 65 in the PAYG in and the BP economies in 2068.

	Age	Workers 65+
PAYG	65.12	9.08
BP	69.88	18.11

Other important effects come indirectly through taxes. Government expenditures with retirement pensions is zero and the payroll tax rate is only 2.8% in the BP economy. On the expenditure side, government transfers increase as more low income households qualify. On the revenue side, capital income tax collection as a share of output is constant (since capital income as a share of output is constant, due to the small open economy assumption). Household consumption is higher in the BP economy, and despite the additional cost of debt in the BP economy, the consumption tax rate required to clear the government budget is lower: $\tau_c = 18$ compared to 25% in PAYG.

Table 15: Government Budget in the PAYG and BP simulations of 2068 (% of output, Y, at factor cost).

	Public Expenditure				Public Revenues			
	T_r	P	U	rB	T_c	T_k	T_y	T_p
PAYG	0.83	23.03	1.38	0.00	9.62	2.55	7.45	24.41
BP	0.92	0.00	1.29	3.13	10.02	2.55	10.31	1.29

G: government consumption, T_r: minimum income, P: pension payments, U: unemployment benefits expenditures; T_c: consumption tax collections, T_k: capital income taxes, T_y: household income tax revenue, T_p: payroll tax revenue.

Table 16: Policy Parameters and tax revenues in the PAYG and in the BP economies.

	Tax Rates (%)	
	PAYG	BP
τ_c	25.4	18.1
τ_p	50.9	2.7
τ_b	-	26.0
τ_e	65.3	44.23

τ_c: consumption tax rate, τ_y: household income tax rate, τ_k: capital income tax rate, τ_p: payroll tax. τ_x fund tax rate; e.g. $x = b, f$, τ_e efficient labour tax (see Footnote 9).

In order to interpret the magnitude of the welfare gains in the reformed economy, we use a consumption equivalent variation measure (CEV) that converts average welfare into consumption units. As explained above, we found the welfare maximizing BP contribution rate relative to the PAYG economy (2068) as the benchmark. To convert the increase in welfare into a CEV, we compute the percentage change in a household's lifetime consumption that equates its expected lifetime utility in the PAYG economy, to that in the reformed economy BP economy. Formally, let $i \in J \times H \times \mathcal{Z} \times \mathcal{L} \times A$ denote the household's type.[27] Define $v^{PAYG}(i, \Delta(i))$ as the equilibrium value function of a household of type i in model economy PAYG, whose equilibrium consumption plan is changed by a fraction $\Delta(i)$ every period and whose leiture (and search) plan is unchanged. Then the CEV measure is found according to:

$$v^{PAYG}(i, \Delta(i)) = v^R(i), \tag{32}$$

where $v^R(i)$ denotes the equilibrium value function of household of type i in the reformed BP economy.

Table 17 displays the large welfare gains at age 20 from entering a $\tau_b = 26\%$ Backpack backpack economy in 2068, relative to entering the PAYG economy in that same year, where in the former

[27]Recall the dimensions of household heterogeneity: age, education, productivity, labor market statys, assets.

the government has to finance the interest on the reform debt (3.1% of GDP). The gains are of the order of magnitude of the decrease in the payroll tax distortion, which is very high with the PAYG pension system. All education types are at least 33% better off in the BP economy, despite the relatively high contribution rate of the BP pension system. The main reason for this is that despite the high BP contribution rate, the effective labor tax is lower, BP savings are capitalized instead of transfered between few workers and many retirees, and workers decide to leave the labor force later. Hence retirement pensions are higher than in the PAYG economy (Table 11), which allows for higher consumption and lower private savings during the entire lifecycle. Consumption is much higher in the BP economy, specially before the first retirement in the PAYG system (age 63), and average work hours are lower. The fact that agents retire later on does not have a significant effect, given the effective discount rate (β times survival probabilities).

Table 17: Consumption Equivalent Variation (Δ, %) in welfare of household of different education levels in the BP economy with $\tau_b = 26\%$, relative to the PAYG economy in 2068.

	Education			
	Dropouts	High School	College	All
CEV	41.30	42.02	40.84	41.71

In this section we have shown that it is possible to implement a reform of the PAYG pension system during a demographic transition that doubles the ratio of individuals older than 65 relative to the 20-64 group, that is welfare improving for all cohorts who enter the economy during the transition period, with the introduction of the fully funded Backapack saving system.

The PAYG and BP long-run economies, compared above, despite sharing many important features (technology, demographics, government tax structure), differ significantly in terms of the retirement pension system available to households – with large aggregate consequences, as discussed above. Nevertheless, there are different dimensions of the pension reform that contribute to its large welfare gains. These can be isolated and analyzed separately: eliminating the pay-as-you-go system, introducing a fully funded pension system, and adding flexibility to this system by allowing worker to use BP contributions during period of involuntary unemployment. The BP reform is the sum of these three elements. By studying these different aspects separatly, we show how the Backpack system delivers higher welfare when compared to standard (i.e. defined contribution) fully funded pensions or the full privatization of savings.

Section 6 presents a comparison with alternative funded pension systems, comparing their performance againts the Backpack system.

6 Comparison with alternative funded pension systems

In order to study the different components of the BP system, we consider below two alternative long-run economies: one where the PAYG pension system is eliminated and workers save for retirement only through private savings (we label it Private Savings economy), and another where the PAYG pension system is substituted by a standard defined contribution funded pension system. Additionally, we discuss effects at the individual level, and compare the different economies in terms of welfare.

In order to do so, we solve the stationary equilibrium of these alternative economies, assuming the 2068 age distribution and the elimination of PAYG pensions. For the defined contribution pension system, we perform a grid search as in the BP exercise and find a welfare maximizing mandatory pension contribution rate of $\tau_f = 19\%$.[28]

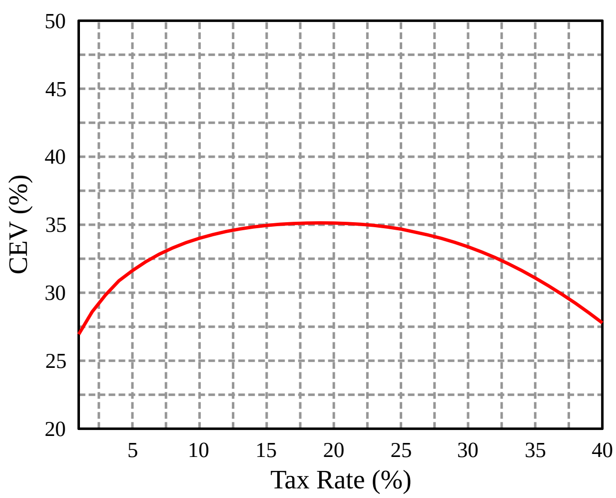


Figure 12: Average increase in welfare (CEV) at age 20 as a function of FF contribution rate (τ_f) in economies with a Fully Funded pension system relative to the PAYG economy, assuming a final debt level of 3.1 times output and $r^* = 1\%$.

PS. In the Private Savings economy there is no explicit retirement pension system, and households support consumption after retirement exclusively using private savings (PS).

FF. The fully-funded, defined contribution, pension scenario is labeled *FF*. In this case, agents save a mandatory contribution as a fixed fraction of their labor earnings, that accumulate in an individual notional account until retirement. At retirement, the capitalized lifetime contributions

[28]Here we assume that the final FF steady state has the same debt level as in the BP reform. We have confirmed that the optimal BP system delivers higher average welfare when compared to the FF system even if we assume that the cost of debt is zero after the FF transition.

are converted in a pension payment as an actuarily fair annuity.

The PS economy is computed after eliminating the public pension system, by setting $p_r = 0$. This implies that aggregate pension payments are zero, $P = 0$. The economy with a fully funded pension system is similar to the BP economy, with the important distiction that worker contributions to the pension system are claimed at retirement, but not after job loss.

The full description of the two economies above is included in Appendix B. Below we clarify the differences relative to the pension system available to workers in each economy.

6.1 Fully-Funded System

The *FF* economy features a standard fully-funded, defined contribution, pension system. Retirement pensions are financed by individual own contributions accumulated while working. Specifically, each worker has a mandatory contribution rate of τ_f of gross labor earnings y. The contributions are remunerated at the rate of return of capital. We assume, as in the Backpack system, that notional returns are taxed at the same rate as private savings returns, τ_k; and that they are not part of the income tax base, as in the BP case. Hence, denoting by m_t the notional account of pension claims of a given worker at the beginning of period t, the evolution is given by:

$$m_{t+1} = \tau_f y + (1 + r(1 - \tau_k))m_t, \tag{33}$$

and

$$m_{t+1} = (1 + r(1 - \tau_k))m_t, \tag{34}$$

in periods out of work. When a worker of age R retires with accumulated pension claims m, he is entitled to a pension payment per year given by:

$$p^F(m) = \frac{(1+r)^{R-T}}{\sum_{j=R}^{T} \psi_j} m. \tag{35}$$

In expectation, at retirement age R, given his capitalized career contributions m, the retiree receives an actuarily fair annuity $p^F(m)$. The aggregate amount of pension claims is invested in the international capital market. As in the BP case, the system is fully funded because pension payments due to retirees who live longer than average are transferred from pension claims of retirees who leave earlier than average, and no other (taxed) resources are necessary to finance pension payments. [29]

6.2 Results

As in the BP policy reform, changing or eliminating the pension system requires an assumption about which tax instrument is changed in order to balance the government budget. We mantain the

[29] Appendix B.2 contains the formal description of the FF economy.

assumption that the payroll tax rate τ_p adjusts to clear the social security budget, which in both the FF and PS economies (as in the BP economy), since there are no government liabilities with retirement pensions, consists only of unemployment benefit expenditures. As before, we assume that the consumption tax rate τ_c adjusts to clear the budget (government debt and government consumption are constant). In the following tables, we include the PAYG 2068 economy results, presented in Section 4, for comparison.

Table 18 shows the main aggregates in the three reformed economies. Since we have discussed above the differences between the PAYG and the BP economies, we will focus here on the main differences across the three reformed economies. The elimination of the PAYG pension system drives most of the differences in macroeconomic aggregates in the three economies: the three reformed economies are closer to each other than any of them is to the PAYG economy. It has a large direct effect on disposable income through the reduction in payroll taxes, and a large direct effect on savings behaviour due to the elimination of pension payments. Unsurprisingly, all the reformed economies have higher asset levels than the PAYG economy. The retirement pension system in the PAYG economy discourages private savings before retirement. In contrast, the PS economy, where retirees finance consumption exclusively through private savings, displays the highest stock of private savings, 11.35. The flexibility of the optimal BP system makes aggregate private savings the lowest among the reformed economies, since workers internalize the possibility to use BP assets during involuntary unemployment.

Table 18: Aggregates in the PAYG, PS, FF, and BP simulations of 2068.

	Y	L	A	K	X	C	h
PAYG	2.04	0.65	2.23	6.86	–	0.76	0.17
PS	2.49	0.80	11.35	8.37	–	1.03	0.22
FF	2.16	0.69	11.19	7.26	7.31	1.15	0.18
BP	2.12	0.68	9.95	7.11	7.84	1.18	0.18

*In this table, variable Y is output at factor cost, A is total assets, and X is total backpack assets in the BP economy and aggregate pension claims in the FF economy.

Household consumption and total savings are higher in all reformed economies but there are important differences between the three scenarios. Households save much more in the PS economy, as private savings are the only means to finance consumption after retirement. Savings continue until later in life, while annuity payments in the FF and BP economies allow agents to start desaving when they are around 60 years old, on average (roughly 10 years earlier than in the PS economy). Consequently, consumption is higher in the FF and BP economies, in particular during the last decades of life. The BP economy features the highest consumption (in all education groups) due to higher pension payments. Since the optimal BP tax rate is 26%, compared to the contribution rate of 19% in the fully funded FF system, pension payments are higher in the BP case. With higher aggregate retirement savings in the BP economy, workers can afford to retire earlier in comparison to the other two reformed economies. In contrast, the PS economy displays the lowest share of retirees and the highest share of workers. This is explained by the average retirement age in each economy.[30]

Table 19: Retirement statistics in the PAYG, PS, FF, and BP economies of 2068.

	R. Age	Workers 65+	Dropouts	High School	College
PAYG	65.12	9.08	44.07	57.90	89.54
PS	96.10	48.84	40.57	53.86	83.87
FF	73.07	23.18	77.76	99.91	137.83
BP	69.88	18.11	83.88	107.80	148.42

Columns from left to right: Average retirement rate, share of workers among population older than 65, average income after retirement as a share of per capita output for dropouts, high school and college educated households.

Table 20: Labor Market Shares in the PAYG, PS and BP simulations of 2068 (% of population).

	W	U	I	R
PAYG	50.77	10.77	3.75	34.70
PS	66.86	13.57	4.55	15.02
FF	56.74	11.15	7.58	24.51
BP	55.48	12.40	3.37	28.73

W: workers, U: unemployed, I: inactive, R: retirees.

Table 21: Government Budget in the PAYG, PS, FF, and BP simulations of 2068 (% of output, Y, at factor cost).

	Public Expenditure				Public Revenues		
	T_r	P	U	rB	T_c	T_y	T_p
PAYG	0.83	23.03	1.38	0.00	9.62	7.45	24.41
PS	1.69	0.00	1.20	0.00	9.35	8.61	1.20
FF	1.11	0.00	1.23	3.13	10.37	10.15	1.23
BP	0.92	0.00	1.29	3.13	10.02	10.31	1.29

T_r: minimum income, P: pension payments, U: unemployment benefits expenditures; rB: interest payments; T_c: consumption tax collections T_y: household income tax revenue, T_p: payroll tax revenue.

Table 21 shows the output shares of the government taxes and revenues in the three scenarios.[31]

[30]Note that the average retirement age in the PS economy is above 96, higher than average life expectancy in the 2068 economy. The reason is that many households stay inactive (do not search but remain in the labor force) until they die, while others decide to retire. In both cases, they do not have access to a retirement pension in the PS economy. Conditional on retiring, the average age at which agents leave the labor force is 96.

[31]Recall that government consumption as a share of output is fixed, and the level unintentional bequest taxed by

Pay-as-you-go pension payments (P) are zero in the reformed economies, whereas they represent 23% of output in the PAYG economy. This difference explains the large decrease in the payroll tax rate in Table 22, from 51% in the PAYG economy to less than 3% in the reformed economies. Despite unemployment increasing once the PAYG system is eliminated, unemployment benefit expenditures as a ratio of output slightly decrease because output increases. Table 22 shows an increase in social income transfers to the poorest agents in the economy once PAYG pensions are eliminated. The reason for this is the following: by eliminating PAYG pensions, some low productivity and low savings workers over 65 eventually loose their job but keep searching, staying unemployed while they don't find one (they would mostly choose to retire with PAYG pensions). After two years of unemployment, unemployment benefits expire and, once falling below the poverty threshold to qualify for social assistance, they start collecting government transfers. In the PS and BP economy, more households reach this state and hence aggregate transfers to low income households are higher. The aggregate amount of tranfers is lower in the BP economy, among the reformed scenarios, because households retire relatively earlier and retirement pensions are higher in that economy, and hence fewer households reach the minimum income level to quality for government assistance. Higher retirement pensions also imply higher income tax collections on the BP economy. This allows for a lower consumption tax rate (Table 22) and but still a relatively large consumption tax base, due to higher aggregate consumption, in order to balance the government budget at the steady state.

Table 22: Policy Parameters and tax revenues in the PAYG, PS, FF and in the BP economy.

	Tax Rates (%)				Revenue Y^* Ratios (%)			
	PAYG	PS	FF	BP	PAYG	PS	FF	BP
τ_c	25.77	22.71	19.48	18.05	9.62	9.35	10.37	10.02
τ_k	25.00	25.00	25.00	25.00	2.55	2.55	2.55	2.55
τ_y	14.18	14.18	14.18	14.18	7.45	8.61	10.15	10.31
τ_p	51.33	2.97	2.73	2.79	24.41	1.20	1.23	1.29
τ_x	0.00	0.00	19.00	26.00			9.79	13.40
τ_e	66.79	32.14	43.78	48.23				

τ_c: consumption tax rate, τ_y: household income tax rate, τ_k: capital income tax rate, τ_p: payroll tax. τ_x fund tax rate; e.g. $x = b, f$, τ_e efficient labour tax (see Footnote 9).
* : As a share of output at factor cost.

the government also fixed. The other components react to any changes in the economy.

Table 23: The Distributions of Earnings, Income, and Wealth

		Bottom	Quintiles					Top
	Gini	10	1st	2nd	3rd	4th	5th	10
The Earnings Distributions (%)								
PAYG	0.38	3.1	7.4	10.4	14.8	22.3	45.0	28.7
PS	0.34	3.5	8.1	11.6	15.3	23.3	41.7	26.5
FF	0.34	3.5	8.4	11.5	15.6	23.2	41.3	26.1
BP	0.34	3.4	8.2	11.6	15.6	23.2	41.4	26.1
The Income Distributions (%)								
PAYG	0.34	1.9	5.5	11.8	19.3	22.8	40.5	24.5
PS	0.44	1.6	4.0	8.6	14.8	23.3	49.2	31.0
FF	0.41	1.4	4.3	10.5	15.4	24.2	45.7	29.0
BP	0.40	1.3	4.5	10.1	15.9	24.1	45.4	28.2
The Wealth Distributions (%)								
PAYG	0.75	0.0	0.0	0.9	4.8	17.5	76.7	55.8
PS	0.54	0.0	0.8	5.8	13.5	24.9	54.9	34.1
FF	0.69	0.0	0.0	1.0	8.1	21.9	69.0	46.9
BP	0.80	0.0	0.0	0.0	1.2	14.7	84.0	60.4

There are also important differences in terms of income and wealth inequality between status quo and reformed economies. Table 23 shows the distribution of income, earnings and wealth in the four economies. Changes in all inequality measures are mainly driven by the longer working lifetime in the reformed ecomomies (PS, BP, and FF) compared to the PAYG economy. In the reformed economies, earnings inequality decreases mainly because the difference in the deterministic labor productivity by educational type, which strongly decreases for the more educated workers as they become older. Recall that people retire later in the reformed economies. In the reformed economies, income inequality increases mainly because the following. Retirees replace public retirememt income (evenly distributed since there are only three types of public retirement pensions) by capital income and/or annuity income which is more unevenly distributed. Wealth inequality refers only to private assets holdings. In the PS economy, dropouts increase by more their saving rates, as there are no public pensions (the main income source for low educated retired people), so wealth inequality decreases. In the BP and FF economies, wealth inequality is higher than in the PS economies, as they deliver a forced saving scheme for the retirement period, so low educated people reduce by more savings during their working lifetime. The higher this compulsory saving, the higher this effect, so the higher the wealth inequality.

Some of these results are sensitive to the small open economy assumption. If changes in the pension system affect interest rate and wages, private savings and labor supply react and the optimal backpack contribution rate (τ_b) and the pension contribution rate (τ_f) also differ. In particular, eliminating the PAYG system encourages private savings in any alternative reformed economy. More savings reduce the interest rate and increase the wage rate in a closed economy, which in turn decreases incentives to save and the return on the fully funded and backpack pension systems savings. This leads to lower optimal contribution rates in the defined contribution systems. We report the closed economy results in Appendix A.

6.3 Welfare effects

We use the same consumption equivalent measure as above (CEV) to quantify the increase in average lifetime utility at age 20 across steady state economies, for each education group.

Table 24: Consumption Equivalent Variation (Δ, %) in the PS, BP and Pension Fund economies, relative to the PAYG economy.

Simulation	Education			
	Dropouts	High School	College	All
PS	35.70	36.02	33.59	35.39
FF	35.60	35.40	34.08	35.13
BP	41.30	42.02	40.84	41.71

Table 25: Consumption Equivalent Variation (Δ, %) in the FF and BP economies, relative to the PS economy.

Simulation	Education			
	Dropouts	High School	College	All
FF	-0.07	-0.47	0.36	-0.23
BP	4.21	4.52	5.53	4.77

Welfare is higher in the BP economy because of the additional flexibility of backpack contributions during periods of unemployment. Because workers can access these funds after job loss, it provides unemployment insurance. This makes it possible to increase the contribution rate (relative to what is socially desirable in the fully funded pension system) and deliver higher retirement pensions, compensating for the distortionary effect of a fixed contribution rate for all workers (irrespective of age, earnings or wealth). Table 26 shows the extent to which unemployed workers use backpack savings in the first period of involuntary unemployment, compared to private savings.

Table 26: Average Private and Backpack (de-)saving rates by age (as a proportion of unemployment benefits), for unemployed workers who search for a job.

	Private Savings	Backpack Savings
Age 30	-22.40	-10.68
Age 40	-39.88	-22.28
Age 50	-41.69	-30.73

The fully funded (FF) delivers lower welfare in the long-run when compared to both the BP system and the laissez-faire PS economies, although very close to the latter. This result should be interpreted with caution. As in the final steady state of the BP economy, the FF economy is reached with a certain level of 'entitlement debt' generated by the elimination of the PAYG system without default. Since we have not solved the full FF transition, and the final level of debt depends on the transition, we assumed for comparison that final debt level is the same as in the BP final steady state ($3.1Y$). We did however compute an upper bound on the long run welfare gains delivered by the FF system, by assuming that the final steady state is reached with zero debt, and found that this bound is numerically identical to the gains found in the BP system (41.7%), accounting for its implied debt level that we obtained after solving for the BP reform, discussed above. As any FF transition, without default on PAYG pensions, would imply a positive level of 'entitlement debt', the main result of our analysis that the BP system delivers higher welfare gains holds.

7 Conclusions

Using an overlapping generations model with labor market frictions, we have shown that there can be important allocative and welfare gains in introducing a Backpack system in an economy with a pay-as-you-go pension system and unemployment insurance. The main mechanism behind these gains is to have a fully funded pension system in a aged population, with partial substitution of private savings by backpack savings (exempt from income taxation), while total savings and capital increase; as a result, effective labour taxes are eighteen points lower. Associated with this change there is a better allocation of employment, with higher share of employed – in particular, a higher percentage of high productive agents within the employed – and a lower share of inactive and retirees. Effectively, there is a more efficient allocation of savings in the economy, with a shift from pure transfers (to the unemployed and retirees) to savings and, therefore, investment in productive capital. Unemployed are better off due to the prospect of higher earnings, and retirees are better off since in our economy pension benefits are linked to productivity, which is higher in the BP economy. The gains are even larger if we consider the Spanish economy as a closed economy, since there is a higher capitalization, with corresponding lower interest rates and higher wages (see Appendix A). This means that, in as much as EU economies are not fully open, the gains from a PAYG reform for a Backpack system are even larger than the ones described here.

The final result is that a substantial Pareto improvement can be achieved by replacing the PAYG system with the BP system. The BP steady-state also dominates the simple elimination of the PAYG, letting agents freely choose their savings for retirement; i.e. the Private Savings (PS) economy. In comparing the two, the PS has a lower effective labour tax, but all the savings are part of the taxable income and retirement income is not insured. Welfare is also higher than in an economy with a fully funded pension fund (FF), since agents can better manage their savings as to insure not only their retirement, but also their unemployment spells beyond what the existing unemployment insurance provides. To our knowledge, we have been the first to analyse employment and welfare effects in comparing alternative social security systems, among them the Backpack.

The immediate question that our steady state results suggest is how to implement a Pareto improving transition: no losers in the generations involved from the current PAYG to the final steady-state BP economy. This would already be challenging in a steady state economy, where pension payments are more than 10% of GDP and the dependency is 31% (as in Spain), if this ratio were to be the same in the decades to come, but it is even more difficult when the country faces an ageing transition in the first half of the 21st Century where the dependency ratio for the PAYG system doubles to 60%; i.e. from 3.2 workers per retiree to 1.6. Nevertheless, we show how a Pareto transition can be based in two elements. First, the large welfare gains that the reform can achieve in the long-run once it has been implemented (42% in average CEV). This suggest the second element, which is the timing of a debt-financed reform: given the fast ageing process in the decades up to the 2050s, a reform that anticipates the ageing transition – where all current and future workers move to the BP system, with debt issuance to finance BP asset transfers for workers with implicit claims to PAYG pensions (and current PAYG pension payments) – minimizes the final cost of the reform: the interest payments that are financed with taxes in the final steady state economy. In our calibrated Spanish economy, the amount of financing debt is large (160% of GDP in the first year, which becomes 310% at the end of the transition) but much lower than in a slow transition (less than half) and it is sustainable with reasonable low interest rates (our benchmark is one percent annual rate, a steady-state cost of 3.1% of GDP if there is no growth).

In our analysis, we have made some assumptions and restrictions. A reform may be (fully or partially) financed by other means (more efficient taxation, broader labour market reform, higher growth, etc.), but the *fast* transition from PAYG fo BP pensions presented in this article is a benchmark of the overall costs and benefits of a reform without losers; short of this, there will be losers with a reform of the PAYG system. The "Next Generation" will be better off with the legacy of a Backpack system with an 'entitlement debt' than with the legacy of a PAYG system without a 'reform debt'.

Appendices

A Closed economy results

A.1 Pay-as-you-go pension system in the long run

Here we present the results comparing different long-run steady state equilibria assuming Spain is a closed economy. This assumption impliess that the wage rate and the interest rate are determined by market clearing conditions of domestic labor and capital markets. We recalibrate the initial steady state as a closed economy. Given the closed economy assumption, any policy change that affects household savings has a direct effect on the supply of capital in the economy and the equilibrium interest rate.

Table 27: Labor Market Shares in the baseline 2018 model economy, and in the PAYG 2068 simulation (% of population).

	W	U	I	R
Model (2018)	58.31	11.68	5.30	24.70
PAYG (2068)	50.46	10.46	4.26	34.80

W: workers, U: unemployed, I: inactive, R: retirees.

Table 28: Macroeconomic Aggregates and Ratios in the baseline 2018 economy, and in the PAYG 2068 simulation*.

	Y	K/Y	L/Y	h^a	C/Y	I/Y	w	$r(\%)$
Model (2018)	2.04	3.23	33.15	32.72	46.31	30.97	1.55	6.40
PAYG (2068)	1.79	2.98	35.81	33.39	50.13	24.11	1.44	7.68

*In this table, variable Y is output at factor cost. The number for K/Y is in model units and not in percentage terms. All the remaining ratios are expressed in percentage terms.
[a]Variable h denotes the average share of disposable time allocated to the market.

Table 29: Government Budget in the 2018 model economy and in the PAYG simulation of 2068 (% of output, Y, at factor cost).

	Public Expenditure			Public Revenues			
	T_r	P	U	T_c	T_k	T_y	T_p
Model (2018)	0.91	11.50	1.29	9.55	2.46	7.73	12.80
PAYG (2068)	0.82	23.04	1.36	11.55	2.72	7.94	24.38

T_r: minimum income, P: pension payments, U: unemployment benefits expenditures; T_c: consumption tax collections, T_k: capital income taxes, T_y: household income tax revenue, T_p: payroll tax revenue.

The main aggregate changes relative to the open economy results are indeed the interest and wage rate. Under the PAYG system, the decrease in aggregate capital increases its marginal product and hence the equilibrium interest rate in 2068. The price change amplifies the decline in output due to the ageing process (12% in a closed economy vs. 6% in an open economy) due to a fall in investment (i.e. savings). Aggregate labor market stocks evolve similarly as in the open economy.

A.2 Backpack Economy

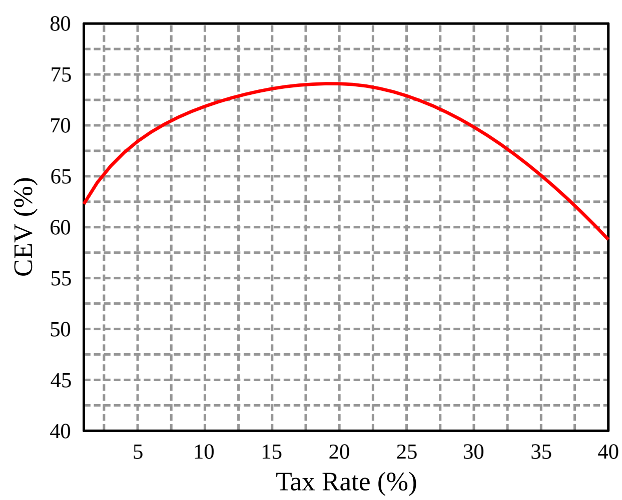


Figure 13: Average increase in welfare (CEV) at age 20 as a function of BP contribution rates (τ_b) in reformed economies with a BP pension system relative to PAYG economy 2068, assuming a final debt level of 2.2 output and $r^* = 1\%$.

We perform the same grid search procedure as in the main text to find the welfare maximizing BP rate in 2068 under a closed economy assumption. When prices react to change in the pension system, the welfare maximizing BP rate is slightly lower than in an open economy assumption.

Intuitively, the interest rate declines as the stock of capital, including BP assets, increases. This makes it optimal to implement a 21% BP rate (instead of 26% in an open economy). The final debt level is 2.2 times output and, as in the main text, we assume an interest rate on the reform debt of one percent per year.

The following tables compare the 21% BP economy with the PAYG economy as closed economies.

Table 30: Aggregates in the PAYG and BP simulations of 2068.

	Y	K/Y^*	L/Y^*	h^a	C/Y^*	I/Y^*	B/Y^*	w	$r(\%)$
PAYG	1.79	2.98	35.81	33.39	50.13	24.11	–	1.44	7.68
BP	2.84	4.41	25.67	31.88	48.90	27.53	3.24	2.08	2.39

[*]In this table, variable Y is GDP at factor cost.
[a]Variable h denotes the average share of disposable time allocated to the market.

In constrast to the results in the main text, BP contributions are invested in productive capital in a closed economy, increasing the aggregate capital stock in the economy. The capital-output is 4.4 in the BP economy and 2.98 in the PAYG economy, with the stock of capitalized BP contributions at 3.24 of output. While aggregate savings are higher in the BP economy (private + BP assets), private savings are lower than in the PAYG economy. Households substitute private assets with BP assets, but the aggregate stock of savings, and therefore productive capital, is still larger in the BP steady state.

The larger capital stock decreases its marginal product, and accordingly the real interest rate falls. The capital-labor ratio more than doubles, making labor more productive, hence the wage rate increases. Government expenditures with retirement pensions is zero, and the payroll tax rate is only 2.93% in the BP economy. On the expenditure side, government transfers increase as more low income households qualify. On the revenue side, capital income tax collection as a share of output falls, because despite the increase in capital stock and an additional source of capital income taxes, coming from BP assets, the return on capital falls and capital income as a share of output falls. Despite higher household consumption in the BP economy, the consumption tax rate is higher ($\tau_c = 28.57\%$ compared to 23.1% in PAYG), in order to balance the government budget. Table 31 shows that the share of retirees is substantially down in the BP closed economy, and the share of workers higher than in the open economy. The effective labor tax is 3 pp. higher in the BP closed economy, but the wage rate is higher by almost a half. This provides an incentive to work until later, and therefore the share of workers is higher (and retirees lower) when compared to the open economy results.

Table 31: Labor Market Shares in the PAYG and BP simulations of 2068 (% of population).

	W	U	I	R
PAYG	50.46	10.46	4.26	34.80
BP	58.84	13.52	3.73	23.91

W: workers, U: unemployed, I: inactive, R: retirees.

Table 32: Policy Parameters and tax revenues in the PAYG and in the BP economies.

	Tax Rates (%)	
	PAYG	BP
τ_c	23.05	28.45
τ_p	51.02	2.93
τ_b	-	21.00
τ_e	64.69	47.46

τ_c: consumption tax rate, τ_y: household income tax rate, τ_k: capital income tax rate, τ_p: payroll tax. τ_x fund tax rate; e.g. $x = b, f$, τ_e efficient labour tax (see Footnote 9).
* : As a share of output at factor cost.

Table 33: Government Budget in the PAYG and BP simulations of 2068 (% of output, Y, at factor cost).

	Public Expenditure				Public Revenues			
	G	T_r	P	U	T_c	T_k	T_y	T_p
PAYG	21.62	0.82	23.04	1.36	11.55	2.72	7.94	24.38
BP	21.62	1.32	0.00	1.30	13.91	1.25	7.56	1.30

G: government consumption, T_r: minimum income, P: pension payments, U: unemployment benefits expenditures; T_c: consumption tax collections, T_k: capital income taxes, T_y: household income tax revenue, T_p: payroll tax revenue.

A.3 Fully Funded pension system and Privatization

Here we again compare the performance of the BP system to the fully funded benchmark and to a full privatization of savings, now under the closed economy assumption.

For the defined contribution pension system (FF), we perform a grid search as in the BP exercise and find a welfare maximizing mandatory pension contribution rate of $\tau_f = 11\%$ in the closed economy.

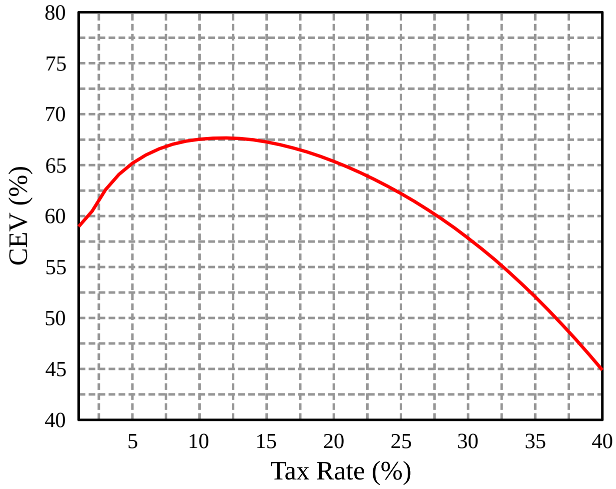


Figure 14: Average increase in welfare (CEV) at age 20 as a function of FF contribution rates (τ_b) in reformed economies with a FF pension system relative to PAYG economy 2068, assuming a final debt level of 2.2 output and $r^* = 1\%$.

A.3.1 Results

In the following tables, we include the PAYG 2068 closed economy results for comparison. Table 34 shows the main aggregates in the three reformed economies. As in the open economy exercises, the elimination of the PAYG pension system drives most of the differences in macroeconomic aggregates across the three economies. It has a large direct effect on disposable income through the reduction in payroll taxes, and a large direct effect on savings behaviour due to the elimination of pension payments. As before, all the reformed economies have higher capital-output ratios than the PAYG economy.

Table 34: Aggregates in the PAYG, PS and BP simulations of 2068.

	Y	K/Y^*	L/Y^*	h^a	C/Y^*	G/Y^*	I/Y^*	X/Y^*	w	$r(\%)$
PAYG	1.79	2.98	35.81	33.39	50.13	21.62	24.11	–	1.44	7.68
PS	3.03	4.18	26.01	31.94	38.48	21.62	35.02	–	1.98	2.99
FF	3.09	4.56	24.01	31.86	42.59	21.62	32.79	2.24	2.14	2.04
BP	2.84	4.41	25.67	31.88	48.90	21.62	27.53	3.24	2.08	2.39

$X = B, M$ in the BP and FF economies, respectively. B denotes aggregate backpack savings, while M denotes aggregate pension savings in the FF economy. *In this table, variable Y is GDP at factor cost.
[a]Variable h denotes the average share of disposable time allocated to the market.

The capital-output ratio is higher in the FF economy, and the equilibrium interest rate lower.

Despite this, the stock of pension savings is lower than in the BP economy. This means that households substitute private savings with pension savings at a lower rate.

Table 35: Labor Market Shares in the PAYG, PS and BP simulations of 2068 (% of population).

	W	U	I	R
PAYG	50.46	10.46	4.26	34.80
PS	67.16	13.89	4.66	14.29
FF	62.33	13.25	5.05	19.37
BP	58.84	13.52	3.73	23.91

W: workers, U: unemployed, I: inactive, R: retirees.

Table 36: Government Budget in the PAYG, PS and BP simulations of 2068 (% of output, Y, at factor cost).

	Public Expenditure				Public Revenues			
	G	T_r	P	U	T_c	T_k	T_y	T_p
PAYG	21.62	0.82	23.04	1.36	11.55	2.72	7.94	24.38
PS	21.62	2.24	–	1.23	15.20	1.49	6.92	1.23
FF	21.62	1.80	–	1.26	15.05	1.11	7.06	1.26
BP	21.62	1.32	–	1.30	13.91	1.25	7.56	1.30

G: government consumption, T_r: minimum income, P: pension payments, U: unemployment benefits expenditures; T_c: consumption tax collections, T_k: capital income taxes, T_y: household income tax revenue, T_p: payroll tax revenue.

Table 37: Policy Parameters and tax revenues in the PAYG, PS, FF and in the BP economy.

	Tax Rates (%)				Revenue Y^* Ratios (%)			
	PAYG	PS	BP	FF	PAYG	PS	BP	FF
τ_c	23.05	39.45	28.45	35.33	11.55	15.19	13.91	15.05
τ_y	11.28	11.28	11.28	11.28	7.94	6.92	7.56	7.06
τ_k	11.88	11.88	11.88	11.88	2.72	1.49	1.26	1.10
τ_p	51.02	3.04	2.93	2.98	24.38	1.23	1.30	1.26
τ_x	0	0	21.00	11.00				
τ_e	64.69	38.31	47.46	43.61				

τ_c: consumption tax rate, τ_y: household income tax rate, τ_k: capital income tax rate, τ_p: payroll tax. τ_x fund tax rate; e.g. $x = b, f$, τ_e efficient labour tax (see Footnote 9).
* : As a share of output at factor cost.

Table 38: The Distributions of Earnings, Income, and Wealth

		Bottom	Quintiles					Top
	Gini	10	1st	2nd	3rd	4th	5th	10
The Earnings Distributions (%)								
PAYG	0.37	3.2	7.5	10.4	15.0	22.5	44.6	28.1
PS	0.34	3.5	8.2	11.6	15.4	23.3	41.6	26.3
BP	0.34	3.5	8.3	11.6	15.5	23.2	41.4	26.1
FF	0.34	3.6	8.4	11.6	15.5	23.3	41.3	26.1
The Income Distributions (%)								
PAYG	0.37	2.0	5.4	11.1	17.0	23.8	42.7	27.0
PS	0.43	1.9	4.8	8.9	14.6	23.2	48.5	30.5
BP	0.40	1.6	4.8	10.4	15.5	24.3	45.1	28.4
FF	0.41	1.8	4.6	10.3	14.9	24.2	45.9	29.2
The Wealth Distributions (%)								
PAYG	0.63	0.0	0.7	3.9	8.9	21.6	64.9	44.1
PS	0.54	0.0	0.7	5.5	13.4	25.5	54.8	33.6
BP	0.79	0.0	0.0	0.0	1.4	16.6	81.9	57.7
FF	0.65	0.0	0.0	2.0	9.5	23.7	64.7	42.3

A.3.2 Welfare effects

Table 39 displays the welfare gains at age 20 from eliminating PAYG pensions (PS), and eliminating PAYG pensions and introducing a FF or a BP pension system, in the 2068 steady state, assuming the economy is closed.

As in the open economy results, the gains are of the order of magnitude of the decrease in the payroll tax, necessary to finance the PAYG pension system. Welfare gains are even higher in the closed economies because not only the effetive labor tax is much lower in the reformed economies, as in the open economy results, but the wage rate is roughly 50% higher. Because households in the bottom half of the wealth distribution have low (private) savings, most of their income is derived from labor. These is also the group with lower consumption, hence the increase in income brought by lower taxes and higher wages is valued marginally by more. This amplifies the CEV measure according the utilitarian welfare criteria.

Table 39: Consumption Equivalent Variation (Δ, %) in the PS, BP and Pension Fund economies, relative to the PAYG economy.

Simulation	Education			
	Dropouts	High School	College	All
PS	58.08	59.54	57.86	58.93
FF	66.71	67.75	67.46	67.66
BP	72.05	74.31	74.23	74.09

Table 40: Consumption Equivalent Variation (Δ, %) in the FF and BP economies, relative to the PS economy.

Simulation	Education			
	Dropouts	High School	College	All
FF	5.45	5.14	6.21	5.49
BP	8.84	9.25	10.50	9.53

Table 41: Private saving rates by education and level of private savings at age 30 and 50 (as a proportion of unemplyment benefits), for unemployed workers who lost the job in the previous period.

	Private Savings, a	Education		
		Dropouts	High School	College
Age 30	0.2 of per capita GDP	-98.4	-93.3	-63.7
	1.0 of per capita GDP	-110.0	-107.4	-80.2
	2.0 of per capita GDP	-116.8	-105.5	-97.3
Age 50	0.2 of per capita GDP	-92.6	-85.4	-57.2
	1.0 of per capita GDP	-129.6	-121.6	-86.8
	2.0 of per capita GDP	-64.0	-33.6	-103.5

Table 42: Backpack (de-)saving rates by education and level of private savings at age 30 and 50 (as a proportion of unemplyment benefits), for unemployed workers who lost the job in the previous period.

	Private Savings, a	Education		
		Dropouts	High School	College
Age 30	0.2 of per capita GDP	-4.8	-30.7	-19.8
	1.0 of per capita GDP	-7.7	-6.0	-4.1
	2.0 of per capita GDP	-9.7	-7.5	-5.1
Age 50	0.2 of per capita GDP	-5.4	-4.2	-2.8
	1.0 of per capita GDP	-8.9	-6.9	-4.7
	2.0 of per capita GDP	-11.5	-8.9	-6.1

B Detailed description of model economies

B.1 Backpack economy: individual decision problem

In this subsection we present the model equations that describe the BP economy. An individual who is currently employed solves the following optimization problem:

$$W(j,h,z,a,b) = \max_{c,l,a'} \left\{ u(c,l) + \beta\psi_j \sum_{z' \in Z} \Gamma(z'|z) \Big[(1-\sigma_j) J(j+1,h,z',a',b') + \sigma_j U(j+1,h,z',a',b',0) \Big] \right\} \tag{36}$$

subject to:

$$(1+\tau_c)c + a' + \tau_y \hat{y} + (\tau_p + \tau_b)y \leq (1 + r(1-\tau_k))a + y + TR(y), \tag{37}$$

the backpack law of motion,

$$b' = \tau_b y + (1 + r(1-\tau_k))b, \tag{38}$$

and a no-borrowing constraint:

$$a' \geq 0. \tag{39}$$

Gross labor income is $y = \omega\epsilon z l$, $l \in [0,1]$, income tax base $\hat{y} = (1-\tau_p-\tau_b)y + r(1-\tau_k)a$ and government transfers for low income households are denoted by $TR(y) = t_r \mathbb{1}_{TR}(y)$, where $\mathbb{1}_{TR}(y) = 1$ if $y < \bar{t_r}$ and zero otherwise. z' evolves according to the Markov process Γ.

An agent who has been separated from a job and hasn't restarted work yet solves the following problem:

$$U(j,h,z,a,b,d) = \\ = \max_{c,a',b',e} \Bigg\{ u(c) - \gamma e + \\ \beta\psi_j \sum_{z' \in Z} \Gamma(z'|z) \Big[e\Big(\lambda_j^{\mathbf{u}} J(j+1,h,z',a',b',d+1) + (1-\lambda_j^{\mathbf{u}}) U(j+1,h,z',a',b',d+1)\Big) \\ + (1-e)\Big(\lambda_j^{\mathbf{n}} J(j+1,h,z',a',b',d+1) + (1-\lambda_j^{\mathbf{n}}) N(j+1,h,z',a',b')\Big)\Big] \Bigg\} \tag{40}$$

subject to

$$(1+\tau_c)c + a' + b' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))(a+b) + UB(d,e) + TR(y), \tag{41}$$

and

$$e \in \{0,1\}, \tag{42}$$

$$a' \geq 0, \tag{43}$$

$$b' \leq (1 + r(1-\tau_k))b. \tag{44}$$

The income tax base is given by $\hat{y} = r(1-\tau_k)a$. The unemployed worker may be entitled to unemployment benefits: $UB(d,e) = u_b \mathbb{1}_{UB}(d,e)$, with $\mathbb{1}_{UB}(d,e) = 1$ indicating eligibility for unemployment benefits. Formally:

$$\mathbb{1}_{UB}(d,e) = \begin{cases} 1 & \text{if } e = 1 \text{ and } d \leq \bar{d}, \\ 0 & \text{otherwise.} \end{cases} \tag{45}$$

The state variable d evolves deterministically according to $d' = d+1$ if the worker continues unemployed in the following period, and $d = 0$ in the period immediately after a separation shock.

Finally, an agent may start the period without a job because he has previously decided not to work and has not started a new job yet. In this case, he solves the following problem:

$$N(j,h,z,a,b) = \\ \max_{c,a',e} \Bigg\{ u(c) - \gamma e + \beta\psi_j \sum_{z' \in Z} \Gamma(z'|z) \Big[e\Big(\lambda_j^{\mathbf{u}} J(j+1,h,z',a',b') + (1-\lambda_j^{\mathbf{u}}) N(j+1,h,z',a',b')\Big) + \\ (1-e)\Big(\lambda_j^{\mathbf{n}} J(j+1,h,z',a',b') + (1-\lambda_j^{\mathbf{n}}) N(j+1,h,z',a',b')\Big)\Big] \Bigg\}, \tag{46}$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + TR(y), \tag{47}$$

and

$$a' \geq 0, \tag{48}$$

$$b' = (1 + r(1-\tau_k))b. \tag{49}$$

As before, $\hat{y} = r(1-\tau_k)a$. In this case the non-employed worker is not eligible for unemployment benefits, and he also cannot use backpack assets.

We consider now the the problem of the retiree after the retirement decision.

$$V(j,h,a,b) = \max_{c,a'} \left\{ u(c) + \beta\psi_j \Big[V(j+1,h,a',b) \Big] \right\}, \tag{50}$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + p^B(b) + TR(y). \tag{51}$$

Pension payments are part of the income tax base: $\hat{y} = r(1-\tau_k)a + p^B(b)$. After retirement, labor market productivity is always zero and hence expectations take into account only the survival risk.

To close the description of the household's problem, we define the job acceptance and retirement decisions. These jointly pin down the value of having a job offer at the beginning of a period:

$$J(j,h,z,a,b,d) = \max \Big\{ V(j,h,a,b), \max\{W(j,h,z,a,b), N(j,h,z,a,b)\} \Big\}. \tag{52}$$

The outermost max operator represents the retirement decision, while the inner operator is the job acceptance decision.

B.2 Fully Funded pensions economy: individual decision problem

In this subsection we present the model equations that describe the economy with a fully funded (defined contribution) pension scheme. Current worker pension claims are denoted by m. An individual who is currently employed solves the following optimization problem:

$$W(j,h,z,a,m) = \max_{c,l,a'} \left\{ u(c,l) + \beta\psi_j \sum_{z' \in Z} \Gamma(z'|z) \Big[(1-\sigma_j) J(j+1,h,z',a',m') + \sigma_j U(j+1,h,z',a',m',0) \Big] \right\} \tag{53}$$

subject to:

$$(1+\tau_c)c + a' + \tau_y \hat{y} + (\tau_p + \tau_b)y \leq (1 + r(1-\tau_k))a + y + TR(y), \tag{54}$$

and pension claims evolve according to,

$$m' = \tau_f y + (1 + r(1 - \tau_k))m. \tag{55}$$

The no-borrowing constraint is:

$$a' \geq 0. \tag{56}$$

Gross labor income is $y = \omega \epsilon z l$, $l \in [0,1]$, income tax base $\hat{y} = (1 - \tau_p - \tau_f)y + r(1 - \tau_k)a$ and government transfers for low income households are denoted by $TR(y) = t_r \mathbb{1}_{TR}(y)$, where $\mathbb{1}_{TR}(y) = 1$ if $y < \bar{t_r}$ and zero otherwise. z' evolves according to the Markov process Γ.

An agent who has been separated from a job and hasn't restarted work yet solves the following problem:

$$\begin{aligned} &U(j,h,z,a,m,d) = \\ &= \max_{c,a',b',e} \Bigg\{ u(c) - \gamma e + \\ &\qquad \beta\psi_j \sum_{z' \in Z} \Gamma(z'|z) \Big[e\Big(\lambda_j^{\mathbf{u}} J(j+1,h,z',a',m',d+1) + (1-\lambda_j^{\mathbf{u}}) U(j+1,h,z',a',m',d+1)\Big) \\ &\qquad\qquad + (1-e)\Big(\lambda_j^{\mathbf{n}} J(j+1,h,z',a',m',d+1) + (1-\lambda_j^{\mathbf{n}}) N(j+1,h,z',a',m')\Big)\Big] \Bigg\} \end{aligned} \tag{57}$$

subject to

$$(1 + \tau_c)c + a' + m' + \tau_y \hat{y} \leq (1 + r(1 - \tau_k))(a + m) + UB(d,e) + TR(y), \tag{58}$$

and

$$e \in \{0, 1\}, \tag{59}$$

$$a' \geq 0, \tag{60}$$

$$m' \leq (1 + r(1 - \tau_k))m. \tag{61}$$

The income tax base is given by $\hat{y} = r(1 - \tau_k)a$. The unemployed worker may be entitled to unemployment benefits: $UB(d,e) = u_b \mathbb{1}_{UB}(d,e)$, with $\mathbb{1}_{UB}(d,e) = 1$ indicating eligibility for unemployment benefits. Formally:

$$\mathbb{1}_{UB}(d,e) = \begin{cases} 1 & \text{if } e = 1 \text{ and } d \leq \bar{d}, \\ 0 & \text{otherwise.} \end{cases} \tag{62}$$

The state variable d evolves deterministically according to $d' = d + 1$ if the worker continues unemployed in the following period, and $d = 0$ in the period immediately after a separation shock.

Finally, an agent may start the period without a job after he has decided not to work and has not started a new job yet. In this case he solves the following problem:

$$N(j,h,z,a,m) = \max_{c,a',e}\Big\{u(c) - \gamma e + \beta\psi_j \sum_{z'\in Z}\Gamma(z'|z)\Big[e\Big(\lambda_j^{\mathbf{u}} J(j+1,h,z',a',m') + (1-\lambda_j^{\mathbf{u}})N(j+1,h,z',a',m')\Big) + (1-e)\Big(\lambda_j^{\mathbf{n}} J(j+1,h,z',a',m') + (1-\lambda_j^{\mathbf{n}})N(j+1,h,z',a',m')\Big)\Big]\Big\}, \tag{63}$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + TR(y), \tag{64}$$

and

$$a' \geq 0, \tag{65}$$

$$m' = (1 + r(1-\tau_k))m. \tag{66}$$

As before, $\hat{y} = r(1-\tau_k)a$. In this case the unemployed worker is not eligible for unemployment benefits.

We consider now the the problem of the retiree after the retirement decision, with the final pension claim m.

$$V(j,h,a,m) = \max_{c,a'}\Big\{u(c) + \beta\psi_j\Big[V(j+1,h,a',m)\Big]\Big\}, \tag{67}$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + p^F(m) + TR(y). \tag{68}$$

Pension payments are again part of the income side of the budget constraint: $\hat{y} = r(1-\tau_k)a + p^F(m)$. After retirement, labor market productivity is always zero and hence expectations take into account only the survival risk.

To close the description of the household's problem, we define the job acceptance and retirement decisions. These jointly pin down the value of having a job offer at the beginning of a period:

$$J(j,h,z,a,m,d) = \max\Big\{V(j,h,a,m), \max\{W(j,h,z,a,m), U(j,h,z,a,b,m)\}\Big\}. \tag{69}$$

The outermost max operator represents the retirement decision, while the inner operator is the job acceptance decision.

B.3 Baseline and PAYG economy: individual decision problem

In the Baseline and PAYG economies workers have access to a PAYG pension system. Therefore the state vector does not include variable recording pension claims. Workers solve the following optimization problem:

$$W(j,h,z,a)=\max_{c,l,a'}\left\{u(c,l)+\beta\psi_j\sum_{z'\in Z}\Gamma(z'|z)\Big[(1-\sigma_j)J(j+1,h,z',a')+\sigma_j U(j+1,h,z',a',0)\Big]\right\} \quad (70)$$

subject to:

$$(1+\tau_c)c+a'+\tau_y\hat{y}+\tau_p y\leq(1+r(1-\tau_k))a+y+TR(y), \quad (71)$$

and a no-borrowing constraint:

$$a'\geq 0. \quad (72)$$

An unemployed worker solves the following problem:

$$\begin{aligned}&U(j,h,z,a,d)=\\&=\max_{c,a',e}\Bigg\{u(c)-\gamma e+\\&\qquad\beta\psi_j\sum_{z'\in Z}\Gamma(z'|z)\Big[e\Big(\lambda_j^{\mathbf{u}}J(j+1,h,z',a',d+1)+(1-\lambda_j^{\mathbf{u}})U(j+1,h,z',a',d+1)\Big)\\&\qquad+(1-e)\Big(\lambda_j^{\mathbf{n}}J(j+1,h,z',a',d+1)+(1-\lambda_j^{\mathbf{n}})N(j+1,h,z',a')\Big)\Big]\Bigg\}\end{aligned} \quad (73)$$

subject to

$$(1+\tau_c)c+a'+\tau_y\hat{y}\leq(1+r(1-\tau_k))a+UB(d,e)+TR(y), \quad (74)$$

and

$$e\in\{0,1\}, \quad (75)$$

$$a'\geq 0. \quad (76)$$

An agent who starts the period without a job, after having quit a job before solves the following problem:

$$\begin{aligned}&N(j,h,z,a)=\\&\max_{c,a',e}\Bigg\{u(c)-\gamma e+\beta\psi_j\sum_{z'\in Z}\Gamma(z'|z)\Big[e\Big(\lambda_j^{\mathbf{u}}J(j+1,h,z',a')+(1-\lambda_j^{\mathbf{u}})N(j+1,h,z',a')\Big)+\\&\qquad(1-e)\Big(\lambda_j^{\mathbf{n}}J(j+1,h,z',a')+(1-\lambda_j^{\mathbf{n}})N(j+1,h,z',a')\Big)\Big]\Bigg\},\end{aligned} \quad (77)$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + TR(y), \tag{78}$$

and

$$a' \geq 0. \tag{79}$$

A retiree with age j, education level h and private savings a solves the following problem:

$$V(j,h,a) = \max_{c,a'} \left\{ u(c) + \beta\psi_j \Big[V(j+1,h,a') \Big] \right\}, \tag{80}$$

subject to

$$(1+\tau_c)c + a' + \tau_y \hat{y} \leq (1 + r(1-\tau_k))a + p_h + TR(y). \tag{81}$$

Pension payments p_h depend on education level h and are part of the income side of the budget constraint. In this case, $\hat{y} = r(1-\tau_k)a + p_h$. After retirement, labor market productivity is always zero and hence expectations take into account only the survival risk.

Pension payments depend on the education level and on the average of labor market earnings in group h during the N_b years prior to the first retirement age R_0, the minimum statutory retirement age. Specifically, pension payments are given by:

$$p_h = p_r \bar{y}_h = p_r \frac{1}{N_b} \sum_{i=j-N_b}^{j-1} \bar{y}_{j,h}. \tag{82}$$

To close the description of the household's problem, we define the job acceptance and retirement decisions. These jointly pin down the value of having a job offer at the beginning of a period:

$$J(j,h,z,a,d) = \max \Big\{ V(j,h,a), \max\{W(j,h,z,a), U(j,h,z,a,d)\} \Big\}. \tag{83}$$

The outermost max operator represents the retirement decision, while the inner operator is the job acceptance decision.

B.4 Private Savings economy: individual decision problem

The description of the decision problems in the PS economy is as in the previous subsection, but with $p_h = 0$ for all h.

C Definition of a stationary equilibrium in the BP economy

Let $j \in J$, $h \in H$, $z \in \mathcal{Z}$, $l \in \mathcal{L}$, $d \in \mathcal{D}$, $a \in A$, and $b \in B$ and let $\mu_{j,h,z,l,d,a,b}$ be a probability measure defined on $\Re = J \times H \times \mathcal{Z} \times \mathcal{L} \times \mathcal{D} \times A \times B$.[32] Then, a stationary competitive equilibrium for this economy is a government policy, $\{G, P, T_r, U, T_k, T_s, T_y, T_c, E\}$, a household policy, $\{c(j,h,z,d,a,b)$, $l(j,h,z,d,a,b)$, $s(j,h,z,d,a,b)$, $r(j,h,z,d,a,b)$, $a^{'}(j,h,z,d,a,b)$, $b^{'}(j,h,z,d,a,b)\}$, a measure, μ, factor prices, $\{r, w\}$, macroeconomic aggregates, $\{C, Y, K, L\}$, and a function, Q, such that:

(i) The government policy satisfy the consolidated government described in Expressions (8)-(9).

(ii) Firms behave as competitive maximizers. That is, their decisions imply that factor prices are factor marginal productivities $r = f_1(K, HL) - \delta$ and $\omega = f_2(K, HL)$.

(iii) Given the government policy, and factor prices, the household policy solves the households' decision problem defined in Expressions (13), through (28).

(iv) The stock of assets, consumption, the aggregate labor input, pension payments, unemployment benefit payments, lump-sum transfers, tax revenues, and accidental bequests are obtained aggregating over the model economy households as follows:

$$
\begin{aligned}
A &= \int a + b \; d\mu \\
C &= \int c \; d\mu \\
L &= \int \epsilon_{jh} z l \; d\mu \\
U &= \int ub \; d\mu \\
T_r &= \int t_r \; d\mu \\
T_c &= \int \tau_c c \; d\mu \\
T_k &= \int \tau_k r a \; d\mu \\
T_p &= \int \tau_p y \; d\mu \\
T_y &= \int \tau_y \hat{y} \; d\mu \\
E &= \int (1 - \psi_j)(1 + r) a^{'} \; d\mu
\end{aligned}
$$

where all the integrals are defined over the state space $\Re$.

[32]For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript.

(vi) The goods market clears:

$$C+\int(a^{'}+b^{'}-(1-\delta)(a+b))d\mu+G+NX=F(K,HL). \tag{84}$$

where the last term of the left-hand side of this expression, NX, are net exports.

(vii) The law of motion for μ_j is:

$$\mu_{j+1}=\int_{\Re}Qd\mu_j. \tag{85}$$

Describing function Q formally is complicated because it specifies the transitions of the measure of households along its five dimensions: age, education level, productivity, employment status, and assets holdings. An informal description of this function is the following: We assume that new-entrants, who are 20 years old, enter to the economy as workers, unemployed, or inactive, following the shares of these groups for the 20-24 cohort in the Spanish economy in 2018, and that they own zero assets. Moreover, workers enter the economy with a job opportunity, that they draw the stochastic component of their endowment of efficiency labor units from its invariant distribution. Their educational shares are exogenous. The evolution of μ_{jh} is exogenous, it replicates the the distribution by age and education of the Spanish population in our calibration target year, 2018. The evolution of μ_z is governed by the conditional transition probability matrix of its stochastic component. The evolution of μ_l, is governed by the exogenous probabilities of find/loss a job, by the endogenous employment and search decisions, and by the optimal decision to retire. The evolution of μ_a is determined by the optimal savings decision and by the changes in the population. The evolution of μ_b is determined by the backpack law of motion. The evolution of μ_d is given by the deterministic evolution of unemployment spell duration.

D Calibration

D.1 Initialising the steady-state

In order to determine the steady-state, first we choose as an initial distribution of households $\mu_0=\mu_{2018}$; that is, we take μ_{jh} at year 2018 directly from the Encuesta de Población Activa from the Spanish National Institute of Statistics. We also take from INE the conditional probabilities of surviving from age j to age $j+1$, ψ_j, at that same year. The labor market flow data used to calibrate the job finding and job destruction probabilities were provided by Lalé and Tarasonis (2017). The initial distribution of households imply an initial value for the capital stock. This value is $K_{2018}=7.2484$. The initial distribution of households and the initial survival probabilities determine the initial value of unintentional bequests, E_{2018}. Finally, we must also specify the value for the productivity process, H. Since H determines the units which we use to measure output and does nothing else, we choose $H=1.0$.

D.2 Parameters

Once the initial conditions are specified, to characterize our model economy fully, we must choose the values of a total of 42 parameters. Of these 42 parameters, 5 describe the household preferences, 21 the process on the endowment of efficiency labor units, 3 the production technology, 6 the pension system rules, and 9 the remaining components of the government policy. The functional form for the utility function is $u(c,l) = \frac{c^{(1-\sigma)}}{1-\sigma} - \alpha \frac{l^{1+1/\varphi}}{1+1/\varphi}$. To choose the values of these 42 parameters we need 42 equations or calibration targets which we describe below.

D.3 Equations

To determine the values of the 42 parameters that identify our model economy, we do the following. First, we determine the values of a group of 36 parameters directly using equations that involve either one parameter only, or one parameter and our guesses for (Y, L). To determine the values of the remaining 6 parameters we construct a system of 6 non-linear equations. Most of these equations require that various statistics in our model economy replicate the values of the corresponding Spanish statistics in 2018. We describe the determination of both sets of parameters in the subsections below.

D.3.1 Parameters determined solving single equations

The life-cycle profile of earnings. We measure the deterministic component of the process on the endowment of efficiency labor units independently of the rest of the model. We estimate the values of the parameters of the three quadratic functions that we describe in Expression (86), using the age and educational distributions of hourly wages reported by the *Instituto Nacional de Estadística* (INE) in the *Encuesta de Estructura Salarial* (2010) for Spain.[33] This procedure allows us to identify the values of 9 parameters directly.

$$\epsilon_{jh} = \xi_{1h} + \xi_{2h} j - \xi_{3h} j^2 \tag{86}$$

We take directly the values for both the stochastic component of the process on the endowment of efficiency labor units and the conditional transition probabilities across them, from Brogueira de Souza et al. (2020). The rationale for this is because this process proved to generate in our model economy income and earning inequality consistent with the Spanish data.

The pension system. In 2018 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 45,014 euros of annual gross labor income. Since we omit the tax

[33] Since we only have data until age 64, we estimate the quadratic functions for workers in the 20–64 age cohort and we project the resulting functions from age 65 onwards.

Table 43: Parameters determined solving single equations

	Parameter	Value
Parameters determined directly		
Earnings Life-Cycle		
	$\xi_{1,1}$	0.9189
	$\xi_{1,2}$	0.8826
	$\xi_{1,3}$	0.5064
	$\xi_{2,1}$	0.0419
	$\xi_{2,2}$	0.0674
	$\xi_{2,3}$	0.1648
	$\xi_{3,1}$	0.0006
	$\xi_{3,2}$	0.0008
	$\xi_{3,3}$	0.0021
	$z(1)$	1.0000
	$z(2)$	2.3490
	$z(3)$	5.9042
	z_{11}	0.9821
	z_{12}	0.0177
	z_{13}	0.0001
	z_{21}	0.0291
	z_{22}	0.9709
	z_{23}	0.0000
	z_{31}	0.0001
	z_{32}	0.0003
	z_{33}	0.9995
Preferences		
Curvature	σ	2.0000
Labor elasticity	φ	0.1000
Technology		
Capital share	θ	0.4846
Public Pension System		
Number of years of contributions	N_b	21
First retirement age	R_0	62
Parameters determined by guesses for (Y, L)		
Public Pension System		
Payroll Tax Rate	τ_p	0.2597
Government Policy		
Government consumption	G	0.3894
Capital income tax rate	τ_k	0.1188
Consumption tax rate	τ_c	0.2064
Income tax Rate	τ_y	0.1128

cap, we impose that all gross earnings pay pension contributions. We also impose that payroll tax collections are used to finance both pension payments and unemployment benefits. This implies that the payroll tax rate in our model economy is 0.2609.

Our choice for the number of years used to compute the retirement pensions in our benchmark model economy is $N_b = 21$. This is because in 2018 the Spanish *Régimen General de la Seguridad Social* took into account the last 21 years of contributions prior to retirement to compute the pension. Finally, our choice for the first retirement ages is $R_0 = 62$.

Government policy. To specify the government policy, we must choose the values of government consumption, G_t, the level of public debt, D_t, the share of accidental bequest that is confiscated by the government, E, the replacement rate b_1, of the tax rate on capital income, τ_k, of the tax rate on income, τ_y, and of the tax rate on consumption, τ_c.

We target the output shares of G, E, and T_y so that they replicate the GDP shares of Government Consumption, Inheritance Taxes, and Individual Income taxes. According to the INE, in 2018, Government Consumption was 208,875 million euros, and the Inheritance Tax, and the Individual Income tax collected 2,687, and 93.247 million euros, respectively.[34] Consequently, the ratios of these variables to GDP at market prices are 17.40, 0.20, and 7.05 percent.

For simplicity, we also assume that the initial level of public debt, as a share of GDP, is 0 percent. The rationale for this choice is because the government in our model economy has not the same number of tax instruments that the Spanish government, so we opt to discard this specific public expenditure. Otherwise, the consumption tax rate need to balance the government budget should increase to unrealistic levels. According to Cañón et al. (2016) the ratio of per capita public transfers to the threshold level was 75%, so we set $b_1 = 0.75$. Regarding the capital income tax rate, and according to the OECD, in Spain in 2018, this number was 25%, Consequently, we set $\tau_k = 0.25$. Finally, the government budget is an additional equation that allows us to obtain residually the consumption tax rate.

Preferences. Of the four parameters in the utility function, we choose the value of σ and φ directly. Specifically, we choose $\sigma = 2.0$ and $\varphi = 0.1$.

Technology. According to the Spanish National Institute of Statistics data (INE), the capital income share in Spanish GDP was 0.4846 in 2018. Consequently, we choose $\theta = 0.4846$.

To determine the value of the interest rate, we proceed as follows. According to the INE, Corporate Profit Tax collections amounted 29,711 million euros in 2018, or 2.24 percent of GDP

[34] We exclude from Government Consumption the expenditure in Subsidies and Investment Aid.

at market prices. Then,

$$0.0224 = \tau_k r \frac{K}{Y^*} \tag{87}$$

According to the BBVA database, in 2016 the value of the Spanish capital stock was 3,281,631 million euros.[35] According to the INE in 2016 the Spanish Gross Domestic Product at market prices was 1,113,840 million euros. Dividing these two numbers, we obtain $K/Y^* = 2.94$. We already set $\tau_k = 0.25$. Consequently, the value for the interest rate is $r = 3.0476\%$, which is our target value for the model economy interest rate.

Finally, we obtain the value for the depreciation rate, δ. According to the firm's optimality conditions:

$$r = \theta \frac{Y}{K} - \delta \tag{88}$$

where Y is output at factor cost. According to the INE, in Spain in 2018 this number was 977,345. Consequently, the value for the depreciation rate is $\delta = 0.1138$.

Adding up. So far we have determined the values of 35 parameters either directly or as functions of our guesses for (Y, L) only. We report their values in Table 43.

D.3.2 Parameters determined solving a system of equations

We still have to determine the values of 6 parameters. To find the values of those 6 parameters we need 6 equations, where these 6 equations require that model economy statistics replicate the value of the corresponding statistics for the Spanish economy in 2018.

Table 44: Macroeconomic Aggregates and Ratios in 2018 (%)

	P/Y^{*a}	U/Y^{*b}	T_r/Y^*	GW^c	W^d	I^e
Spain	10.47	1.32	0.83	0.67	59.59	5.16

[a]Variable Y^* denotes GDP at market prices.
[b]The ratio U/Y^* is the Unemployment benefits as a share of Output at market prices.
[c]Variable GW is the Gini Index of wealth.
[d]Variable W is the share of workers in the Spanish population with 20+ years old.
[e]Variable I is the share of inactive in the Spanish population with 20+ years old.

Aggregate Targets. According to the INE, unemployment benefits amounted 17,469 million. That same year, and according to the Spanish Instituto Nacional de la Seguridad Social, pension payments were 125,899 million euros. Finally, and according to Cañón et al. (2016), the sum of different

[35]This number can be found at *http://www.fbbva.es/TLFU/microsites/stock09/fbbva_stock08_index.html*.

subsidies aimed to protect those people who do not receive any public benefit amounted 8,976 million euros in 2015.[36] Consequently, the ratios of these variables to GDP at market prices are 1.32, 10.47 and 0.83 percent.

According to Anghel et al. (2018), the Gini Index of wealth in 2014 in Spain was 0.67. Finally, and according to the Encuesta de Población Activa (INE), in Spain in 2018 there were 32,433,800 people aged 20+ years old.[37] That same survey reports that 19,327,700 were workers and 3,479,100 were unemployed. Consequently, these numbers imply that the share of workers was 59.59 percent and the share of unemployed were 10.72 percent.

The Parameters. The 6 parameters determined by the system are the following:

- Preferences: β, α, and γ.
- Pension system: p_r.
- Fiscal policy: b_0, and $\overline{t_r}$.

Table 45 provides the values for the 6 parameters.

Table 45: Parameters, values and targets.

Parameter	Value	Target
β	0.9965	GW
α	28×10^4	W
γ	1.7281	I
p_r	0.7650	P/Y^*
$\overline{t_r}$	0.8300	T_r/Y^*
b_0	0.3518	U/Y^*

E The Spanish Social Security

The Spanish contributory pension system, is the most important program of social protection in Spain, where public contributory pensions are provided by the following three programs. First, the *Régimen General de la Seguridad Social* covers the private sector employees and the members of cooperative firms and the employees of most public administrations other than the central governments. Second, the *Regímenes Especiales de la Seguridad Social* cover the self-employed

[36]These types of subsidies were the minimum income program, the agricultural and income programs, the Active Insertion Income, the temporary program of protection for unemployment and insertion, and the Activation Program for Employment.

[37]We exclude students and people who do household chores.

workers and professionals.[38] And third, the scheme for government employees, or *Régimen de Clases Pasivas* covers public servants employed by the central government and its local branches.

In this article we focus exclusively on the retirement pensions payed by the *Régimen General de la Seguridad Social*. Consequently, this section describes the key features of this system and its 2011 and 2013 reforms.

Financing and elegibility. The Régimen General de la Seguridad Social is a mandatory pay-as-you-go scheme. The payroll tax rate is proportional to covered earnings, which are defined as total earnings, excluding payments for overtime work, between a floor and a ceiling that vary by broadly defined professional categories. The payroll tax rate is 28.3 percent, of which 23.6 percent is attributed to the employer and the remaining 4.7 percent to the employee.

Entitlement to an old-age pension requires at least 15 years of contributions. The retirement age that entitles workers to receive a full retirement pension is 65 for workers who have contributed at least 36 years and three months. Previous to the 2011 Pension reform, every worker aged 61 or older could retire earlier paying an early retirement penalty, as long as they had contributed to the pension system for at least 30 years. Exceptionally, workers who had entered the system before 1967 could retire at age 60. The 2011 Reform of the Spanish pension system delayed the early retirement age from 61 to 63 for those workers who decide to retire on a voluntary basis, and it also delayed the full entitlement retirement age from 65 to 67. The delay in the early retirement age was immediate, and the delays in the normal retirement are gradual: one month per year between 2013 and 2018, and two months per year between 2019 and 2027. Consequently, the full entitlement retirement age in Spain will be 66 in 2021 and 67 in 2027.

Retirement Pensions. The main component of the retirement pension is the *Regulatory Base*, defined as the average covered earnings of the last 21 years before retirement. Labor income earned in the last two years prior to retirement enters the calculation in nominal terms, and the covered earnings of the remaining years are revaluated using the rate of change of the Spanish Consumer Price Index. The 2011 Reform of the Spanish pension system extended the number of years of earnings used by the Regulatory Base up to the last 25 years before retirement. The extension of the number of years used to compute the pensions was phased in gradually and it will end in 2022. In addition, the Regulatory Base in multiply by a percentage which depends on the age of the retirees and on the number of years of contributions. And, each year worked after the full entitlement retirement age increases the Regulatory Base in 2 or 3 percentage points depending on the length of the contributory career. Finally, retirement pensions are bound by a minimum and a maximum pension, where minimum pensions depend the pensioner's age and on the composition of the household.

[38]This program includes self-employed, agricultural workers and small farmers, domestic workers, sailors, and coal miners.

The Revaluation of pensions. In 2018, the Spanish pension system returned to a full price indexation of pensions.[39]

The Pension Reserve Fund. Since 2000, part of the surpluses generated by the pension system are deposited in a Pension Reserve Fund. However, and since the stock of assets of this fund only represented 0.4 percent of GDP at the end of 2018, which is our calibration target year, we assume that there is no Pension Fund in our model economy.

E.1 Changes in the Fiscal and Pension Policies between the initial and the final steady states

- In the final steady state, the legal retirement ages are 63 and 67 years, rather than 62 and 66 years old as it is the case in the initial steady state
- In the final steady state, the number of years of labor income used to compute the pension are the last 25 years before retirement, rather than the last 21 as it is the case in the initial steady state.
- The above changes follow the 2011 Spanish pension reform. The extension of the retirement ages and the number of years used to compute the pensions was phased in gradually.
- Finally, we assume that the final steady state does not introduce the last reform related to the Spanish Minimum Income scheme, approved by the Spanish government in 2020.

[39] The two main measures of the 2013 Pension Reform, the Sustainability Factor and the Pension Revaluation Index, have recently been eliminated by the Spanish government.

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