Falling Interest Rates and Credit Reallocation: Lessons from General Equilibrium

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Abstract

We show that in a canonical model with heterogeneous entrepreneurs, financial frictions, and an imperfectly elastic supply of capital, a fall in the interest rate has an ambiguous effect on aggregate economic activity. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing financial constraints and by prompting relatively less productive entrepreneurs to invest. In general equilibrium, however, this higher demand for capital raises its price and crowds out investment by more productive entrepreneurs. When this reallocation is strong enough, a fall in the interest rate reduces aggregate output. A numerical exploration of the model suggests that this reallocation effect is quantitatively significant and – in response to persistent changes in the interest rate – stronger than the traditional balance-sheet channel. We provide evidence of the reallocation effect using US firm-level data.

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1 Introduction

One distinctive feature of recent decades has been the sustained and significant decline in real interest rates across the globe. Although conventional wisdom holds that declining interest rates should stimulate economic activity, there are mounting concerns that such declines – at least when they are persistent – may have undesirable side effects, such as endangering financial stability (Rajan, 2015; Martinez-Miera and Repullo, 2017; Coimbra and Rey, 2017; Brunnermeier and Koby, 2018; Bolton et al., 2021) or slowing-down the pace of technological innovation and long-term growth (Liu et al., 2019; Quadrini, 2020; Benigno et al., 2020). Moreover, recent evidence suggests that periods of fast credit growth fueled by low interest rates can also result in lackluster productivity performance (Reis, 2013; Gopinath et al., 2017; Doerr, 2018; García-Santana et al., 2020).

We contribute to this debate by proposing a novel mechanism through which declining interest rates can foster the proliferation of *socially* unproductive activities. We consider a canonical economy populated by entrepreneurs who have the ability to invest in capital. We make three assumptions. First, entrepreneurs are heterogeneous in their productivity, i.e., they differ in their effectiveness at using capital to produce consumption goods. Second, entrepreneurs face financial frictions, i.e., they cannot pledge the entire surplus from their activities to outsiders. Finally, the supply of capital is imperfectly elastic. The first two assumptions imply that, in equilibrium, there is heterogeneity in the marginal return to investment across entrepreneurs; the third assumption introduces general equilibrium effects. We show that, in this environment, a fall in the interest rate can further distort the allocation of capital and can therefore have an ambiguous effect on aggregate output.[1]

Our findings challenge the common notion that, in the presence of financial frictions, lower interest rates stimulate economic activity by raising both entrepreneurs' willingness and ability to invest. In our framework, the conventional view holds in partial equilibrium. In general equilibrium, however, declining interest rates may stimulate investment by the wrong mix of entrepreneurs. The logic goes as follows. Lower interest rates make it attractive for less productive entrepreneurs to invest, which raises the price of capital and crowds out investment by more productive (and financially constrained) entrepreneurs. As a result, capital is reallocated from more to less productive entrepreneurs. We formalize this general-equilibrium *reallocation effect* and show that it attenuates the stimulative effect of declining interest rates on output.

[1] We focus throughout most of the paper on a small-open economy that takes the world interest rate as given. Our results extend to a closed economy where interest rate changes are driven by changes in fundamentals (see Appendix C.2).

The strength of the reallocation effect depends on entrepreneurial heterogeneity, on the elasticity of the capital supply and on the severity of the financial friction. In particular, when the supply of capital is sufficiently inelastic and the financial friction is severe enough, we show that the reallocation effect becomes so strong that a fall in the interest rate is contractionary. Moreover, the reallocation effect is tied to an inefficiency.

Since the capital supply is not perfectly elastic, entrepreneurial investment affects the equilibrium price of capital. Yet, individual entrepreneurs do not internalize this effect, which gives rise to a *pecuniary externality*. In the absence of heterogeneity or financial frictions, the marginal return to investment would be equalized across entrepreneurs and this pecuniary externality would have no first-order effect on output or welfare. With heterogeneity and financial frictions, however, this is no longer the case. Simply put, less productive entrepreneurs do not internalize the crowding-out effect of their investment on that of more productive entrepreneurs. A benevolent social planner, even if she is also subject to financial frictions, would limit the investment of less productive entrepreneurs in order to free up capital and foster investment by more productive entrepreneurs. One implication of this is that a fall in the interest rate is always expansionary in the constrained efficient allocation.

We view our main contribution as conceptual: in the presence of financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned) by general-equilibrium reallocation effects. This mechanism requires that the price of capital depend on local economic conditions (i.e., domestic demand and supply for capital), which is a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets (e.g., Kiyotaki and Moore (1997); Krishnamurthy (2003); Lorenzoni (2008); Brunnermeier and Sannikov (2014)). Indeed, just as balance-sheet effects would vanish if the price of capital were independent of local economic conditions, so would the reallocation effects that we emphasize in this paper.

In order to incorporate balance-sheet effects and study their interaction with our reallocation effects, we extend our analysis to an infinite-horizon economy. As in most macro-finance models, rising asset prices in this economy boost entrepreneurial wealth and thus relax the financial constraints of more productive entrepreneurs. This channel should amplify the stimulative effect of declining rates. We show that this intuition is only partially correct. The reason is that balance-sheet effects are driven by unexpected changes in the price of capital and are thus inherently transitory, whereas the general-equilibrium forces that drive reallocation last for as long as interest rates remain low. As a result, if the reallocation effect is strong enough, the response of output to a persistent fall in the interest rate features a transitory boom followed by a persistent bust: the balance-sheet channel temporarily raises the investment of more productive entrepreneurs, but its effect gradually wears off as the contractionary reallocation effect begins to dominate.

To explore the quantitative significance of reallocation, we calibrate the model to US data. To do so, we follow the macro-finance literature and interpret capital as land or real estate (e.g., Kiyotaki and Moore (1997)) and set its supply elasticity to 2. This value is consistent with the average long-term real-estate supply elasticity across US metropolitan areas estimated by Saiz (2010). We evaluate the effects of a gradual fall in the interest rate, from approximately 2% in the mid-1980s to 0.6% in 2018, which is consistent with the smoothed estimate of the long-term real interest rate in Bauer and Rudebusch (2020). The resulting reallocation effect is quantitatively large: whereas this fall in the interest rate would raise output by 1.8% if productivity were constant, it actually reduces output by 5.2% once the adverse general-equilibrium effects on productivity are taken into account. The key takeaway is not so much the ultimate size of the reallocation effect – which may be overestimated by assuming that all capital in the economy takes the form of land or real estate – but rather that it is of a similar magnitude as balance-sheet effects. Given that macroeconomists have devoted a substantial amount of effort to documenting the latter (Peek and Rosengren, 2000; Gan, 2007; Chaney et al., 2012), our findings suggest that they should also pay close attention to the former.

These findings are consistent with recent evidence on the macroeconomic effects of credit booms driven by low interest rates. In particular, they shed further light on the experience of several Southern European economies during the early 2000s, when a reduction in interest rates coincided with local booms in credit and asset prices coupled with a decline in aggregate productivity (Gopinath et al., 2017; García-Santana et al., 2020). Our findings suggest that this decline in productivity may have been driven not just by the expansion of less productive activities, which is not bad in itself, but also by the crowding-out of more productive activities through general-equilibrium effects. Indeed, evidence of such adverse reallocation is found by Banerjee and Hofmann (2018), who show that – for a set of advanced economies during recent decades – an increase in the share of "zombie" (i.e., less productive) firms in a given sector has been associated with a decline in investment and employment by "non-zombie" (i.e., more productive) firms in that same sector.[2]

To provide a more direct test of our theory, we analyze the effects of interest rate changes on firm-level investment across geographical regions in the US. We focus on one key insight of the theory: whereas a fall in the interest rate always expands the investment of less productive

[2]For related work on "zombie lending", see also Caballero et al. (2008); Adalet McGowan et al. (2018); Tracey (2019); Acharya et al. (2021).

firms, i.e., firms with low marginal product of capital (MPK), it may actually reduce the investment of more productive firms, i.e., firms with high MPK, due to general-equilibrium effects. We test this insight by using Compustat data to assess whether falling interest rates have a stronger impact on the investment of low-MPK (vis-à-vis high-MPK) firms in regions where general-equilibrium effects are stronger. To proxy for general-equilibrium effects at the local level, we use existing measures of real-estate supply elasticities for the US. This allows us to disentangle the direct effect of interest rates on firm investment from their indirect effect, which operates through real-estate prices. Our findings suggest that the general-equilibrium component is significant: the investment response to a 1.8 percentage point increase in real estate prices (equivalent to its standard deviation) is 0.23 percentage points stronger for firms that have a one standard deviation lower MPK. This is a substantial effect given that the average investment rate in the sample is 0.5 percent.

Our paper contributes to the literature that studies the negative side effects of credit booms on aggregate productivity (Reis, 2013; Gopinath et al., 2017; Gorton and Ordonez, 2020). Gopinath et al. (2017), in particular, have argued that declining interest rates can lead to a fall in productivity if the ensuing rise in credit is channeled to less productive firms. However, in their setting – as in our partial-equilibrium analysis – such an expansion in credit is beneficial because even less productive firms add value from a social perspective. Our contribution is to show how declines in interest rates can actually destroy social value, potentially reducing aggregate output, once general equilibrium effects are taken into account.[3]

Our paper is also related to the growing literature on macroeconomics with heterogeneous agents. Much of this literature has studied how heterogeneity shapes an economy's response to monetary policy. Although this research focuses mostly on heterogeneity among households (e.g., Cloyne et al. (2020), Kaplan et al. (2018), Slacalek et al. (2020)), a growing body of work also analyzes heterogeneity among firms (e.g., Anderson and Cesa-Bianchi (2020), Cloyne et al. (2018), Manea (2020), Ottonello and Winberry (2020), Jeenas (2020)).[4] Within this work, we are closest to Monacelli et al. (2018) and González et al. (2020), who study how interest rates shape productivity through their effects on the incentives and financial constraints of heterogeneous firms: differently from them, however, our focus is on the reallocation of resources

[3]Kiyotaki et al. (2021) have recently developed a model in which – due to financial frictions – entrepreneurs borrow predominantly against their near-term incomes, which means that a fall in the interest rate may actually tighten financial constraints.

[4]Relatedly, Leahy and Thapar (2019) study how age-structure shapes the impact of monetary policy: they find that monetary policy is most potent in regions with a larger share of middle-aged due to their higher propensity for entrepreneurship. On the other hand, Caggese and Pérez-Orive (2020) show how lower interest rates may become less expansionary in economies where intangible investments become more important.

across firms through general-equilibrium effects.

Finally, our model is related to work that stresses inefficiencies in the allocation of factors of production due to financial frictions. A recurring theme in this work is that individual firms do not internalize the effect of their demand on factor prices, which may lead to inefficient outcomes in the presence of financial constraints. Biais and Mariotti (2009), Ventura and Voth (2015), Martin et al. (2018), Asriyan et al. (2021), Buera et al. (2021) and Lanteri and Rampini (2021) provide examples of this work. In a related vein, Coimbra and Rey (2017) study the allocation of risky capital across financial intermediaries that are subject to financial constraints and are heterogeneous in their tolerance for risk. They show how, by reallocating capital towards riskier intermediaries, a decline in the interest rate may increase financial instability.

The paper is organized as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium effects of declining interest rates, as well as the normative properties of equilibrium. Section 4 extends the analysis to an infinite-horizon economy and performs a numerical exploration. Section 5 provides supporting evidence, and Section 6 concludes.

2 Baseline model

Consider an economy that lasts for two periods, $t = 0, 1$. There are two goods: a perishable consumption good (c) and capital (k). There are two sets of risk-neutral agents, entrepreneurs and capitalists, each of unit mass.

Preferences. The preferences of all agents are given by:

$$U = E_0[c_1],$$

where c_1 is the consumption at $t = 1$ and $E_0[\cdot]$ is the expectations operator at $t = 0$.

Endowments. Each entrepreneur is endowed with w units of the consumption good at $t = 0$, while capitalists have no endowment.

Technology. Each capitalist has access to a production technology that converts $\chi(k)$ units of the consumption good into k units of capital at $t = 0$, where χ is quasi-convex and weakly increasing in k.[5] Capital can be used for production by entrepreneurs. Specifically, each entrepreneur has access to a production technology that converts one unit of capital at $t = 0$ into A units of the consumption good at $t = 1$. We refer to A as entrepreneurial productivity

[5]The case where capitalists are simply endowed with $\bar{K}$ units of capital is captured by the following cost function: $\chi(k) = 0$ for $k \leq \bar{K}$ and $\chi(k)$ is infinite thereafter.

and assume that it is distributed independently across entrepreneurs, with distribution G that has an associated density g with full support on the interval $[0,1]$.

Markets. The economy is small and open and there is an international financial market where agents can borrow and lend consumption goods at a world interest rate R. Here, we introduce a central friction of our analysis by supposing that an entrepreneur can always walk away with a fraction $1-\lambda$ of her output at $t=1$. This pledgeability friction will endogenously limit the borrowing and investment that each entrepreneur can undertake. There is also a competitive capital market, where agents can trade capital at a unit price q in period $t=0$. Note that, as the economy ends at $t=1$, capital is no longer valuable after production at that date.

2.1 Optimization and equilibrium

Since agents can borrow and lend consumption goods in the international financial market at the interest rate R, only the clearing of the capital market is crucial for equilibrium. To characterize this market clearing condition, we analyze next the demand and supply of capital.

Capital demand. Let b_A and k_A respectively denote total borrowing and capital demand by an entrepreneur with productivity A. Given prices $\{q,R\}$, the entrepreneur makes her optimal borrowing and investment decisions to maximize her expected consumption:

$$\max_{\{b_A,k_A\}} \quad A\cdot k_A - R\cdot b_A, \tag{1}$$

subject to budget, borrowing and feasibility constraints:

$$q\cdot k_A \leq \; w + b_A, \tag{2}$$

$$R\cdot b_A \leq \; \lambda\cdot A\cdot k_A, \tag{3}$$

$$0 \leq k_A \; . \tag{4}$$

Note that the price of capital q enters the budget constraint (2) but not the borrowing constraint (3): the reason, as already explained, is that the price of capital at $t=1$ equals zero. This will change when we extend the analysis to an infinite-horizon economy in Section 4.

Optimization leads to the following capital demand:

$$k_A(q,R) \begin{cases} = 0 & \text{if } \frac{A}{q} < R \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w\right] & \text{if } R = \frac{A}{q} \\ = \frac{1}{q - \frac{\lambda \cdot A}{R}} \cdot w & \text{if } \frac{\lambda \cdot A}{q} < R < \frac{A}{q} \\ = \infty & \text{if } R \leq \frac{\lambda \cdot A}{q} \end{cases}, \tag{5}$$

which has an associated level of borrowing:

$$b_A(q,R) = q \cdot k_A(q,R) - w. \tag{6}$$

Equation (5) states that an entrepreneur's demand of capital is decreasing in the interest rate, R. When the interest rate is greater than her return to capital, A/q, an entrepreneur finds it optimal to invest in financial markets and demands no capital. When both returns are equalized, the entrepreneur is indifferent between investing in capital and not doing so. When the interest rate is smaller than the return to capital but greater than its pledgeable return, the entrepreneur demands capital until her borrowing constraint binds.

Equation (5) also implies that the demand function $k_A(q,R)$ is decreasing in q. For an entrepreneur who is unconstrained, i.e., $A \leq q \cdot R$, lower values of q raise the return to investing in capital. For an entrepreneur who is constrained, lower values of q relax the borrowing constraint and enable her to expand her borrowing and her purchases of capital. Finally, the demand function $k_A(q,R)$ is weakly increasing in λ, because a higher pledgeability of output enables constrained entrepreneurs to expand their borrowing and thus their purchases of capital.

We denote the aggregate demand for capital by entrepreneurs by:

$$K^D(q,R) \equiv \int_0^1 k_A(q,R) \cdot dG(A). \tag{7}$$

Capital supply. Given the price of capital, each capitalist chooses his supply of capital to maximize profits. Formally, we use $K^S(q)$ to denote a solution to:

$$\max_{k \geq 0} \; q \cdot k - \chi(k). \tag{8}$$

Since all capitalists are identical, $K^S(q)$ denotes both the individual and the aggregate supply of capital, which is weakly increasing in q.[6]

[6]E.g. if $\chi(\cdot)$ is increasing and convex with $\chi(0) = \chi'(0) = 0$, then $K^S(q) = \chi'^{-1}(q)$ and is increasing in q.

Market clearing. The price of capital, q, ensures that the capital market clears:

$$K^S(q) = K^D(q, R). \tag{9}$$

Aggregate domestic output is given by:

$$Y(q, R) = \int_0^1 A \cdot k_A(q, R) \cdot dG(A). \tag{10}$$

Naturally, output depends not just on the total demand of capital $K^D(q, R)$ but also on the way it is distributed across entrepreneurs.

Equilibrium. Given interest rate R, an equilibrium consists of a price of capital q and allocations $\{\{k_A, b_A\}_A, K^S, Y\}$, such that $\{k_A, b_A\}_A$ satisfy Equations (5) and (6), K^S is a solution to (8), the capital market clears according to Equation (9), and Y satisfies Equation (10).

3 Equilibrium effects of changes in interest rates

We want to understand the equilibrium effects of changes in the interest rate R. For now, we interpret changes in R as being induced by exogenous factors, such as changes in the world interest rate or in capital inflows. In Online Appendix C.2, we show that our results also hold when R is endogenous and its decline is driven by changes in model fundamentals.

Figure 1 depicts the distribution of capital across entrepreneurs for given prices $\{q, R\}$. To determine the aggregate effects of a change in R, we need to understand how this distribution of capital responds to such a change. In what follows, we refer to those entrepreneurs who find it optimal not to invest (i.e., $A < q \cdot R$) as "infra-marginal", to those that invest until their borrowing constraint binds (i.e., $A > q \cdot R$) as "supra-marginal", and to those that are indifferent (i.e., $A = q \cdot R$) as "marginal" entrepreneurs.

At first sight, the effect of a change in R on investment and output seems trivial. It follows immediately from Equation (5) that, for a given price q, all entrepreneurs raise their demand of capital when R falls. Supra-marginal entrepreneurs demand more capital because a lower value of R raises the present value of pledgeable output, thus relaxing their borrowing constraints. Moreover, some infra-marginal entrepreneurs start investing because a lower value of R raises the present value of their investment. This partial-equilibrium effect of a decline in R is depicted through a shift from the solid-blue to the dashed curve in panels (a) and (b) of Figure 2.

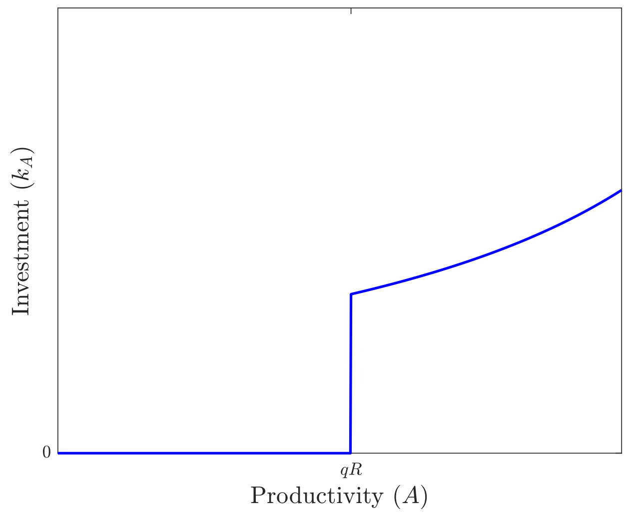


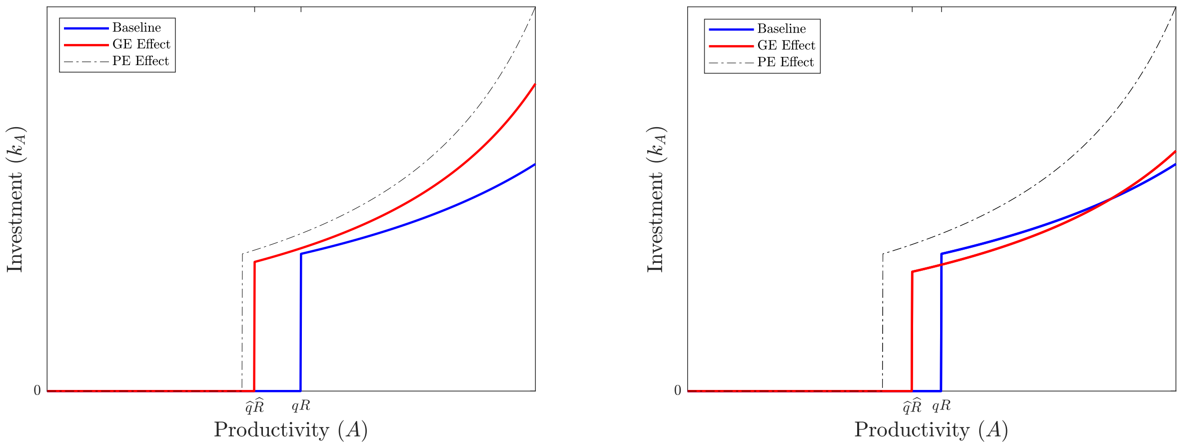
Figure 1: **Distribution of capital across entrepreneurs given prices** $\{q, R\}$**.**

As long as the supply of capital is not perfectly elastic, however, the effects of a decline in R do not end here. There is also a general-equilibrium effect because the price of capital q must increase to ensure capital-market clearing. This effect makes capital less attractive and reduces investment along all margins, but it cannot be so strong as to raise the productivity of the marginal entrepreneur, $q \cdot R$: otherwise, all entrepreneurs would reduce their demand of capital, which is a contradiction. This general-equilibrium effect of a decline in R is depicted through a shift from the dashed to the solid-red curve in panels (a) and (b) of Figure 2.

This discussion suggests that a decline in the interest rate must necessarily raise the investment of some infra-marginal entrepreneurs, although it may reduce the investment of some supra-marginal entrepreneurs. Formally, the change in the investment of a supra-marginal entrepreneur with productivity A is given by:

$$\frac{dk_A}{dR} = \frac{\left|\frac{dq}{dR}\right| - \frac{\lambda \cdot A}{R^2}}{q - \frac{\lambda \cdot A}{R}} \cdot k_A, \tag{11}$$

which has both a partial- and a general-equilibrium component. The second term in the numerator represents the partial-equilibrium effect, by which a decline in R increases the pledgeable return to capital and thus entrepreneurs' ability to invest. The first term in the numerator captures instead the general-equilibrium effect, by which a decline in R raises the price of capital thereby reducing entrepreneurial demand of capital. Equation (11) suggests that the investment of supra-marginal entrepreneurs may decline when the interest rate falls, provided that the general-equilibrium effect is strong and the partial-equilibrium effect is weak enough.



(a) Weak GE Effects

(b) Strong GE Effects

Figure 2: **Partial- and general-equilibrium effects of a fall in the interest rate.**

The strength of the general-equilibrium effect depends largely on the elasticity of capital supply, while the strength of the partial-equilibrium effect depends on the tightness of the financial friction. As $\lambda \to 0$, for instance, the partial-equilibrium effect disappears altogether. In panels (a) and (b) in Figure 2, the overall effect of a fall in the interest rate on investment is captured by the shift from the solid blue to the solid red line. In panel (a), general-equilibrium effects are weak and all supra-marginal entrepreneurs invest more when the interest rate falls; in panel (b), instead, general-equilibrium effects are strong and some supra-marginal entrepreneurs invest less when the interest rate declines.

The effect of a change in the interest rate on output is shaped by the behavior of investment across all entrepreneurs. Formally, we can combine Equations (9) and (10) to obtain:

$$\frac{dY}{dR} = \underbrace{\int_{q\cdot R}^{1} (A - q\cdot R)\cdot \frac{dk_A}{dR}\cdot dG(A)}_{\text{Capital-reallocation effect } \equiv\, \mathcal{R}} + \underbrace{q\cdot R\cdot \frac{dK^S(q)}{dR}}_{\text{Capital-supply effect } \equiv\, \mathcal{K}} , \tag{12}$$

where dk_A/dR is given by Equation (11). The first term in Equation (12), which we denote by $\mathcal{R}$, captures the capital-reallocation effect: the change in output driven by changes in the investment of supra-marginal entrepreneurs. As we have already noted, this effect can be positive or negative depending on the relative strength of the general- and partial- equilibrium effects. In what follows, we say that the capital-reallocation effect is *stronger* when $\mathcal{R}$ is more positive. The second term in Equation (12), which we denote by $\mathcal{K}$, captures instead the capital-supply effect: the change in output driven by adjustments in the aggregate capital stock. This effect is always (weakly) negative, since lower interest rates raise the demand for capital and thus the equilibrium stock of capital. In what follows, we say that the capital-supply effect is *stronger* when $\mathcal{K}$ is more negative.

Equation (12) illustrates the role of both heterogeneity and financial frictions in shaping the aggregate response of output to changes in the interest rate. In the absence of heterogeneity, all entrepreneurs would have the same productivity; in the absence of financial frictions, only the most productive entrepreneur would invest. In either case, the capital-reallocation effect would disappear and the response of output to the interest rate would be negative and driven only by the capital-supply effect, i.e., on the economy's ability to adjust the capital supply to the shifting demand. With heterogeneity and financial frictions, however, the response of output to changes in the interest rate depends not just on the behavior of aggregate investment but also on its reallocation across entrepreneurs. In fact, the capital reallocation effect can be so strong that falling interest rates may become contractionary.

To illustrate this possibility, consider a simple example in which there is no borrowing (i.e., $\lambda = 0$) and the capital stock is fixed (i.e., $K^S(q) = \bar{K}$). The lack of borrowing means that the investment of supra-marginal entrepreneurs is equal to w/q, and thus independent of the interest rate: this maximizes the strength of the reallocation effect, $\mathcal{R}$. The fixed capital supply, in turn, completely eliminates the capital supply effect, $\mathcal{K}$. Therefore, a decline in the interest rate must necessarily reduce aggregate output. By boosting the investment of unconstrained infra-marginal entrepreneurs, a lower interest rate raises the equilibrium price of capital and thus reduces supra-marginal investment, which is productive but constrained.

This example is of course stark but, as the next proposition shows, the result is more general and does not rely on such extreme scenarios.

Proposition 1 *Consider two economies that have the same equilibrium allocations and are identical in all respects except for the capital supply schedule. Let ε denote the elasticity of capital supply with respect to the price of capital q in equilibrium. Then, in the economy with lower ε:*

- *the capital-reallocation effect, $\mathcal{R}$, is stronger;*
- *the capital-supply effect, $\mathcal{K}$, is weaker;*
- *the response of output to a change in the interest rate, dY/dR, is greater;*

moreover, for low enough ε, there is a threshold $\bar{\lambda}_\varepsilon > 0$ such that dY/dR is positive if $\lambda < \bar{\lambda}_\varepsilon$.

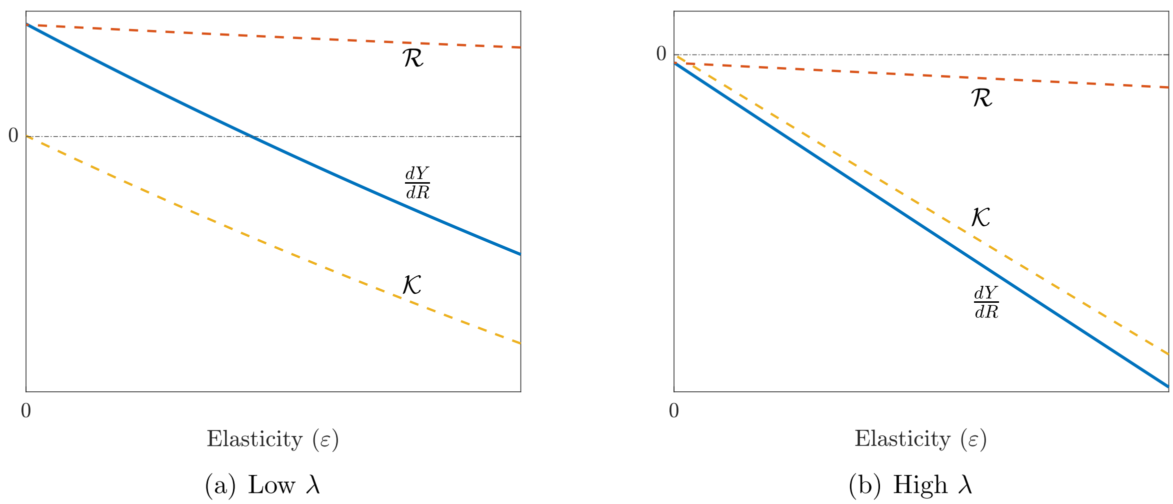


Figure 3: **Effect of the elasticity of capital supply on the response of output to a change in the interest rate.**

Proposition 1 states that the response of output to the interest rate is decreasing in the elasticity of the capital supply, ε, which governs the strength of the general-equilibrium effect.[7] This is illustrated in Figure 3, which plots dY/dR against ε for low and high values of λ, respectively. Both panels show that dY/dR increases as ε decreases. Lower values of ε weaken the capital-supply effect and, by reinforcing the general-equilibrium response of the price of capital, strengthen the capital-reallocation effect. When λ is low, moreover, the reallocation effect becomes positive and – for low values of ε – a fall in the interest rate leads to a decline in aggregate output (see panel (a)).[8]

Proposition 1 shows that, in general-equilibrium, the interaction of heterogeneous productivity and financial frictions gives rise to capital-reallocation effect that can mitigate or even overturn the expansionary effects of declining interest rates. Although it has been derived in a fairly stylized environment, we discuss next how it extends to more general settings.

3.1 Robustness

In our baseline model, entrepreneurs operate a linear production technology. As a result, an entrepreneur's marginal productivity is constant and coincides with her average productivity.

[7]The parametrization of the cost function $\chi(\cdot)$ used for Figure 3 is provided in the proof of Proposition 1.

[8]The result in Proposition 1 is local, in the sense that it characterizes dY/dR at a given equilibrium. In particular, if the equilibrium changes – e.g. due to a change in R – so does the strength of the reallocation effect and thus the threshold $\bar{\lambda}_\varepsilon$. This implies that, for given parameter values, the sign of dY/dR need not be the same for all levels of R.

Moreover, entrepreneurs who operate larger firms (i.e., those with higher A) are also those who are more financially constrained. We next show that our main insights extend naturally to settings with diminishing returns, where marginal and average productivity need not coincide, and where large productive firms may be unconstrained.

We make the following two adjustments to the setup in Section 2. First, we assume that entrepreneurs have a diminishing-returns technology $k \mapsto A \cdot f(k)$, with $f'(\cdot) > 0 > f''(\cdot)$ and $\lim_{k \to 0} f'(k) = \infty$. Second, we allow for the possibility of large, unconstrained firms by letting endowments vary across entrepreneurs. We denote the joint distribution of (A, w) by G.

Given prices (q, R), let $k_A^*(q, R)$ denote the first-best investment scale for an entrepreneur with productivity A, which satisfies $\frac{A \cdot f'(k_A^*)}{q} = R$. Optimization leads to the following capital demand:

$$k_{(A,w)}(q, R) = \begin{cases} k_A^*(q, R) & \text{if } q \cdot k_A^*(q, R) \leq w + \frac{\lambda \cdot A \cdot f\left(k_A^*(q,R)\right)}{R} \\ k \; : \; q \cdot k = w + \frac{\lambda \cdot A \cdot f(k)}{R} & \text{otherwise} \end{cases}. \tag{13}$$

This economy features two sets of entrepreneurs: those who are financially unconstrained, with an MPK, $A \cdot f'(k)$, *equal* to $q \cdot R$; and those who are financially constrained, with MPK *above* $q \cdot R$.[9] Note that an entrepreneur is unconstrained if her endowment, w, is large relative to her first-best investment scale, k_A^*.[10]

The capital price q ensures market clearing:

$$K^S(q) = \int k_{(A,w)}(q, R) \cdot dG, \tag{14}$$

and aggregate output is given by:

$$Y = \int A \cdot f\left(k_{(A,w)}(q, R)\right) \cdot dG. \tag{15}$$

The response of output to a change in the interest rate can be expressed as follows:

$$\frac{dY}{dR} = \int_{(A,w) \in \mathcal{C}} \left(A \cdot f'(k_{(A,w)}(q, R)) - q \cdot R\right) \cdot \frac{dk_{(A,w)}(q, R)}{dR} \cdot dG + q \cdot R \cdot \frac{dK^S(q)}{dR}, \tag{16}$$

where $\mathcal{C} = \{(A, w) : A \cdot f'(k_{(A,w)}(q, R)) > q \cdot R\}$ is the set of constrained entrepreneurs.

[9] In our baseline model, the marginal entrepreneurs are also unconstrained, as their marginal product of capital is $q \cdot R$. However, in contrast to this setting, they have an infinitesimal mass.

[10] Alternatively, we could allow entrepreneurs to differ in the severity of the financial friction that they face, i.e., heterogeneous λ's (capturing, e.g., more or less reputable firms), where those with sufficiently high λ are unconstrained.

Equation (16) is essentially the same as Equation (12), with the only difference that now entrepreneurs' marginal productivity of capital, $A \cdot f'(k)$, depends on their scale of production. It is then straightforward to extend the result in Proposition 1, where the reallocation effect now refers to the reallocation of capital from higher MPK (i.e., more constrained) towards lower MPK (i.e., less constrained) entrepreneurs in response to a fall in the interest rate. This setting generalizes our baseline model and allows for the presence of large-unconstrained firms (as in Khan and Thomas (2013)) if we suppose that entrepreneurs with high productivity have sufficiently high endowments (i.e., there is a positive correlation between A and w). However, these productive, large entrepreneurs have a low-MPK because they are unconstrained and thus invest until their marginal product of capital equals $q \cdot R$.

In the Online Appendix, we explore other extensions of our baseline setting. Online Appendix C.1 explores an extension with credit risk, in which entrepreneurs differ in their probability of success and can thus default. Our main result is unaffected, although it is now the heterogeneity in entrepreneurs' *expected* productivity that matters for the capital-reallocation effect. Online Appendix C.2 explores a closed-economy version of our setting, in which the interest rate is determined endogenously and its fall is prompted by an increased desire to save (i.e., a savings glut). Thus, our findings are consistent with one of the most common hypothesis to explain the sustained decline in interest rates over the past several decades (e.g., Bernanke et al. (2005); Caballero et al. (2008)).

Perhaps the main limitation of the baseline economy is that it is essentially static, as it lasts for only two periods, and entrepreneurial wealth is exogenous. In a fully dynamic economy, entrepreneurial wealth would naturally be endogenous to (i) productivity, as more productive entrepreneurs may accumulate wealth faster; and potentially (ii) asset prices, due to the well-known balance-sheet effects à la Kiyotaki and Moore (1997). Section 4 extends the analysis to an infinite-horizon economy and shows how the reallocation effect highlighted here interacts with wealth accumulation and balance-sheet effects. Before moving to the fully dynamic model, however, we turn to the normative properties of equilibrium.

3.2 Normative properties

We now analyze the extent to which the competitive equilibrium is constrained (in)efficient. As we show, the general-equilibrium induced reallocation that we identified in the previous section is closely linked to an externality, by which the investment of some less productive entrepreneurs is excessive from a social point of view.

Consider the problem of a social planner whose objective is to maximize aggregate consumption at $t = 1$.[11] The planner is constrained to only choosing investment $\{k_A\}$ for entrepreneurs, but agents make all the other decisions on their own. This implies, in particular, that the planner must respect individual budget and financial constraints. To simplify the exposition, we assume throughout that $\chi(\cdot)$ is strictly convex.

Formally, the social planner solves the following maximization problem:

$$\max_{\{k_A\}} \int_0^1 A \cdot k_A \cdot dG(A) - R \cdot [\chi(K^S) - w], \tag{17}$$

subject to:

$$R \cdot (q \cdot k_A - w) \leq \lambda \cdot A \cdot k_A \quad \text{and} \quad 0 \leq k_A \quad \forall A, \tag{18}$$

and to capitalists' optimization and market clearing:

$$\chi'^{-1}(q) = K^S = \int_0^1 k_A \cdot dG(A). \tag{19}$$

The objective function of the planner in (17) says that aggregate consumption at $t = 1$ is equal to aggregate output net of repayments to international lenders, which are in turn given by R times the difference between the cost of capital production and aggregate endowment at $t = 0$. Equations (18) and (19) state that the planner must respect individual budget and financial constraints, feasibility, and market clearing. In particular, the planner is not able to make transfers so as to overcome financial frictions.

To understand the solution to the planner's problem, consider the (social) net present value (NPV_A^{SP}) of a unit of investment, k_A, by entrepreneur with productivity A:

$$\text{NPV}_A^{SP} \equiv \frac{A}{R} - q - \left[\chi''\left(K^S\right) \cdot \int \gamma_{\widehat{A}} \cdot k_{\widehat{A}} \cdot dG(\widehat{A})\right] \tag{20}$$

where γ_A denotes the multiplier on the borrowing constraints of entrepreneurs with productivity A. The first observation is that, since NPV_A^{SP} is linear and increasing in A, there exists a marginal entrepreneur $\widetilde{A}$ with $\text{NPV}_{\widetilde{A}}^{SP} = 0$, such that only entrepreneurs with productivity above $\widetilde{A}$ invest and they do so until their borrowing constraints bind. The second observation is that, since the term in brackets is positive, the planner perceives a higher social cost (or a lower social benefit) of investment than individual entrepreneurs, who only compare A/R to q. This is because the planner internalizes that each additional unit of investment raises the

[11] Since preferences are linear, such an objective is equivalent to maximizing the equally-weighted aggregate welfare. We thus abstract from distributional considerations.

equilibrium price of capital (as $\chi''(K^S) > 0$) and thus tightens the borrowing constraints of all entrepreneurs with productivity above $\widetilde{A}$. Since the borrowing constraints bind for all such entrepreneurs (as $\gamma_A > 0 \ \forall A > \widetilde{A}$), this entails a first-order welfare loss. Consequently, the planner restricts investment by some entrepreneurs by setting $\widetilde{A} > q \cdot R$.

The following proposition summarizes the above discussion and also states its main implication for the response of output to changes in interest rates.

Proposition 2 *Let $\widetilde{A}$ denote the productivity of the marginal entrepreneur at the social planner allocation, and q^{CE} denote the price of capital in the competitive equilibrium. Then:*

$$\widetilde{A} > R \cdot q^{CE},$$

i.e., relative to the competitive equilibrium, the planner restricts investment by some supra-marginal entrepreneurs. Moreover, letting Y^{SP} denote output in the social planner allocation, it holds that:

$$\frac{dY^{SP}}{dR} < 0,$$

i.e., a fall in the interest rate is always expansionary in the social planner's allocation.

The first part of Proposition 2 follows directly from our previous discussion. It illustrates that the planner forbids some entrepreneurs from investing altogether: by doing so, she enables more productive entrepreneurs to expand their investment, as their constraints are relaxed when capital becomes less scarce (i.e., when the price of capital falls). The second part of Proposition 2 follows directly from the first. To see this, simply note that a fall in the interest rate can only reduce output if it reallocates capital from supra- to infra-marginal entrepreneurs (see Equation (12)). But the planner can always keep these reallocation effects under control by adjusting the productivity of the marginal entrepreneur, $\widetilde{A}$, when the interest rate changes.

This type of planner intervention, which prevents some entrepreneurs from investing altogether, may seem far-fetched and informationally demanding for the planner (i.e., she needs to know which entrepreneurs to exclude). However, it is straightforward to show that the planner allocation can be decentralized through a simple Pigouvian subsidy τ on savings, financed with lump-sum taxes on capitalists. By choosing the subsidy appropriately, the planner can ensure that all entrepreneurs with productivity lower than $\widetilde{A}$ prefer to save their endowments at the market interest rate and collect the subsidy rather than investing in capital.

These results are reminiscent of the literature on "zombie" firms, which emphasizes that low interest rates incentivize unproductive activities (Caballero et al., 2008; Adalet McGowan et al., 2018; Banerjee and Hofmann, 2018; Tracey, 2019; Acharya et al., 2021). In much of that literature, however, the emphasis is on distortions that provide incentives to keep some firms operational even though they are not profitable (e.g. evergreening by banks). We show instead that investment can be socially excessive despite having a positive net present value from a private standpoint, because individual entrepreneurs do not internalize the crowding-out effect that they have on more productive investment.

3.3 How do we interpret capital?

The main insight of our paper is that, in the presence of heterogeneity and financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned) by general-equilibrium reallocation effects. This mechanism requires the price of capital to depend on local economic conditions (i.e., domestic demand and supply for capital), which is a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets (e.g., Kiyotaki and Moore (1997); Krishnamurthy (2003); Lorenzoni (2008); Brunnermeier and Sannikov (2014)). Indeed, just as balance-sheet effects would vanish if the price of capital were independent of local economic conditions, so would our reallocation effect.

Going beyond the conceptual insight, how can we interpret capital in the data? The literature has often thought of physical assets, which are owned by firms and give rise to balance sheet effects (e.g., Kiyotaki and Moore (1997)). Within these assets, moreover, the literature has mostly focused on land or real estate, where the importance of general equilibrium effects (e.g., as induced by an imperfectly elastic supply) seems most natural. This is in opposition to machinery, for instance, whose price is less dependent on local economic conditions because it can be traded internationally. In the remainder of the paper, we follow the literature and interpret capital as land or real estate. By doing so, we can incorporate the classic balance-sheet effect and show how it interact with the reallocation effect that we emphasize here.

Our key insight is not limited to physical assets, however, and could arise in relation to any factor of production that requires credit. One example is skilled labor, which (i) requires working capital insofar as firms pay wages before output is realized, and (ii) is relatively scarce, at least in the medium-term. Under this interpretation, there are no balance sheet effects (as firms do not own their workers) but there still are reallocation effects: by increasing the demand for skilled labor by relatively less productive firms, a decline in interest rates would raise skilled wages thereby crowding out labor demand by relatively more productive firms.[12]

[12]Moreover, these two interpretations are not mutually exclusive. Even firms that do not use real estate directly as an input of production do so indirectly as the value of residential real estate is a key determinant of

4 Dynamics of wealth accumulation

We now extend the analysis to a dynamic economy. There are two main reasons for doing so. The first is conceptual: in a dynamic setting, changes in the interest rate are likely to have additional effects through wealth accumulation. In particular, productive entrepreneurs may stand to gain from a falling interest rate, as they benefit both through lower costs of borrowing and – potentially – through higher asset prices and their associated balance-sheet effects. The second reason is quantitative: a dynamic setting lends itself to a numerical exploration that can inform us about the quantitative relevance of the reallocation effect highlighted in the theory.

Throughout this section, we follow the literature and adopt a continuous-time setting. The advantage of doing so is that it enables us to obtain analytical expressions for the stationary distribution of wealth, which is a key component of equilibrium characterization. We show, however, that all of our results go through in a discrete-time economy à la Kiyotaki and Moore (1997) in Online Appendix C.3.

4.1 A dynamic economy

Suppose that time t is continuous. To simplify notation, we omit time subscripts whenever possible. As in the baseline model of Section 3, the economy is populated by a continuum of entrepreneurs with mass one. The key difference relative to the baseline is that entrepreneurs differ not only in their productivity, which varies stochastically, but also in their wealth, which evolves endogenously.

Individual productivity alternates stochastically according to an idiosyncratic Poisson process with common arrival rate θ. If an entrepreneur with productivity A is hit by the shock, she draws a new productivity A' from the distribution $g(A'|A)$, where we denote $g(A)$ as the corresponding stationary density that has full support on some interval $[\underline{A}, \bar{A}]$. Otherwise, her productivity remains unchanged. An individual's wealth w in turn evolves endogenously according to her equilibrium rate of return and her consumption choices. Absent the Poisson shock, the law of motion of entrepreneurial wealth is given by:

$$\dot{w} = y + \dot{q} \cdot k - \delta \cdot q \cdot k - r \cdot b - c, \tag{21}$$

where $y = A \cdot k$ is the output flow obtained from operating capital stock $k \geq 0$; $\dot{q}$ is the rate of

wages. Under this interpretation, a decline in the interest rate raises labor demand by relatively less productive firms, boosting the local demand for residential real estate and prompting an increase in wage that crowds out relatively more productive firms.

change of the price of capital $q > 0$; $\delta \geq 0$ is the depreciation rate of physical capital; $r > 0$ is the interest rate on debt b, where $b < 0$ denotes savings; and $c \geq 0$ is the consumption flow.

As in the baseline economy, entrepreneurs may be financially constrained. We follow Moll (2014) and assume that, immediately after issuing debt, an entrepreneur can default, walk away with a portion $1-\lambda \in (0,1)$ of her capital and re-enter financial markets.[13] Formally, this gives rise to the following financial constraint:

$$b \leq \lambda \cdot q \cdot k. \tag{22}$$

Capital is supplied by a unit mass of capitalists. In particular, they have the ability to produce $I \geq 0$ units of capital per unit of time at a cost of $\chi(I) \geq 0$ in terms of the consumption good, where $\chi(\cdot)$ is increasing and quasi-convex. Thus, the aggregate capital stock K evolves according to:

$$\dot{K} = I(q) - \delta \cdot K, \tag{23}$$

where $I(q)$ is derived from the capitalists' optimization problem and is given by:

$$I(q) \equiv \arg\max_{I \geq 0} \left\{ q \cdot I - \chi(I) \right\}. \tag{24}$$

All agents have logarithmic preferences for consumption and discount future consumption at rate $\rho > r$.[14] Since in this small open economy capitalists only affect equilibrium dynamics through the supply of capital, we ignore their consumption-savings choice in what follows.

4.2 Equilibrium

In any period t, entrepreneurs choose their consumption c, their capital stock k, and their debt b, given the path of asset prices and the interest rate. The optimal choice of entrepreneurs with productivity A is:

$$k = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{w}{q} & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ 0 & \text{otherwise} \end{cases}, \tag{25}$$

[13] As in the literature, switching to a continuous-time setting requires a slight change in the nature of the financial constraint in order to maximize tractability. As we mentioned earlier, however, Online Appendix C.3 extends our results to a discrete-time setting à la Kiyotaki and Moore (1997) where the financial constraint is analogous to the one in the baseline model of Section 2.

[14] As is standard, entrepreneurs must be impatient relative to international lenders because otherwise they would eventually outgrow financial constraints.

and

$$b = q \cdot k - w, \tag{26}$$

$$c = \rho \cdot w. \tag{27}$$

Equation (25) says that, just as in the baseline model, there is a threshold entrepreneur who is indifferent between saving or investing in capital. The only difference is that this threshold is now given by $(r + \delta) \cdot q - \dot{q}$, as part of the cost of capital is in the form of depreciation and part of the return to capital accrues in the form of capital gains. Entrepreneurs above the threshold (i.e., supra-marginals) borrow and invest as much as possible, whereas those below (i.e., infra-marginals) save at the interest rate r. Equation (27) says that, due to log-preferences, entrepreneurs consume a portion ρ of their wealth at each instant.

Substituting these optimal choices into law of motion (21), it follows that individual wealth w evolves according to:

$$\dot{w} = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \delta - \lambda \cdot r\right) \cdot \frac{1}{1-\lambda} - \rho\right] \cdot w & \text{if } A + \dot{q} \geq (r + \delta) \cdot q \\ (r - \rho) \cdot w & \text{otherwise} \end{cases}. \tag{28}$$

Equation (28) captures the endogeneity of wealth accumulation in the dynamic economy, which depends on the interest rate and the return to capital. In particular, more productive entrepreneurs accumulate wealth at a faster pace due to their higher return on capital. Moreover, a lower interest rate has a positive effect on the wealth accumulation of supra-marginal entrepreneurs (who are borrowers) and a negative effect on that of infra-marginal entrepreneurs (who are savers). Lastly, higher capital gains boost the wealth accumulation of those entrepreneurs that invest more and thus have a higher exposure to capital.

To characterize the equilibrium, it is convenient to aggregate all entrepreneurs with the same productivity A and define their aggregate wealth as:

$$W_A \equiv \int w \cdot f(A, w) \cdot dw, \tag{29}$$

where $f(A, w)$ is the share of entrepreneurs with productivity A and wealth w. We denote aggregate entrepreneurial wealth by:

$$W \equiv \int W_A \cdot dA. \tag{30}$$

By combining Equation (29) with the stochastic structure of productivity shocks, it follows that W_A evolves according to:

$$\dot{W}_A = \int \dot{w} \cdot f(A, w) \cdot dw + \theta \cdot \int \left[g(A|A') \cdot W_{A'} - g(A'|A) \cdot W_A \right] \cdot dA' \,.$$

The first term in this expression aggregates the evolution of individual wealth across all entrepreneurs with productivity A. The second term reflects how this pool of entrepreneurs evolves due to productivity shocks. Together with Equation (28), we obtain:

$$\dot{W}_A = \begin{cases} \left[\left(\frac{A+\dot{q}}{q} - \delta - \lambda \cdot r \right) \cdot \frac{1}{1-\lambda} - \rho - \theta \right] \cdot W_A + \theta \cdot \int g(A|A') \cdot W_{A'} \cdot dA' & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ (r - \rho - \theta) \cdot W_A + \theta \cdot \int g(A|A') \cdot W_{A'} \cdot dA' & \text{otherwise} \end{cases} . \tag{31}$$

Per-capita investment of entrepreneurs with productivity A is then given by:

$$k_A = \begin{cases} \frac{1}{1-\lambda} \cdot \frac{W_A}{q} \cdot \frac{1}{g(A)} & \text{if } A + \dot{q} \geq (r+\delta) \cdot q \\ 0 & \text{otherwise} \end{cases} , \tag{32}$$

so that the market-clearing condition for capital can be expressed as:

$$K = \int_{A \geq (r+\delta) \cdot q - \dot{q}} k_A \cdot g(A) \cdot dA, \tag{33}$$

where K satisfies Equation (23), and aggregate output is:

$$Y = \int_{A \geq (r+\delta) \cdot q - \dot{q}} A \cdot k_A \cdot g(A) \cdot dA. \tag{34}$$

Given a path of interest rates $\{r\}$, an equilibrium consists of paths of prices $\{q\}$ and allocations $\{\{W_A, k_A\}_A, W, K, Y, I\}$ such that Equations (23)-(24) and (30)-(34) are satisfied in all periods.

4.3 An illustration: reallocation and balance-sheet effects

To illustrate how a fall in the interest rate affects the equilibrium in the dynamic economy, we focus first on a special case in which the capital supply is fixed at $\bar{K}$ (thus $\delta = 0$) and the productivity process satisfies $g(A'|A) = g(A')$ for all A' and A, i.e., the likelihood that an entrepreneur transitions to productivity A' is independent of her current productivity. This enables us to analytically characterize the steady-state effects of a permanent fall in the interest rate. It also allows us to isolate the interplay between the capital-reallocation effect — the novel transmission channel of real interest rates in our analysis — and the traditional balance-sheet effect.

4.3.1 Reallocation effects between steady states

In steady state, prices and aggregate variables are constant over time, i.e., $\dot{q} = 0$ and $\dot{W}_A = 0$ for all A. Given our assumptions on productivity, we can express the wealth of entrepreneurs with productivity A as a share of aggregate wealth W:

$$\frac{W_A}{W} = \begin{cases} \frac{\theta}{\theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g\left(A\right) & \text{if } A \geq r\cdot q \\ \frac{\theta}{\theta+\rho-r}\cdot g\left(A\right) & \text{otherwise} \end{cases}. \tag{35}$$

Equation (35) shows that the positive link between productivity and wealth extends to the steady state. Since wealth shares must add up to one, it follows that:

$$\int_{r\cdot q}^{1} \frac{\theta}{\theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g(A)\cdot dA = 1 - \int_{0}^{r\cdot q}\frac{\theta}{\theta+\rho-r}\cdot g(A)\cdot dA, \tag{36}$$

while the market clearing condition for capital can be written as:

$$\bar{K} = \left[\int_{r\cdot q}^{1} \frac{\theta}{\theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g(A)\cdot dA\right]\cdot\frac{1}{1-\lambda}\cdot\frac{W}{q}. \tag{37}$$

Equations (35)-(37) jointly determine the steady-state values of $\{W_A\}$, W, and q as a function of r.

Equation (35) shows that, for given aggregate wealth W and price of capital q, a permanent fall in r reduces the relative wealth of infra-marginal entrepreneurs: lower interest rates transfer wealth from creditors to debtors. Together with Equation (36), this implies that the productivity of the marginal entrepreneur, $r \cdot q$, must fall as savings become less attractive relative to investment. Note that Equation (36) pins down the price of capital q to ensure that wealth shares are consistent with a stationary equilibrium. Finally, Equation (37) implies that – given the wealth share of entrepreneurs – aggregate wealth in terms of capital (W/q) must adjust to preserve market clearing.

As in the baseline economy, the reallocation effect is driven by general-equilibrium forces in the capital market. The main difference is that aggregate wealth W is endogenous in the dynamic economy, so that reallocation is not just driven by the price of capital q but also by the ratio W/q.[15]

The effect of a fall in r on steady-state output can be obtained by combining Equations (33) and (34):

$$\frac{dY}{dr} = \int_{r \cdot q}^{1} (A - r \cdot q) \cdot \frac{dk_A}{dr} \cdot g(A) \cdot dA, \tag{38}$$

which is the equivalent of Equation (12) when the capital supply is fixed. As in the baseline model, the capital-reallocation effect can be positive or negative because it captures both the reallocation of capital from supra- to infra-marginal entrepreneurs (which reduces aggregate output) and the reallocation of capital among supra-marginal entrepreneurs (which may increase aggregate output).

4.3.2 Reallocation effects in non-stationary dynamics

We consider an economy that is in steady state until time t_0, when it is hit by an unanticipated shock to the path of the interest rate. The subsequent path is deterministic and eventually converges to a stationary value.[16] We examine the economy's response to both a permanent (Figure 4) and a temporary (Figure 5) fall in the interest rate.

On impact, a fall in the interest rate increases the price of capital, which in turn raises the wealth of supra-marginal entrepreneurs –who owned the capital before the shock,– while leaving the wealth of infra-marginal entrepreneurs unchanged. Formally, letting t_0 and t_0^+ denote the instants immediately before and after the shock respectively, the wealth of entrepreneurs at time t_0^+ is given by:

$$W_{A,t_0^+} = \begin{cases} \left(\frac{q_{t_0^+}}{q_{t_0}} - \lambda\right) \cdot \frac{1}{1-\lambda} \cdot W_{A,t_0} & \text{if } A \geq (r_{t_0} + \delta) \cdot q_{t_0} \\ W_{A,t_0} & \text{otherwise} \end{cases}. \tag{39}$$

Combining Equation (39) with market-clearing condition (33) at time t_0^+, it is possible to

[15] In fact, the effect of a fall in r on the price of capital q is in general ambiguous in this economy. For instance, if the losses inflicted on infra-marginal entrepreneurs by a lower value of r are not compensated by the gains of supra-marginals (e.g. if λ is small), then Equation (36) implies that q must fall in response to a fall in r. Online Appendix C.4 displays the steady state in closed form for a simpler case in which productivity is uniformly distributed on the unit interval across entrepreneurs.

[16] Similar results follow if the shock is instead anticipated, but we keep our approach for tractability.

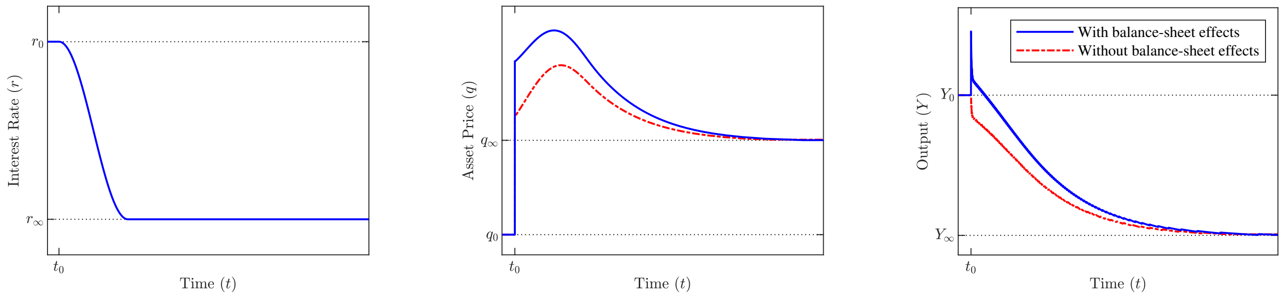


Figure 4: **A permanent fall in the interest rate.**

characterize the jump in the price of capital on impact,

$$\frac{q_{t_0^+}}{q_{t_0}} = \lambda \cdot \frac{\int_{A \geq r \cdot q_{t_0^+} - \dot{q}_{t_0^+}} W_{A,t_0} \cdot dA}{\int_{A \geq r \cdot q_{t_0^+} - \dot{q}_{t_0^+}} W_{A,t_0} \cdot dA - (1-\lambda) \cdot \int_{A \geq r \cdot q_{t_0}} W_{A,t_0} \cdot dA}\,, \tag{40}$$

where from time t_0^+ onward, the evolution of aggregate variables $\{\{W_A, k_A\}_A, W, q, Y\}$ is characterized by Equations (30)-(34).[17]

Equations (39)-(40) capture the balance-sheet effect and, together with Equations (32)-(34), characterize its implications for the allocation of capital and for aggregate output. On impact, a fall in the interest rate increases the wealth of supra-marginal entrepreneurs more than proportionally to the change in the price of capital, which improves the allocation of capital and expands output.[18] This *positive* reallocation eventually vanishes, as there are no more unexpected changes to the price of capital, while the *negative* reallocation effect emphasized in our baseline model persists as long as the interest rate remains depressed.

Figure 4 illustrates these dynamics by depicting the evolution of asset prices (middle panel) and output (right panel) in response to a decline in the path of the interest rate (left panel). As the solid lines show, asset prices jump on impact and they continue to rise for a while after the shock due to the expectation of further interest-rate declines going forward. Output first expands relative to the baseline as the balance-sheet effect dominates, but it eventually contracts as the negative reallocation effect takes over.

To further isolate the effects of the balance-sheet channel, the dashed lines in Figure 4

[17]We omit variables $\{K, I\}$ and Equations (23)-(24) in this characterization because the capital stock is fixed. To solve the dynamics from time t_0^+ onward, one needs to guess a value for $\dot{q}_{t_0^+}$ and verify that the obtained solution using (30)-(34) and (39)-(40) converges to the final steady state.

[18]This positive reallocation induced by the balance-sheet channel critically depends on leverage: it becomes arbitrarily small with $\lambda \simeq 0$, as the wealth of supra-marginal entrepreneurs becomes proportional to the value of capital.

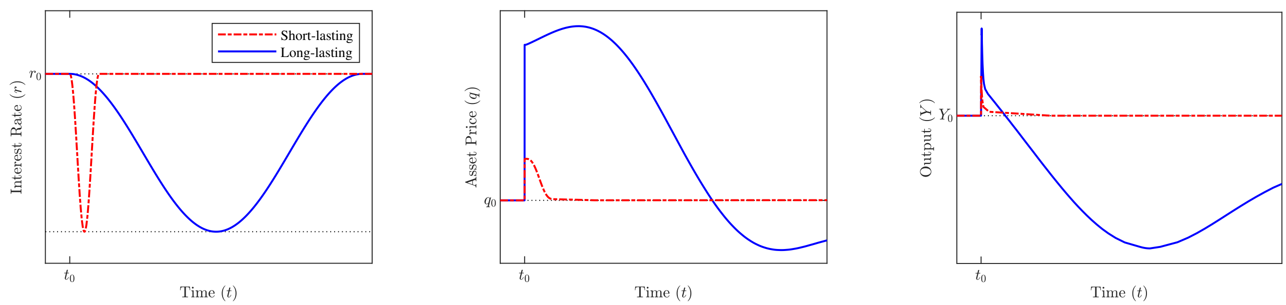


Figure 5: **A temporary fall in the interest rate.**

depict a counterfactual economy in which the balance-sheet channel is fully shut down by assuming that the wealth of entrepreneurs increases proportionally to the price of capital (i.e., $W_{A,t_0^+}/q_{t_0^+} = W_{A,t_0}/q_{t_0}$ for all supra-marginal entrepreneurs). In this case, the lower path of the interest rate still generates an upward jump in the price of capital but this now has no effect on output. The reason is intuitive: given that the wealth of supra-marginal entrepreneurs increases alongside the price of capital, their demand of capital is unaffected by the shock.[19].

The same forces are at work if the fall in the interest rate is instead transitory, but the overall response of output may be qualitatively different. This is illustrated in Figure 5, which depicts the evolution of asset prices and output in response to a short- vs. a long-lived decline in the interest rate. If the decline is sufficiently short-lived, balance-sheet effects dominate throughout and thus output expands at all horizons. As the shock becomes more persistent, the balance-sheet effect eventually vanishes, generating a boom-bust response of output.

4.4 A numerical exploration

We explore the model's quantitative implications by calibrating it to an annual frequency. The functional form for the production costs of capital is:

$$\chi(I) = \frac{\delta^{-\frac{1}{\varepsilon}}}{1 + \frac{1}{\varepsilon}} \cdot I^{1+\frac{1}{\varepsilon}},$$

where $\varepsilon \geq 0$ is the price-elasticity of the capital supply. Under this functional form, $I(q) = \delta \cdot q^{\varepsilon}$ and thus, in steady state, $K = q^{\varepsilon}$.

We set parameter values to match empirical estimates of variables that are directly affected

[19]This finding is in line with the literature on the balance-sheet channel, which shows that balance-sheet effects rely critically on entrepreneurs' inability to hedge their positions from asset-price fluctuations (see e.g. Krishnamurthy (2003), Di Tella (2017), Asriyan (2021)).

by them (see Table 1 for details), and feed into the model a path of the interest rate consistent with empirical estimates in the literature (see the left-hand panel of Figure 6). We also perform a sensitivity analysis to assess the relative contribution of the three key ingredients of our model: heterogeneous productivity, financial frictions, and the elasticity of the capital supply.

Parameter	**Value**	**Source**
Frequency of transition of log productivity	$\theta = 1$	Yearly frequency
Persistence of disturbance to log productivity	$\gamma_A = 0.8$	Foster et al. (2008)
Volatility of disturbance to log productivity	$\sigma_A = 0.45$	Bloom et al. (2018)
Borrowing limit (ratio of debt to assets)	$\lambda = 0.65$	Kalemli-Ozcan et al. (2012)
Elasticity of capital-supply	$\varepsilon = 2$	Saiz (2010)
Depreciation rate of capital	$\delta = 6.5\%$	Khan and Thomas (2008)
Subjective time discount rate	$\rho = 4\%$	Lustig et al. (2013)

Table 1: **The parameter values used for the numerical exercise.**

In the model, idiosyncratic productivity changes when a Poisson shock arrives, in which case productivity evolves according to the transition density $g(A'|A)$. We set the grid of idiosyncratic productivity and its transition density to match the evolution of firm-level productivity in the data, which is well captured in logarithmic form by an AR(1) process with a persistence of $\gamma_A = 0.8$ and a volatility of disturbance $\sigma_A = 0.45$ (Foster et al., 2008; Bloom et al., 2018). The arrival rate of the Poisson shock is set to $\theta = 1$, so that productivity transitions every year. In the sensitivity analysis, we allow γ_A to vary between 0.7 and 0.9, values that are respectively closer to the estimates used by Khan and Thomas (2008, 2013). We also allow σ_A to vary between 0.4 and 0.5, which respectively correspond to the estimates of the interquartile range and the standard deviation of the disturbance to log of productivity in Bloom et al. (2018). Given parameter value $\gamma_A = 0.8$, the former value is also closer to the value required to match the cross-sectional dispersion of log of productivity estimated by Asker et al. (2014).[20]

For the severity of the financial friction, we set $\lambda = 0.65$ to match the ratio of liabilities to assets for non-financial companies estimated by Kalemli-Ozcan et al. (2012). We allow λ to vary between 0.6 and 0.7 in the sensitivity analysis to accommodate these authors' estimates for the US economy and the euro area, respectively. The former value is closer to the estimate of 0.56 for non-financial companies in emerging market economies by Alter and Elekdag (2020), while the latter value is closer to the leverage multiple of 4 commonly targeted in macroeconomic

[20]Under the AR(1) process, the cross-sectional dispersion of log of productivity is given by $\sigma_A/\sqrt{1-\gamma_A^2}$. Using firm-level data comparable to those in Foster et al. (2008) and Bloom et al. (2018), Asker et al. (2014) estimate a cross-sectional dispersion for the US economy of 0.63. Then, given $\gamma_A = 0.8$, $\sigma_A = 0.63\sqrt{1-0.8^2} \simeq 0.38$ is required to match the estimated dispersion.

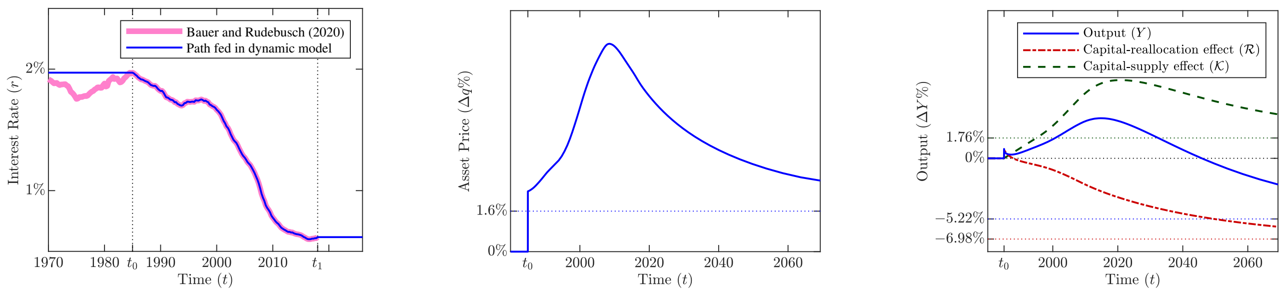


Figure 6: **Quantitative effects from declining real interest rates.**

models with a financial sector (e.g., Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)).

Since we think of capital as representing land or real estate, we set the supply elasticity to $\varepsilon = 2$. This value lies between the population-weighted average (1.75) and the simple average (2.5) of the long-term real-estate supply elasticity across US metropolitan areas estimated by Saiz (2010). In the sensitivity analysis, we allow this elasticity to vary between 0 and 4, the latter value being the highest in the range of the estimated elasticity distribution that results from removing its original 2%-upper tail. Lastly, we set $\delta = 6.5\%$, following Khan and Thomas (2008, 2013), and $\rho = 4\%$, which is consistent with the price-dividend ratio on equity (26.0) and the discount rate of a claim to aggregate consumption (3.51%) estimated by Lustig et al. (2013).[21]

We feed into the model a declining path of the interest rate based on the smoothed estimate of the long-term real interest rate in Bauer and Rudebusch (2020). We use a long-term estimate because the model is not suited for analysis at the business cycle frequency (the only aggregate shock is an unanticipated one to the interest rate). In the numerical exercise, therefore, we interpret changes in the interest rate as changes in its long-term trend.

The left panel in Figure 6 illustrates the path for the interest rate that we consider. Before the mid-1980s, the interest rate is fixed at a stationary value slightly below 2%; from the mid-1980s to the late-2010s, it follows the estimated series in Bauer and Rudebusch (2020); after the late 2010s, it is fixed again at a stationary value slightly above 0.6%. The initial and final moments of the transition between the two stationary values respectively correspond to t_0 =1985:Q3 —the moment in which the estimated series from 1980:Q1 onwards reaches its

[21] In the model, wealth-to-consumption ratio $W/C = 1/\rho$ can be interpreted as the price-dividend ratio on equity, because wealth W only includes internal equity of corporate businesses and the personal consumption of entrepreneurs can be thought of as dividend distributions (as in Bianchi and Bigio (2022)). The stochastic discount factor (SDF) of a representative entrepreneur whose consumption is given by C_t can be constructed as $\Lambda_t \equiv e^{-\rho \cdot t}/C_t$. Under this SDF, in steady state, the discount rate of a claim to aggregate consumption is $-\dot{\Lambda}/\Lambda = \rho$.

maximum,— and to t_1 =2018:Q3 —the endpoint of the series.[22] This path is consistent with historical estimates of the long-term trend in the world real interest rate by Del Negro et al. (2019) —who find that this trend has hovered slightly below 2% in the 1880-1980 period,— and with the widespread view first espoused by Bernanke et al. (2005) that the world real interest rate has experienced a steady and persistent decline starting in the mid-1980s. In the exercise, we assume that the economy is at its initial steady state at t_0 =1985:Q3, at which time it is hit by an unanticipated shock that reveals the entire path of the interest rate going forward. Thus, this specification implies an unanticipated, permanent decline in the interest rate of around 1.4 percentage points with a gradual transition of approximately four decades.[23]

We find that the reallocation effect is quantitatively significant, as shown in Figure 6. The fall in the interest rate initially leads to an expansion in output, with a positive reallocation of capital across entrepreneurs, a positive and increasing capital-supply effect, and a high and increasing price of capital. Eventually, however, the expansion turns into a contraction, with a negative and decreasing reallocation effect, a positive but decreasing capital-supply effect, and a high but decreasing asset price. The capital reallocation effect in the long term is about -7% and explains approximately 80% of the long-run change in output.[24] Moreover, this finding is robust to changes in key parameter values, as discussed below.

We first analyze the sensitivity to changes in the distribution of entrepreneurial productivity as captured by persistence (γ_A) and volatility (σ_A). The key takeaway is that, while volatility is quantitatively relevant for the size of the reallocation effect, persistence is not (see first two columns of Figure 7). Whereas both persistence and volatility strengthen the reallocation effect by increasing the dispersion of productivity, persistence has a countervailing effect by allowing productive entrepreneurs to accumulate wealth faster.

Second, we consider the impact of changes in the financial friction (λ) and the elasticity

[22]The stationary values themselves correspond to these moments, $r_0 = 1.97\%$ and $r_1 = 0.62\%$.

[23]In Online Appendix C.5, we perform two sensitivity exercises. First, we allow for the same overall decline in the interest rate but with an instantaneous transition. Second, we allow for the same overall decline in the interest rate but assume that it is transitory, in the sense that it is expected to return to its original stationary value according to a path that for simplicity is symmetric at t_1.

[24]As the change in the interest rate considered in this numerical exercise is discrete, we adjust Equation (12) to compute the reallocation and capital-supply effects by replacing the productivity of the marginal entrepreneur, $q \cdot r$, by the average productivity of those infra-marginal entrepreneurs that demand capital after the shock:

$$\frac{Y_t - Y_0}{Y_0} = \underbrace{\frac{\int_{A \geq \hat{A}_0} \left(A - \bar{A}_t\right) \cdot \left(k_{A,t} - k_{A,0}\right) \cdot g\left(A\right) \cdot dA}{Y_0}}_{\equiv \mathcal{R}_t} + \underbrace{\bar{A}_t \cdot \frac{K_t - K_0}{Y_0}}_{\equiv \mathcal{K}_t}, \tag{41}$$

where $\bar{A}_t \equiv \int_{\hat{A}_t}^{\hat{A}_0} A \cdot k_{A,t} \cdot g\left(A\right) \cdot dA / \int_{\hat{A}_t}^{\hat{A}_0} k_{A,t} \cdot g\left(A\right) \cdot dA$ and $\hat{A}_t = \left(r_t + \delta\right) \cdot q_t - \dot{q}_t$.

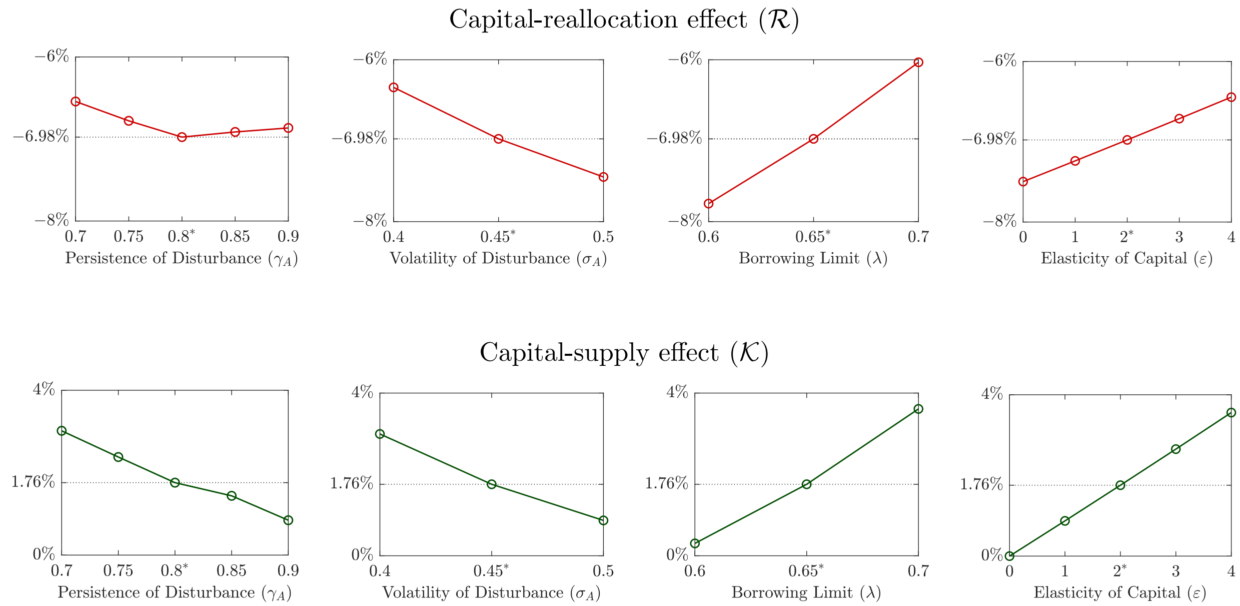


Figure 7: **Sensitivity analysis.**

of the capital supply (ε). As expected, a more severe financial friction and a lower elasticity of capital supply strengthen the reallocation effect and weaken the capital-supply effect (see third and fourth columns of Figure 7). However, the quantitative effect of the financial friction on reallocation is substantially stronger than that of the elasticity. This is partly explained by the fact that, in steady state, the price of capital and hence the distribution of wealth is independent of the capital-supply elasticity (see Equations (35)-(36)).

5 Supporting evidence

The main insight of the theoretical analysis is that, in the presence of financial frictions, the expansionary effect of a decline in the interest rate is weakened (or even overturned) by general-equilibrium reallocation effects. Whereas a fall in the interest rate always expands the investment of unconstrained (low-MPK) firms, it may actually reduce the investment of constrained (high-MPK) firms due to the general-equilibrium increase in the price of capital.[25] We test this mechanism in US firm-level data by assessing whether falling interest rates have a stronger impact on the investment of low-MPK (vis-à-vis high-MPK) firms when general-equilibrium effects are stronger.

[25]This is captured in our baseline model by Equation (11).

5.1 Empirical strategy

To fix ideas, consider the following regression on firm-level investment:

$$\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta \Delta \log q_{kt} + \gamma \Delta r_t + \Gamma' Z_{it-1} + \varepsilon_{it}, \tag{42}$$

where $\Delta \log k_{it+1}$ is the investment of firm i during period t, α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is a measure of asset-price growth in region k at time t, Δr_t is the interest rate change at time t, and Z_{it-1} is a vector of firm-level controls.

Ideally, we would want γ in regression model (42) to capture the direct effect of interest-rate changes on investment, and β to capture the general-equilibrium effect that operates through changes in asset prices. Bringing regression model (42) to the data immediately raises two questions. First, what is the right asset on which to measure general-equilibrium effects? Second, how can we isolate the changes in asset prices that are induced by changes in the interest rate?

To address these questions, we follow the macro-finance literature and focus on real estate (Chaney et al., 2012). The main advantage of doing so is that there are readily available measures of real-estate supply elasticities for the US (Saiz, 2010), which we can use to obtain exogenous variation for the strength of general-equilibrium effects across regions. This allows us to estimate Equation (42) using two-stage least squares (2SLS) estimation, where the first-stage regression is:

$$\Delta \log q_{kt} = \alpha_k + \alpha_t + \rho H_k \Delta r_t + \varepsilon_{kt}, \tag{43}$$

where H_k is the real estate supply elasticity in region k. We expect $\rho > 0$ and significant, implying that real estate prices are more sensitive to interest rate changes in regions where real estate supply is less elastic. The second-stage regression then replaces real estate prices with their predicted values from the first-stage regression.

To test directly the prediction of our model, we estimate the following extension of regression model (42) using 2SLS:

$$\begin{aligned} \Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta_1 \Delta \log q_{kt} + \gamma_1 \Delta r_t \\ + \beta_2 mpk_{it-1} \Delta \log q_{kt} + \gamma_2 mpk_{it-1} \Delta r_t + \Gamma' Z_{it-1} + \varepsilon_{it}, \end{aligned} \tag{44}$$

where mpk_{it-1} is the log of the marginal product of capital (MPK) of firm i at time $t-1$. The reallocation effect emphasized by the theory, by which changes in the interest rate affect lower- and higher-MPK firms differentially, is captured by β_2 and γ_2. Moreover, the theory predicts that $\beta_2 < 0$, so that the general-equilibrium component of the reallocation effect raises the relative investment of lower-MPK (vis-à-vis higher-MPK) firms.

5.2 Data

To estimate regression models (42) and (44), we use firm-level data from the US Compustat database at quarterly frequency. The main advantages of this dataset is that it offers quarterly data for a long time period and that it contains detailed data on firm financials. A drawback is that it only covers publicly listed firms and may therefore underestimate the relevance of financial frictions in the overall economy. We provide next a brief description of the main variables used in the analysis, and relegate a more detailed explanation to Appendix B.

We measure investment $\Delta \log k_{it+1}$ as the logarithmic change in k_{it+1}, where k_{it+1} is the real book value of the tangible capital stock of firm i at the end of period t.

We obtain quarterly real estate prices from the Federal Housing Finance Agency (FHFA) for each Metropolitan Statistical Area (MSA), and compute $\Delta \log q_{kt}$ as the logarithmic change in real estate prices in each MSA k in period t. We use real-estate supply elasticities at the MSA-level from Saiz (2010).

To proxy for the change in the interest rate Δr_t we use the real interest rate on 30-year mortgages from Freddie Mac's Primary Mortgage Market Survey, as in Chaney et al. (2012). The advantage of this measure is that long-term interest rates are arguably more relevant for borrowing decisions of firms than short-term rates. In Appendix B, we also use the monetary shocks of Jarociński and Karadi (2020), who identify monetary surprises based on high-frequency data around monetary policy announcements. The advantage of this variable is that it is by construction exogenous to investment.

We compute MPK at the firm level as the product of the output-elasticity of capital times the ratio of the firm's real value added and its real capital stock. To obtain the output-elasticity of capital at the sector (one-digit SIC industry) level, we estimate production functions using Wooldridge (2009)'s generalized method of moments.

Our final dataset covers the 1990:Q1-2019:Q2 period.

5.3 Empirical results

We estimate equations (42) and (44) and report the results in Table 2. To ease interpretation, all explanatory variables except real estate price growth and the interest rate change are standardized over the entire sample. This increases the standard errors compared to not clustering

Table 2: **Investment response to real estate (RE) prices and interest rates**

	(1)	(2)	(3)	(4)	(5)
RE price growth	0.078***	0.066	0.058**	-0.145	-0.195
	(0.023)	(0.876)	(0.023)	(0.884)	(0.800)
MPK x RE price growth			-0.053***	-0.127***	-0.095***
			(0.017)	(0.039)	(0.037)
MPK x Interest rate change			0.021	0.022	0.004
			(0.052)	(0.058)	(0.058)
Observations	129,027	100,263	129,027	100,263	100,263
R-squared	0.149	0.151	0.169	0.170	0.179
Estimation	OLS	2SLS	OLS	2SLS	2SLS
Firm controls	no	no	no	no	yes
Time sector FE	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes

Notes: In Columns (1) and (2) we report regression results from estimating the following specification: $\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta \Delta \log q_{kt} + \varepsilon_{it}$ where α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is real RE price growth in MSA k at time t, using OLS and 2SLS respectively. In Columns (3) and (4) we report regression results from estimating the following specification: $\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta_1 \Delta \log q_{kt} + \beta_2 mpk_{it-1} \Delta \log q_{kt} + \gamma_2 mpk_{it-1} \Delta r_t + \Gamma' Z_{it-1} + \varepsilon_{it}$ where α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is real RE price growth in MSA k at time t, mpk_{it-1} is the log of the marginal product of capital (MPK) of firm i at time $t-1$, Δr_t is the change in real mortgage rates at time t, and Z_{it-1} is a vector containing mpk_{it-1}, using OLS and 2SLS respectively. In column (5), we re-estimate the 2SLS regression in column (4) with an enlarged set of firm-level control variables, Z_{it-1}, containing MPK, sales growth, size, current assets as a share of total assets, total debt as a share of total assets, and the interaction of MPK with lagged real GDP growth. All explanatory variables (notably MPK) except RE price growth and the interest rate change are standardized over the entire sample. Standard errors are two-way clustered by firms and quarter. Standard errors in columns (2), (4) and (5) are bootstrapped based on 500 simulations.

and should therefore be seen as a conservative estimate.

Columns (1) and (2) respectively present the OLS and 2SLS estimations of regression model (42) without an extended set of control variables. The effect of Δr_t is absorbed by the sector-time fixed effects. We find that, on average, real estate price growth is positively associated with firm investment, although not significantly so in the 2SLS estimation. One possible interpretation of this correlation is that balance-sheet effects are dominant for the average firm.

Column (3) presents the OLS estimation results of regression model (44), also without an extended set of control variables. Consistent with our theory, general-equilibrium effects as captured by real estate prices appear to hurt the relative investment of higher-MPK firms.

In contrast, the direct effect of changes in interest rates appears to be similar for lower- and higher-MPK firms.

Column (4) presents our main specification, a 2SLS estimation of regression model (44). In the first stage, we instrument real estate price growth using the interaction between the supply elasticity and the change in the real mortgage rate, as in equation (43). The first-stage results are presented in column (1) of Appendix Table 4. The strong F-test supports the choice of our instrument and $\rho > 0$ is significant, indicating that – as expected – real estate prices are more sensitive to changes in interest rates in regions with a lower supply elasticity. In the second stage, we continue to find that the relative investment of higher-MPK firms falls in response to real estate price increases. If anything, the estimated effect is larger once the potential bias in the OLS estimate is accounted for. Column (5) shows that these results are robust to the inclusion of firm controls including sales growth, size, current assets as a share of total assets, total debt as a share of total assets, and MPK and its interactions with lagged real GDP growth. Finally, Table 5 in Appendix B shows that results are also robust to using high-frequency monetary shocks from Jarociński and Karadi (2020).

These findings suggest that the general-equilibrium component plays an important role in shaping reallocation effects.[26] In particular, the estimates in column (4) imply that the investment response to a 1.8 percentage point increase in real estate prices (equivalent to its standard deviation) is 0.23 percentage points stronger for firms that have a one standard deviation lower MPK. This is a substantial effect given that the average investment rate in the sample is 0.5 percent. This effect is reduced to 0.18 percentage points when adding all the firm controls in column (5), which is still substantial.

Finally, we analyze the dynamic impact of the general-equilibrium effect on investment by estimating a type of local projection of Equation (44) á la Jordà (2005):

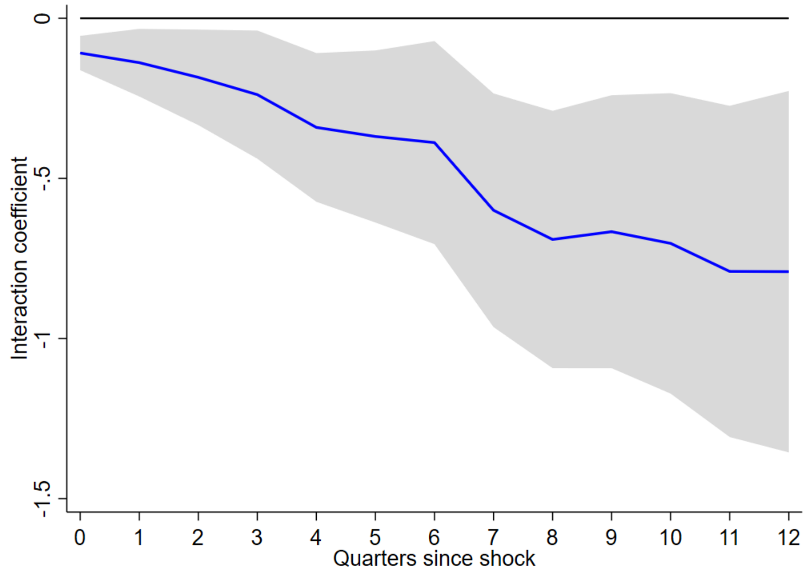
$$\begin{aligned}\log k_{it+h} - \log k_{it-1} = \alpha_{ih} + \alpha_{jth} + \beta_{1h}\Delta \log q_{kt} + \gamma_{1h}\Delta r_t \\ + \beta_{2h} mpk_{it-1}\Delta \log q_{kt} + \gamma_{2h} mpk_{it-1}\Delta r_t + \Gamma_h' Z_{it-1} + \varepsilon_{ith}, \quad (45)\end{aligned}$$

for $h \geq 0$, where we use our 2SLS procedure. The regression coefficients β_{2h} indicate the cumulative general-equilibrium effect of changing interest rates, over the period t to $t+h$, on the relative investment of higher-MPK firms. We include the full set of control variables as in column (5) of Table 2.

The results of the local projection exercise are depicted in Figure 8. Crucially, the main

[26] Note that our methodology does not allow us to measure the total effect (either directly or through general equilibrium) of interest rate changes on investment.

Figure 8: **Dynamics of differential response of investment to real estate price changes**



Notes: The figure displays the regression coefficients β_{2h} (the solid line) obtained when estimating the following specification: $\log k_{it+h} - \log k_{it-1} = \alpha_{ih} + \alpha_{jth} + \beta_{1h}\Delta \log q_{kt} + \beta_{2h} mpk_{it-1}\Delta \log q_{kt} + \gamma_{1h}\Delta r_t + \gamma_{2h} mpk_{it-1}\Delta r_t + \Gamma'_h Z_{it-1} + \varepsilon_{ith}$ for $h = 0, 1, 2, ..., 12$ and Z_{it-1} is a vector containing mpk_{it-1}. Regression is estimated using 2SLS with the interaction between the real estate supply elasticity and the real mortgage rate as instrument for real estate price growth in the first stage. The shaded area depicts the 90% confidence interval. All variables are as defined in Table 2. Standard errors are two-way clustered by firms and quarter.

finding reported in Table 2 – that the general-equilibrium effect of a fall in the interest rate reduces the relative investment of higher-MPK firms – is persistent and remains significant up to twelve quarters after the shock. The strength of this cumulative effect increases over time to around -0.8, though it is estimated with large standard errors.

6 Conclusions

In a canonical economy with heterogeneous entrepreneurs, financial frictions, and an imperfectly elastic supply of capital, a fall in the interest rate has ambiguous effects on economic activity. In partial equilibrium, a lower interest rate raises aggregate investment both by relaxing borrowing constraints and by prompting relatively less productive entrepreneurs to increase their investment. In general equilibrium, however, this higher demand for capital raises its price and crowds-out investment by more productive (financially constrained) entrepreneurs: ultimately, there is a reallocation of capital from more to less productive entrepreneurs. When this general-equilibrium effect is strong enough, a fall in the interest rate can become contractionary.

Our mechanism requires that the price of capital depend on local economic conditions, a common feature in most macroeconomic models that emphasize the role of financial markets and balance sheets. Like most of the literature, we have focused here on real estate to show that the reallocation effect is quantitatively significant and to provide empirical evidence using US firm-level data. The theory is more general, however, as our key insight could arise in relation to any factor of production that requires credit and that is in scarce supply (e.g., skilled labor that requires working capital).

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A Proofs

Proof of Proposition 1. From the capital market clearing condition and the definition of aggregate output, we have:

$$Y = \int_{q\cdot R}^{1} (A - q \cdot R) \cdot k_A(q,R) \cdot g(A) \cdot dA + q \cdot R \cdot K^S(q), \tag{46}$$

where $k_A(q,R)$ is given by (5). Thus, the derivative of aggregate output, Y, with respect to the interest rate, R, is given by:

$$\frac{dY}{dR} = \int_{q\cdot R}^{1} (A - q \cdot R) \cdot \frac{dk_A(q,R)}{dR} \cdot g(A) \cdot dA + q \cdot R \cdot \frac{dK^S(q)}{dR}, \tag{47}$$

where $\frac{dk_A(q,R)}{dR}$ is given by (11). This yields Equation (12) in the text.

Next, assume $\chi(\cdot)$ is twice differentiable at the equilibrium capital supply, so that the capital supply elasticity at equilibrium is given by:

$$\varepsilon = \frac{q}{\chi''\left(\chi'^{-1}(q)\right) \cdot \chi'^{-1}(q)}. \tag{48}$$

From the capital market clearing condition and Equation (5), therefore, we have:

$$\left|\frac{dq}{dR}\right| = \frac{q \cdot \frac{w}{1-\lambda} \cdot g(q \cdot R) + q \cdot \int_{q\cdot R}^{1} \frac{\frac{\lambda \cdot A}{R^2}}{\left(q - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot g(A) \cdot dA}{\varepsilon \cdot K^S(q) + R \cdot \frac{w}{1-\lambda} \cdot g(q \cdot R) + q \cdot \int_{q\cdot R}^{1} \frac{1}{\left(q - \frac{\lambda \cdot A}{R}\right)^2} \cdot w \cdot g(A) \cdot dA}. \tag{49}$$

That is, all else equal (i.e., holding the equilibrium before the interest rate change fixed), $\left|\frac{dq}{dR}\right|$ is decreasing in the capital supply elasticity, ε. Note that changing the capital supply elasticity at equilibrium entails rotating the capital supply schedule around the equilibrium point (K^S, q); see below for an example.

Now, consider two economies that have the same equilibrium allocations and are identical in all respects except for the capital supply elasticity at equilibrium, ε. Since $\frac{dk_A(q,R)}{dR}$ is increasing in $\left|\frac{dq}{dR}\right|$, it follows that all else equal the capital-reallocation effect (the first term in Equation (47)) is stronger in the economy with lower ε. Since $\frac{dK^S(q)}{dR} = -\frac{\varepsilon \cdot K^S(q)}{q} \cdot \left|\frac{dq}{dR}\right|$, by combining with Equation (49), it follows that the capital-supply effect (the second term in Equation (47)) is weaker in the economy with lower ε. Hence, $\frac{dY}{dR}$ is less negative in the economy with lower ε.

We are left to prove the last statement of the proposition. For this, consider the case of $\lambda = 0$. Equation (12) becomes:

$$\frac{dY}{dR} = \int_{q\cdot R}^{1} (A - q \cdot R) \cdot \frac{w}{q^2} \cdot \left|\frac{dq}{dR}\right| \cdot g(A) \cdot dA + q \cdot R \cdot \frac{dK^S(q)}{dR}. \tag{50}$$

Therefore, for $\lambda = 0$ and ε small enough, $\frac{dY}{dR} > 0$. For any such ε, by continuity of $\frac{dY}{dR}$ in λ,

$\exists\bar{\lambda}_{\varepsilon} > 0$ such that $\frac{dY}{dR} > 0$ if $\lambda < \bar{\lambda}_{\varepsilon}$.

Finally, to produce Figure 3, we consider the following parameterization of the capital-supply schedule (which is equivalent to parameterizing the cost of capital production):

$$K^S(q;\varepsilon) = \max\left\{0,\ \bar{K}\cdot\left(1+\varepsilon\cdot\frac{q-\gamma}{\gamma}\right)\right\},\ \varepsilon\in[0,\infty), \tag{51}$$

where γ is a parameter that, given the interest rate R, satisfies:

$$\bar{K} = \int_{\gamma\cdot R}^{1} \frac{1}{\gamma - \frac{\lambda\cdot A}{R}}\cdot w\cdot dG(A) \tag{52}$$

Note that at the interest rate R the equilibrium allocations are independent of ε, the elasticity of the capital supply; in particular, $K^S = \bar{K}$ and $q = \gamma$. ■

Proof of Proposition 2. Consider the problem of the social planner:

$$\max_{\{k_A\}} \int_0^1 A\cdot k_A\cdot dG(A) - R\cdot\left(\chi\left(\int_0^1 k_A\cdot dG(A)\right) - w\right)$$

subject to:

$$R\cdot\left(\chi'\left(\int_0^1 k_A\cdot dG(A)\right)\cdot k - w\right) \le \lambda\cdot A\cdot k_A \quad (\gamma_A\cdot g(A)),$$
$$0 \le k_A \quad (\omega_A\cdot g(A)).$$

In parentheses, we denote the multipliers on the constraints. We also suppose that the cost of capital production, $\chi(\cdot)$, is strictly convex.

The first-order conditions to the planner's problem are given by:

$$\frac{A}{R} - q - \chi''\left(K^S\right)\cdot\int_0^1 \gamma_{\widehat{A}}\cdot k_{\widehat{A}}\cdot dG\left(\widehat{A}\right) = \gamma_A\cdot\left(q - \frac{\lambda\cdot A}{R}\right) - \omega_A \quad \forall A, \tag{53}$$

which together with the Kuhn-Tucker conditions characterize the solution to the problem. Since at the planner's allocation $q = \chi'\left(K^S\right) \ge \frac{\lambda}{R}$, it follows that there exists $0 < \widetilde{A} < 1$ such that $\omega_A > 0$ for all $A < \widetilde{A}$, $\gamma_A > 0$ for all $A > \widetilde{A}$, and $\gamma_{\widetilde{A}} = \omega_{\widetilde{A}} = 0$. After some algebra:

$$\widetilde{A} = q^{SP}\cdot R + \frac{\chi''\left(K^S\right)\cdot\int_{\widetilde{A}}^1 \left(A - q^{SP}\cdot R\right)\cdot\frac{1}{\left(q^{SP}-\frac{\lambda\cdot A}{R}\right)^2}\cdot w\cdot dG(A)}{1+\chi''\left(K^S\right)\cdot\int_{\widetilde{A}}^1 \frac{1}{\left(q^{SP}-\frac{\lambda\cdot A}{R}\right)^2}\cdot w\cdot dG(A)}, \tag{54}$$

where:

$$q^{SP} = \chi'\left(K^S\right) = \chi'\left(\int_{\widetilde{A}}^1 \frac{1}{q^{SP}-\frac{\lambda\cdot A}{R}}\cdot w\cdot dG(A)\right). \tag{55}$$

It follows immediately that $\widetilde{A} > q^{CE}\cdot R$ and $q^{CE} > q^{SP}$.

Next, note that when R falls, the planner could always ensure the same equilibrium allocations (with unchanged $\widetilde{A}$ and q^{SP}), in which case all supra-marginal entrepreneurs' financial constraints become weakly slack. Moreover, the planner would never reduce K^{SP} and Y^{SP} in response to a fall in R, since the aggregate productivity of capital is higher than R times the marginal cost of producing capital, χ', before the change in R and because R declines. Thus, the adjustment in $\widetilde{A}$ and q^{SP} (given by Equations (54) and (55)) must be such that the fall in R is expansionary. ■

B Empirical Appendix

This Appendix describes the dataset and definitions of the variables used in the empirical analysis in Section 5 and presents several extensions and robustness checks of our main results.

B.1 Dataset construction and definition of variables

The dataset is created at the firm level using data from the US Compustat database at quarterly frequency, combined with information on real estate prices, housing supply elasticities, and interest rates.

Real capital. We compute investment as $\Delta log(k_{it+1})$, where k_{it+1} is the real capital stock of firm i at the end of quarterly period t. We measure the real capital stock using the perpetual inventory method, following Ottonello and Winberry (2020). For each firm, we set the initial value of k_{it+1} equal to gross plant, property, and equipment (PPEGTQ) for the first period this variable is reported in the US Compustat database. For subsequent periods, we then compute k_{it+1} by adding the cumulative change in net investment to this initial value. We measure net investment for each period as the change in net plant, property, and equipment (PPENTQ). If firms have missing observations for PPENTQ in between two periods with non-missing observations, we estimate the missing value using a linear interpolation of the two non-missing observations. We construct real values of capital and investment by deflating using the output price deflator for all workers from the Bureau of Labor Statistics (BLS), obtained from: `https://fred.stlouisfed.org/series/IPDNBS`.

MPK. We estimate the marginal product of capital for each firm in two steps. First, we estimate a Cobb-Douglas production function for each firm using the method in Wooldridge (2009). This approach uses material inputs as a proxy variable to control for unobserved productivity and extends the method in Levinsohn and Petrin (2003) by using GMM estimation to estimate the individual equations. Specifically, we use GMM-IV estimation to run a regression of $log(y_t)$ on the following explanatory variables: $log(k_t)$, $log(l_t)$, $log(k_{t-1})$, $log(m_{t-1})$, $log(k_{t-1})log(m_{t-1})$, $log(k_{t-1}^2)$, $log(m_{t-1}^2)$, $log(k_{t-1}^2)log(m_{t-1})$, $log(k_{t-1})ln(m_{t-1}^2)$, $log(k_{t-1}^3)$, $log(m_{t-1}^3)$, a time effect τ_t, and an error term ϵ_t, where y is real value added, k is real capital, l is the real wage bill, and m is real materials. The real wage bill $log(l_t)$ is chosen to be the endogenous variable and this is instrumented using its lagged value $log(l_{t-1})$. Estimation is done with GMM across a pooled sample of all firms in a one-digit SIC industry, to allow for different production functions across industries. Standard errors are clustered at the firm level. This estimation then generates the elasticities for capital and labor. We refer to Wooldridge (2009) for details. Let α_j be the industry-specific elasticity for capital. In the second step, we then use this estimate of α_j to compute the marginal product of capital MPK_{it} for each firm. Starting with a Cobb-Douglas production function $Y_{it} = A_{it}K_{it}^{\alpha_j}L_{it}^{1-\alpha_j}$, where Y is real value added, K is real capital and L is the wage bill, the marginal product of capital MPK_{it} can be computed as follows: $dY_{it}/dK_{it} = \alpha_j A_{it}K_{it}^{\alpha_j-1}L_{it}^{1-\alpha_j} = \alpha_j Y_{it}/K_{it}$.

We compute the wage bill as the number of employees from Compustat times the average wage of all workers for the year obtained from the US Social Security Administration, available at: `https://www.ssa.gov/oact/cola/AWI.html#Series`. We divide this number by four to obtain the wage bill for the quarter. We then compute value added as the sum of operating income before depreciation (oibdp), obtained from Compustat, and the wage bill. Material costs is computed using data from Compustat as Sales (saleq) minus value added. We convert all value added components into real terms using the output price deflator from the BLS.

Real estate prices. As measure of the evolution of real estate prices we use the MSA-level All-transactions house price index at the quarterly level from the Federal Housing Finance Agency (FHFA), downloaded from: `https://www.fhfa.gov/DataTools/Downloads/Documents/HPI/HPI_AT_metro.csv`. We measure real estate price changes as the one-period logarithmic change in this house price index.

Interest rates. As measure of interest rate changes, we use the quarterly change in the real mortgage rate. This variable proxies for the change in real long-term interest rates. We construct this variable by taking the difference between the (annualized) quarterly change in average nominal interest rate on 30-year fixed rate mortgages from Freddie Mac's Primary Mortgage Market Survey, available at: `https://fred.stlouisfed.org/series/MORTGAGE30US`, and the (annualized) CPI inflation rate over the past quarter, available from `https://fred.stlouisfed.org/series/FPCPITOTLZGUSA`. This is our baseline measure. As an alternative proxy for interest rate changes we use the high-frequency monetary shock from Jarociński and Karadi (2020) obtained with median rotation sign restrictions, computed by aggregating daily shocks during each quarter.[27] This shock extracts the first principal component of surprises in interest rate futures with maturities from one month to one year, within a short window of 30 minutes around the times of the Federal Reserve's monetary policy announcements. They extend the approach in Nakamura and Steinsson (2018) by stripping out information shocks from policy shocks using a VAR model with sign restrictions.

Land supply elasticity. Local real-estate supply elasticities at the MSA level, denoted H_k, for a total of 95 MSAs are obtained from Table VI in Saiz (2010), which is available from `https://academic.oup.com/qje/article/125/3/1253/1903664?login=true`. These elasticities capture the amount of local land that can be developed and are estimated using satellite-generated images of the terrain. We transform this dataset into the latest available MSA codes, as of March 2020, as defined by the United States Office of Management and Budget (OMB), by averaging elasticities across merged MSAs in those few cases where MSA definitions have changed because of the merging of several regions.

Control variables. Following Ottonello and Winberry (2020), we construct several firm-level control variables that may be correlated with firm investment using data from the US Compustat database. These include: financial leverage, computed as the ratio of short-term and long-term debt (DLCQ+DLTTQ) divided by the book value of total assets (ATQ); sales growth, computed as the one-period logarithmic change in net sales (SALEQ), after expressing sales in real terms using the output price deflator from the BLS; firm size, computed as the natural logarithm of total assets (ATQ), after expressing total assets in real terms using the output price deflator from the BLS; the ratio of current assets (ACTQ) as a share of total assets (AT); and the interaction between MPK and real GDP growth, where real GDP growth is computed using data from the US Bureau of Economic Analysis (BEA) obtained from FRED at `https://fred.stlouisfed.org/series/GDPC1`. All control variables are included using their lagged terms.

[27]The original dataset called jarocinski-karadi.zip is available from the AEJ Macro website and provides data for the period 1990 to 2016. We are grateful to Peter Karadi for providing us with an updated version of the dataset that extends the monetary shocks until 2019. The dataset provides daily monetary shocks. We transform these daily shocks into quarterly shocks by aggregating the daily shocks for each quarter.

Table 3: **Summary statistics**

	Mean	Median	SD	Min	Max	N
Investment	0.005	-0.002	0.068	-0.662	0.798	175,984
MPK	0.120	-0.033	1.005	-2.385	3.656	175,984
RE price growth	0.008	0.009	0.018	-0.167	0.138	18,442
Change in real mortgage rate	-0.000	-0.000	0.007	-0.033	0.031	118
Monetary shock (in %)	-0.002	0.003	0.080	-0.321	0.198	118
Housing elasticity	1.846	1.650	0.970	0.600	5.450	89

Notes: Investment is the logarithmic change in real capital of the firm. Capital is deflated using the output price deflator. MPK is the marginal product of capital of the firm. Real estate price growth is the real change in the FHFA house price index. Change in real mortgage rate is the change over the quarter in the 30-year mortgage rate from FHFA, adjusted for CPI inflation. Monetary shock is the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over each quarter. Real estate elasticity is the elasticity of real-estate supply at the MSA level from Saiz (2010).

Sectors. All regressions include sector-time fixed effects. To this end, we construct sectoral dummies for the following sectors based on 4-digit SIC classification codes: agriculture, forestry, and fishing (SIC<999); mining (SIC 1000-1499); construction (SIC 1500-1799); manufacturing (SIC 2000-3999); transportation, communications, electricity, gas, and sanitary services (SIC 4000-4999); wholesale trade (SIC 5000-5199); retail trade (SIC 5200-5999), and services (SIC 7000-8999).

Sample selection. We exclude firms incorporated outside the US, and firms in finance, insurance, and real estate sectors (SIC 6000-6799), utilities (SIC 4900-4999), nonoperating establishments (SIC 9995), and industrial conglomerates (SIC 9997). We drop firm-quarter observations that satisfy one of following criteria: negative capital (K) or assets (ATQ); acquisitions (AQCY) larger than 5% of assets; investment rate in the top and bottom 0.5% percent of the distribution; MPK is in the top and bottom 0.5% of the distribution; net current assets as a share of total assets exceeds 10 in absolute value; ratio of net fixed assets (PPENTQ) to total assets (ATQ) is less than 10%; leverage ratio above 10 or negative; quarterly real sales growth exceeds 1 in absolute value; and negative sales or negative ratio of liquid assets (ACTQ-LCTQ) to total assets (ATQ). In addition, we winsorize leverage at each of the 0.5% tails of its distribution. The final dataset covers the period 1990Q1 until 2019Q2, for a total of 175,984 firm-quarter observations across 89 MSAs.

B.2 Descriptive statistics

Table 3 reports the descriptive statistics of our main variables.

In our sample, firm investment shows much heterogeneity, ranging from a rate of -0.66 to $+0.80$, with a median of 0. The marginal product of capital (MPK) also varies strongly

Table 4: **RE price response to land supply elasticity and interest rates**

	(1)	(2)	(3)	(4)
Elasticity · Interest rate change	0.064***		0.122***	-0.021***
	(0.013)		(0.041)	(0.004)
1st quartile of elasticity · Interest rate change		-0.158***		
		(0.039)		
2nd quartile of elasticity · Interest rate change		-0.075*		
		(0.040)		
3rd quartile of elasticity · Interest rate change		-0.052*		
		(0.030)		
$(\text{Elasticity})^2$ · Interest rate change			-0.012*	0.003***
			(0.007)	(0.001)
Observations	9,206	9,206	9,206	9,206
R-squared	0.493	0.493	0.493	0.494
F-test	24.674	5.852	13.597	29.363
p-value (F-test)	0.000	0.001	0.000	0.000

Notes: Regression results from estimating the following specification: $\Delta \log q_{kt} = \alpha_k + \alpha_t + \rho f(H_k)\Delta r_t + \varepsilon_{kt}$. $\Delta \log q_{kt}$ is the change in real real estate prices in MSA k during quarter t, α_k is an MSA regional effect, α_t is a quarterly period fixed effect, and H_k is the elasticity of real estate supply in MSA k. In column (1), we proxy for interest rate changes Δr_t using the change in real mortgage rates. In column (2), we allow for a nonlinear impact of the real estate supply elasticity by using interactions between quartiles of the real estate supply elasticity and the change in real mortgage rates. In column (3), we instrument real estate price changes using interactions between the real estate supply elasticity and its squared term and the change in real mortgage rates. In column (4), we use the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, aggregated over the quarterly period, as proxy for the change in interest rates. Standard errors are two-way clustered by MSA and quarter.

across firms, ranging from -2.4 to $+3.7$, with a median of 0. Real estate price growth across MSAs varies from -16.7 percent to $+13.8$ percent indicating that our sample includes both housing booms and housing busts. The interest rate (shocks) data indicate that our sample period includes both periods of interest rate expansions and contractions. Annual changes in real mortgage rates range from -3.3 to $+3.1$ percentage points, with a median of 0, while short-term monetary shocks range from -0.3 to $+0.2$ percentage points, with a median of 0. Real-estate supply is quite elastic, with an elasticity of 1.9 for the average MSA.

B.3 First-stage results

Table 4 presents the first-stage regressions results. The dependent variable is the real estate price growth at the MSA-quarter level.

In column (1), changes in real estate prices are instrumented using the interaction between housing supply elasticity and the change in the real mortgage rate, following Chaney et al. (2012). This is our baseline first-stage regression. In column (2), we allow for a nonlinear impact of the supply elasticity by using interactions between quartiles of the supply elasticity and the change in real mortgage rates to instrument for real estate price growth. In column (3), we instrument real estate price changes using interactions between the land supply elasticity and its squared term and the change in real mortgage rates. In column (4), real estate price changes are instrumented using interactions between the real estate supply elasticity and its squared term and the high-frequency monetary shock from Jarociński and Karadi (2020), obtained with median rotation sign restrictions, and aggregated over the quarterly period. All regressions are based on 9,206 MSA-quarter observations and include MSA fixed effects and quarterly period fixed effects. Standard errors are two-way clustered by MSA and quarter.

B.4 Robustness tests

Appendix Table 5 presents several robustness checks of our main results. We first consider non-linear specifications of the first stage regressions because existing research has shown that the effects of interest rates on real estate prices is non-linear in the supply elasticity (e.g., Chaney et al. (2012), Kaplan et al. (2017)). In column (1), we report the second-stage results obtained after replacing the real estate supply elasticity with its quartiles in the first-stage regression, following Chaney et al. (2012). The first-stage results of this regression are reported in column (2) of Appendix Table 4. We find that effects of interest rate changes on real estate prices are particularly pronounced in highly inelastic regions.

Table 5: **Investment response to RE prices and interest rates: Robustness checks**

	(1)	(2)	(3)
RE price growth	-0.273	-0.188	0.491
	(0.792)	(0.634)	(0.402)
MPK x RE price growth	-0.095***	-0.095***	-0.091***
	(0.037)	(0.036)	(0.036)
MPK x Interest rate change	0.005	0.004	0.004
	(0.060)	(0.059)	(0.005)
Observations	100,263	100,263	100,263
R-squared	0.179	0.179	0.179
Estimation	2SLS	2SLS	2SLS
Firm controls	yes	yes	yes
Time sector FE	yes	yes	yes
Time clustering	yes	yes	yes

Notes: This table reports regression results from estimating the following specification: $\Delta \log k_{it+1} = \alpha_i + \alpha_{jt} + \beta_1 \Delta \log q_{kt} + \beta_2 mpk_{it-1} \Delta \log q_{kt} + \gamma_2 mpk_{it-1} \Delta r_t + \Gamma' Z_{it-1} + \varepsilon_{it}$ where α_i is a firm fixed effect, α_{jt} is a sector-time fixed effect, $\Delta \log q_{kt}$ is real RE price growth in MSA k at time t, mpk_{it-1} is the log of the marginal product of capital (MPK) of firm i at time $t-1$, Δr_t is the change in interest rates at time t, and Z_{it-1} is a vector containing mpk_{it-1}. All regressions are estimated using 2SLS. In column (1), we use the change in real mortgage rates as proxy for the change in interest rates, and we instrument RE price changes using interactions between quartiles of the land supply elasticity and the change in real mortgage rates. In column (2), we use the change in real mortgage rates as proxy for the change in interest rates, and we instrument RE price changes using interactions between the land supply elasticity and its squared term and the change in real mortgage rates. In column (3), we use the high-frequency monetary shock as proxy for the change in interest rates, and we instrument RE price changes using interactions between the land supply elasticity and its squared term and the high-frequency monetary shock. All regressions include as control variables lagged MPK, sales growth, size, current assets as a share of total assets, total debt as a share of total assets, and the interaction of lagged MPK with lagged real GDP growth. All explanatory variables (notably MPK) except RE price growth and the interest rate shock are standardized over the entire sample. Standard errors are two-way clustered by firms and quarter. Standard errors are bootstrapped based on 500 simulations.

In column (2), we report the results obtained when including the squared term of the housing supply elasticity in the first-stage regression. The first-stage results of this regression are reported in column (3) of Appendix Table 4. Our main results on the interaction between MPK and real estate price growth are robust to both of these changes. In column (3), we use the high-frequency monetary shock as proxy for the change in interest rates, and we instrument real estate price changes using interactions between the land supply elasticity and its squared term and the high-frequency monetary shock. The first stage is reported in column (4) of Appendix Table 4. The first stage results indicates that the effect of interest rates on real estate prices has the expected sign only for regions with highly inelastic supply. Nevertheless, the second stage results are virtually unaltered when using this instrument. We continue to find that the investment response to an increase in real estate prices is stronger for firms with a lower MPK.

C Online Appendix

C.1 Default risk and credit spreads

In this appendix, we extend our analysis to incorporate heterogeneity in entrepreneurial funding costs. To do so, we incorporate idiosyncratic risk into the entrepreneurial production technology. If the entrepreneur invests k at $t = 0$, then she succeeds with probability p at $t = 1$, in which case she receives output $A \cdot k$; otherwise, she fails with probability $1 - p$ and receives output 0. We assume that entrepreneurs differ both in the probability of success p and in the productivity A in case of success. As before, we assume an entrepreneur can pledge at most a fraction λ of her output to outsiders.

Since it is without loss of generality to assume that the entrepreneur does not default upon success, it follows that $R \cdot (1 + \frac{1-p}{p})$ is the interest rate at which an entrepreneur of type (A, p) can raise funds from lenders, where $R \cdot \frac{1-p}{p}$ is the spread over the risk-free rate due to the possibility of default. Let $e(A, p) = p \cdot A$ denote the expected productivity of an entrepreneur of type (A, p). The distributions over p and A induce a distribution $\widetilde{G}(\cdot)$ over the expected productivity e, which we will assume has full support on $[0, 1]$. Observe that the model can accommodate either positive or negative correlation between p and A.

Entrepreneurial demand for capital can be shown to take the form:

$$k_{(A,p)}(q, R) = k_e(q, R) \begin{cases} = 0 & \text{if } \frac{e}{R} < q \\ \in \left[0, \frac{1}{q - \frac{\lambda \cdot e}{R}} \cdot w\right] & \text{if } \frac{e}{R} = q \\ = \frac{1}{q - \frac{\lambda \cdot e}{R}} \cdot w & \text{if } \frac{\lambda \cdot e}{R} < q < \frac{e}{R} \\ \infty & \text{if } q \leq \frac{\lambda \cdot e}{R} \end{cases}; \tag{56}$$

that is, the demand of entrepreneur (A, p) depends only on her expected productivity $e = p \cdot A$, and not on p and A separately. As a result, q must be such that:

$$K^S(q) = K = K^D(q, R) = \int_0^1 k_e(q, R) \cdot d\widetilde{G}(e). \tag{57}$$

Aggregate output is in turn given by:

$$Y = \int_0^1 e \cdot k_e(q, R) \cdot d\widetilde{G}(e). \tag{58}$$

By comparing Equations (56)-(58) with their counterparts in the baseline model, it follows immediately that Proposition 1 holds in this extended setting as well (we only need to relabel A with e). Importantly, note that for a given distribution over expected productivity $\widetilde{G}$, the correlation of p and A is irrelevant for the equilibrium q, K, and Y.

C.2 Closed economy: endogenous interest rates and savings gluts

Throughout our main analysis, we considered a small open economy that experienced an exogenous fall in the world interest rate. In this Appendix, we show that none of our main insights would change if the economy were closed and the fall in the interest rate were the result of a savings glut, i.e., an increase in the economy's desired savings.

Suppose now that the economy is closed, that the agents preferences are given by:

$$U = E_0\{c_0 + \beta \cdot c_1\} \tag{59}$$

for some $\beta \in (0,1)$, and that the capitalists have an endowment $w^C > 0$ of the consumption good at $t = 0$. Given these adjustments, we next show that the main results from our baseline setting can be obtained by raising the desired savings in this economy.

Proposition 3 *The effects of a fall in the interest rate, R, as described in Proposition 1 are isomorphic to those of an increase in w^C and/or β.*

In what follows, we illustrate the proof of this result. First, note that the equilibrium interest rate, R, must be weakly greater than β^{-1}. Otherwise, there would be a positive credit demand but no savings, as all agents who do not invest in capital would want to consume; hence, the credit market would not clear.

Second, observe that, given prices $\{q, R\}$, the aggregate savings of the savers (i.e., the capitalists and entrepreneurs with productivity $A < q \cdot R$) are given by:

$$S(q,R) \begin{cases} = w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R) & \text{if } R > \beta^{-1}, \\ \in [0,\ w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot R)] & \text{if } R = \beta^{-1} \end{cases}. \tag{60}$$

Equation (60) states that if $R > \beta^{-1}$, then the savers save all their resources, which are given by their endowments of the consumption good plus the profits of the capitalists. If $R = \beta^{-1}$, then the savers are indifferent between saving and consuming these resources. As a result, the credit market clearing condition is given by:

$$S(q,R) = \int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A), \tag{61}$$

which together with Equations (5), (6), (8), (9) and (10), characterizes the equilibrium.

Lastly, observe that the aggregate credit demand can be expressed as:

$$\int_{q \cdot R}^{1} b_A(q,R) \cdot dG(A) = q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{62}$$

since the entrepreneurs who invest in capital use all of their endowment plus borrowing to finance purchases of capital.

Therefore, we can immediately see that there are two possibilities in equilibrium.

Case 1. Consider a candidate equilibrium where the interest rate, R, is equal to β^{-1}. For this to be an equilibrium, it must be that:

$$w^C + q \cdot K^S(q) - \chi(K^S(q)) + w \cdot G(q \cdot \beta^{-1}) \geq q \cdot K^S(q) - w \cdot (1 - G(q \cdot R)), \tag{63}$$

which holds if and only if:

$$w^C + w \geq \chi(K^S(q)), \tag{64}$$

where the equilibrium price of capital, q, clears the capital market:

$$K^S(q) = \int_{q \cdot R}^{1} k_A(q, \beta^{-1}) \cdot dG(A). \tag{65}$$

It is therefore immediate that in this case the effects of an increase β on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3. Moreover, observe that this candidate is an equilibrium if w^C and/or β are large enough.

Case 2. Consider a candidate equilibrium where the interest rate R is above β^{-1}. This candidate is an equilibrium if at $R = \beta^{-1}$ the inequality (64) is violated, i.e., if w^C and/or β are small. Hence, in this case, the equilibrium prices $\{q, R\}$ are such that:

$$w^C + w = \chi(K^S(q)), \tag{66}$$

and

$$K^S(q) = \int_{q \cdot R}^{1} k_A(q, R) \cdot dG(A). \tag{67}$$

Here, a rise in w^C raises the capital price (as $\chi(K^S(q))$ is increasing in q) and lowers the interest rate (to offset the effect of a higher q that depresses capital demand). Hence, the effects of an increase in w^C on the aggregate capital and output are isomorphic to those of a fall in R analyzed in Section 3.

Lastly, note that if the equilibrium is initially in Case 2, then an increase in w^C eventually moves the equilibrium into Case 1.

C.3 Our mechanism in the Kiyotaki-Moore model

In this Appendix, we show that the capital-reallocation effects induced by falling interest rates that we emphasized through the main text are also present in the class macro-finance model of Kiyotaki and Moore (1997).

Time is now infinite, $t = 0, 1, ...$ Assume, for simplicity, that all entrepreneurs in the modern sector have the same productivity $A \in (0, 1)$, and that the capital stock is fixed at $\bar{K} > 0$. Thus, aggregate output in any period t depends solely on the allocation of capital between the modern and traditional sectors:

$$Y_t = A \cdot K_t + a \cdot f(\bar{K} - K_t), \tag{68}$$

where K_t denotes the aggregate stock of capital employed in the modern sector at time t.

We focus on equilibria in which the traditional sector is active in all periods and, hence, its demand for capital is given by:

$$\frac{a \cdot f'\left(\bar{K} - K_{t+1}\right) + q_{t+1}}{q_t} = R, \tag{69}$$

i.e., the return to capital within the traditional sector must equal the interest rate.

As in the static model, we introduce a financial friction by assuming that – in any period – an entrepreneur can walk away with a fraction $1 - \lambda$ of her resources, which now include her output and the market value of her capital. It thus follows that entrepreneurs face the following borrowing constraint:

$$R \cdot B_t \leq \lambda \cdot (A + q_{t+1}) \cdot K_{t+1}, \tag{70}$$

where B_t and K_{t+1} respectively denote entrepreneurial borrowing and investment in period t.[28] Note that, since all entrepreneurs are identical, B_t and K_{t+1} also represent aggregate borrowing and investment in the modern sector.

In any period t, the net worth of entrepreneurs equals the sum of their output and the market value of their capital minus repayments to creditors: $A \cdot K_t + q_t \cdot K_t - R \cdot B_{t-1}$. We assume that entrepreneurs consume a fraction $1 - \rho$ of this net worth in every period, where $\rho \cdot R < 1$.[29] This ensures, in the spirit of Kiyotaki and Moore (1997), that the financial constraint holds with equality in all periods. As a result, the modern-sector demand for capital is given by:

$$K_{t+1} = \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot (1 - \lambda) \cdot (A + q_t) \cdot K_t, \tag{71}$$

where we make parametric assumptions to ensure that both sectors are active in a neighborhood of the steady state.[30]

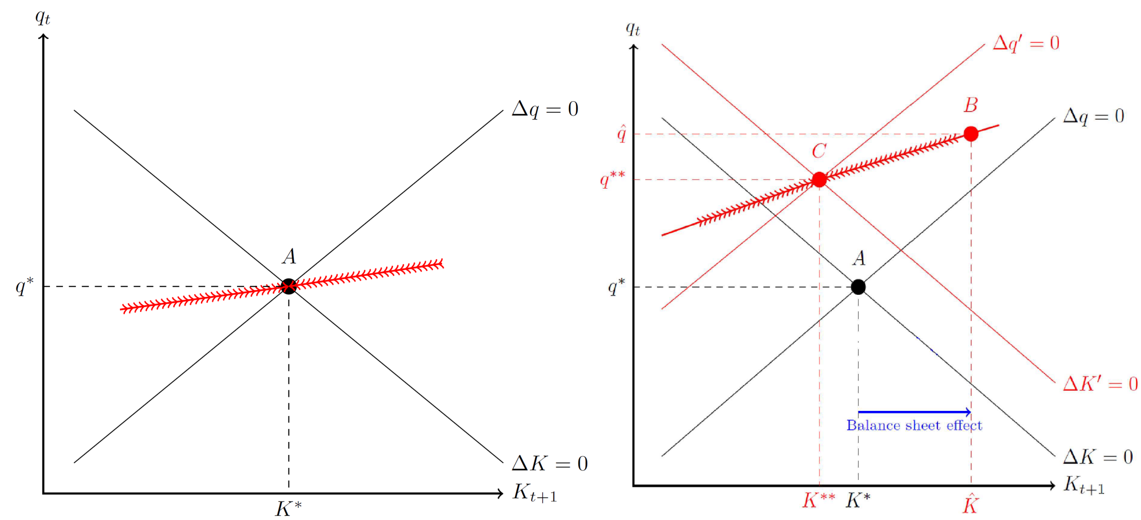
Thus, given an initial value for $K_0 > 0$ and a no bubbles condition on the price of capital, Equations (69) and (71) fully characterize the equilibrium of this economy. Panel (a) of Figure 9 portrays the equilibrium dynamics with the help of a phase diagram in the (K_{t+1}, q_t)-space. The $\Delta q = 0$ locus depicts all the combinations of K_{t+1} and q_t for which Equation (69) is satisfied with $q_t = q_{t+1}$. The locus is upward sloping because a higher level of modern-sector investment, K_{t+1}, is associated with a higher productivity of capital in the traditional sector and – since capital is priced by this sector – with a higher level of q_t. The $\Delta K = 0$ locus depicts instead all the combinations of K_{t+1} and q_t for which Equation (69) is satisfied with $K_t = K_{t+1}$. The locus is downward sloping because a higher level of modern-sector investment,

[28] In Kiyotaki and Moore (1997), the output of investment is not pledgeable but the resale value of capital is fully pledgeable. Although our results would also go through under that specification, we have chosen the current specification in order to preserve symmetry with the baseline model of Section 2.

[29] E.g., it is sufficient to assume that entrepreneurs have log-preferences, i.e., $U^E = \sum_{t=0}^{\infty} \rho^t \cdot log(c_t)$. Note that the preferences of other agents (i.e., capitalists and traditional investors) are irrelevant for the evolution of q_t, K_t and Y_t.

[30] In particular, if K_0 is close to steady state, this requires that:

$$\frac{a \cdot f'(0)}{R - 1} > \frac{R \cdot \rho \cdot (1 - \lambda) + \lambda}{R - R \cdot \rho \cdot (1 - \lambda) - \lambda} \cdot A > \frac{a \cdot f'(\bar{K})}{R - 1}.$$



(a) Convergence to steady-state

(b) Effects of unexpected fall in R

Figure 9: **Equilibrium dynamics and balance sheet effects.** The figure illustrates a phase diagram for the joint evolution of the price of capital and the stock of capital in the modern sector. The saddle path of the system is depicted by a red curve with arrows pointing to the steady state: the left panel depicts the dynamics before the unexpected decline in the interest rate, whereas the right panel depicts the dynamics after it.

K_{t+1}, is only affordable to constrained entrepreneurs if the equilibrium price of capital, q_t, is lower. As the figure shows, the system displays saddle-path dynamics. From an initial condition $K_0 < K^*$, both K and q increase monotonically as the economy transitions to the steady state and modern-sector entrepreneurs accumulate net worth. The opposite dynamics follow from an initial condition $K_0 > K^*$.

The right-hand panel of Figure 9 portrays the response to a permanent and unanticipated decline in R in a given period t_0. In response to a lower R, both loci shift upwards. The $\Delta q = 0$ locus shifts up because the traditional sector's willingness to pay for capital increases alongside the net present value of dividends; the $\Delta K = 0$ also shifts up because entrepreneurs' ability to pay for capital increases as lower interest rates relax their borrowing constraint. The presence of financial frictions, however, mitigates the shift in the $\Delta K = 0$ locus. Thus, as the figure shows, a decline in R triggers an increase in the steady-state price of capital to q^{**}, and a reduction in the capital employed in the modern sector to K^{**}. Hence, a reduction in the interest rate leads to a fall in the steady-state level of output despite the presence of dynamics.

This does not mean, however, that balance sheet effects do not play a role. Indeed, on impact, in response to a decline in the interest rate, the value of capital increases from $q^* \cdot K^*$ to $q_{t_0} \cdot K^*$ while entrepreneurial debt payments - which are pre-determined - remain unaffected and equal to $R \cdot B^*$.[31] Therefore:

$$K_{t+1} = \begin{cases} \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot ((1-\lambda) \cdot (A + q^*) + q_t - q^*) \cdot K^* & \text{if } t = t_0 \\ \frac{1}{q_t - \lambda \cdot \frac{A + q_{t+1}}{R}} \cdot \rho \cdot (1-\lambda) \cdot (A + q_t) \cdot K_t & \text{if } t > t_0 \end{cases} . \tag{72}$$

[31] As in Kiyotaki and Moore (1997), these balance sheet effects require that entrepreneurs' debt payments are not indexed to the price of capital.

The evolution of q_t is still given by Equation (69). This means that the adjustment of K to the new steady-state is not monotonic. As the right-hand panel of Figure 9 shows, K_{t+1} rises to $\widehat{K}$ on impact: this, as stated in the figure, is the balance sheet effect. The expansion of the modern sector is short-lived, though, since from that period onwards the economy evolves along the saddle-path towards its new steady state, which features a higher price of capital but a lower capital stock in the modern sector and thus a lower level of output. This decline from $\widehat{K}$ to K^{**} is, as stated in the figure, due to the reallocation effect: the higher demand of capital by the traditional sector keeps capital prices high, and these slowly erode the net worth of modern-sector entrepreneurs. As a result, the dynamic behavior of aggregate output in this economy resembles closely that of the dynamic economy in Section 4, illustrated in Figure 4.

The key takeaway is that the same reallocation forces that we analyzed in our baseline model of Section 2 are also at work in a dynamic environment. Moreover, these forces are persistent in response to a permanent decline in the interest rate, while the balance-sheet effects that are often highlighted in the literature are transitory. To be sure, an unexpected decline in the interest rate does have an initial balance-sheet effect that benefits productive entrepreneurs and reallocates capital towards them, raising average productivity and output. But this effect is by nature temporary: the reason is that it represents a one-time shock to the level of entrepreneurial net worth, but it does not affect the dynamic evolution of net worth thereafter.

C.4 Closed-form solution to the steady state of the dynamic model

In this Appendix, we derive the steady state of the dynamic model in closed form, for the case in which productivity is i.i.d. over time and uniformly distributed on the unit interval.

As Equation (35) in the text shows, in steady state:

$$W_A = \begin{cases} \frac{\Theta}{\Theta+\rho-\frac{1}{1-\lambda}\cdot\left(\frac{A}{q}-\lambda\cdot r\right)}\cdot g\left(A\right)\cdot W & \text{if } A\geq r\cdot q \\ \frac{\Theta}{\Theta+\rho-r}\cdot g\left(A\right)\cdot W & \text{otherwise} \end{cases}, \tag{73}$$

where $g\left(A\right)=1$ in this case because of the uniform distribution. Let $x\equiv A/(r\cdot q)$, then:

$$\frac{W_{r\cdot q\cdot x}}{W} = \begin{cases} \frac{\Theta}{\Theta+\rho-\frac{x-\lambda}{1-\lambda}\cdot r} & \text{if } x\in\left[1,\frac{1}{r\cdot q}\right] \\ \frac{\Theta}{\Theta+\rho-r} & \text{if } x\in[0,1] \end{cases}, \tag{74}$$

with:

$$\int_0^{\frac{1}{r\cdot q}}\frac{W_{r\cdot q\cdot x}}{W}dx = 1\ . \tag{75}$$

From substituting (74) into (75), it follows that:

$$\frac{1}{r\cdot q} = \lambda+\frac{1-\lambda}{r}\cdot\left[\Theta+\rho-\frac{\Theta+\rho-r}{\exp\left\{\frac{1}{\Theta}\cdot\frac{r}{1-\lambda}\cdot\frac{\rho-r}{\Theta+\rho-r}\right\}}\right]. \tag{76}$$

Equation (76) allows us to express price q as a function of model parameters.

Aggregate output is given by:

$$Y = \frac{1}{\int_1^{\frac{1}{r \cdot q}} W_{r\cdot q\cdot x} \cdot dx} \cdot \left[\int_1^{\frac{1}{r\cdot q}} r \cdot q \cdot x \cdot W_{r\cdot q\cdot x} \cdot dx \right] \cdot \bar{K} \ . \tag{77}$$

Thus, we have that:

$$Y = r \cdot q \cdot \frac{\Theta + \rho - r}{\rho - r} \cdot \Theta \cdot \frac{\left[\left(\Theta + \rho + \frac{\lambda}{1-\lambda} \cdot r\right) \ln\left(\Theta + \rho + \frac{\lambda}{1-\lambda} \cdot r - \frac{r}{1-\lambda} \cdot x\right) + \frac{r}{1-\lambda} \cdot x \right] \Big|_1^{\frac{1}{r\cdot q}}}{\left(\frac{r}{1-\lambda}\right)^2} \cdot \bar{K} \ , \tag{78}$$

which together with Equation (76) allow us to express Y as a function of parameters.

C.5 Sensitivity analysis with respect to the path of the interest rate

In this Appendix, we display the quantitative effects from (i) the permanent decline in the interest rate with the instantaneous transition (Figure 10); and (ii) the temporary decline (Figure 11).

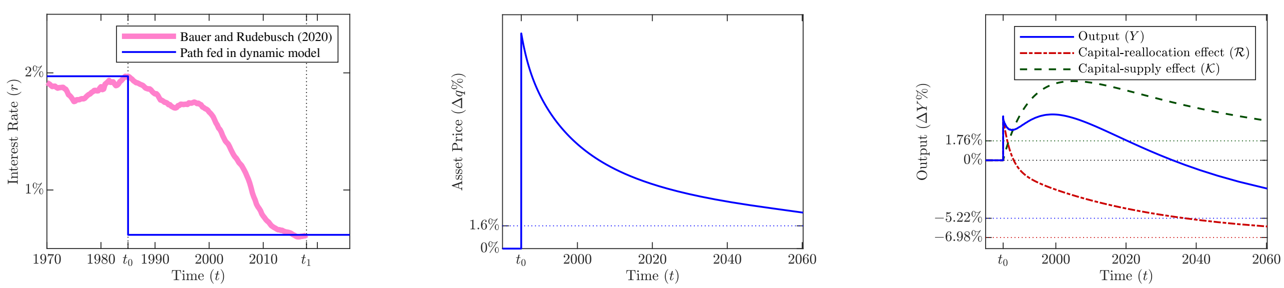


Figure 10: **Permanent decline in the interest rate with the instantaneous transition.**

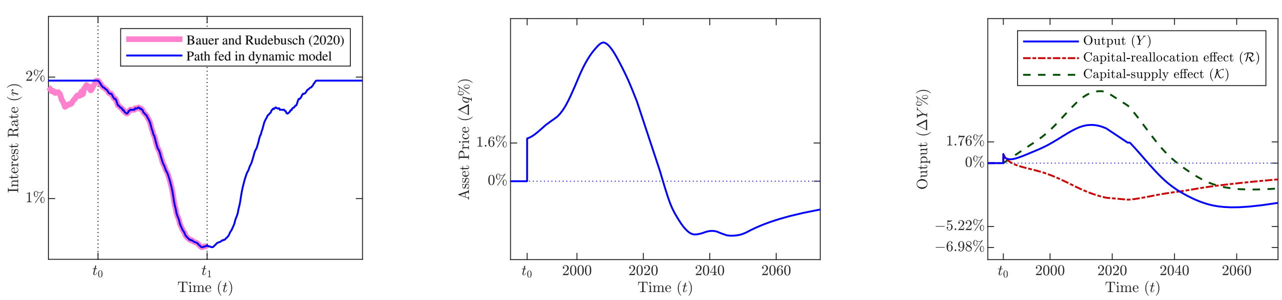


Figure 11: **Temporary decline in the interest rate.**